A trade algorithm for multi-region models subject to spillover externalities

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Abstract

In this paper we present an algorithm that deals with trade interactions within a multi-region model. In contrast to traditional approaches this algorithm is able to handle spillover externalities. We focus on technological spillovers which are due to capital trade. The algorithm of finding a pareto-optimal solution in an intertemporal framework is embedded in a decomposed optimization process. The paper analyzes convergence and equilibrium properties of this algorithm. In the final part of the paper, we apply the algorithm to investigate possible impacts of technological spillovers. While benefits of technological spillovers are significant for the capital-importing region, benefits for the capital-exporting region depend on the type of regional disparities and the resulting specialization and terms-of-trade effects.

JEL classification: C68;O41;F43;F12;C62

keywords: multi-region modeling, technological spillovers, externality, general equilibrium, capital trade

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1 Introduction

In a multi-region setting, investment and trade decisions of each region depend on decisions of each other region. Multi-region modeling becomes a challenging task when different regional interactions are considered. In classical economics and trade theory, prices are the major tool for coordinating regional interactions (cf. Samuelson, 1952; Negishi, 1972). Price-based adjustment algorithms like the standard Walrasian excess demand algorithm can be used for finding equilibrium prices. Early work on algorithms that help to find equilibrium prices numerically was summarized by Scarf (1984). More recently, e.g. Kumar and Shubik (2004) and Luenberger and Maxfield (1995) presented advanced algorithms for the computation of competitive equilibria.

Additional challenges arise from the existence of externalities (Farmer and Lahiri, 2005; Greiner and Semmler, 2002) and the fact that international trade is an inherent dynamic process (Oniki and Uzawa, 1965; Stiglitz, 1970). Our motivation in developing and applying a new method of multi-region modeling is due to the deficits of traditional approaches in dealing with externalities, in particular with technological spillovers. Within this study, the reference point is the Negishi approach - a well-known solution technique for multi-region modeling (Manne and Rutherford, 1994; Leimbach and Toth, 2003). Technically, the Negishi approach merges the regions' optimization problems under a global welfare function. It therefore differs from approaches based on decentralized decision-making. Essentially, it internalizes the coordination function which in decentralized models is played by the market or a virtual auctioneer.

Some well-known models in climate economics, e.g. MERGE (Manne et al., 1995; Manne and Richels, 1995) and RICE (Nordhaus and Yang, 1996), applied the traditional Negishi algorithm in order to find a general equilibrium in an intertemporal optimizing framework. The Negishi approach is numerically quite efficient and in cases without externalities, where the social optimum corresponds to the competitive market equilibrium, often superior to the open loop Nash approach in finding a market solution. However, as Leimbach and Edenhofer (2007) have shown, the Negishi approach is limited in capturing the external effects induced by technological spillovers.

Following the basic idea of Leimbach and Edenhofer (2007), we present an alternative trade algorithm that is able to deal with internalized spillover externalities. We focus on embodied technological spillovers that are induced by capital trade. The presence of physical capital, produced abroad, affects efficiency or productivity levels of domestic firms in a host economy. A host region can boost its productivity by importing capital equipment embodying foreign knowledge. From a theoretical point of view, this paper relates to the literature on economic growth (cf. Barro and Sala-i-Martin, 1995), in particular those taking the impact of trade and externalities into account (e.g. Grossman and Helpman, 1991; Rivera-Batiz and Romer, 1991). In explaining technological change, the theory of endogenous growth (e.g. Romer, 1990) emphasizes the role of knowledge externalities, of investments into R&D, and of innovations that increase the quality or quantity of goods. The concept of embodied spillovers applied here gives rise to a type of technological change which resembles the expansion of product variety, though it generates only transitional growth. While we not account for separated capital stocks, the increase of productivity of the aggregated capital stock assumes that old capital equipment does not become obsolete.

Embodied spillovers represent an external effect. Yet, they have certainly an impact on the investment decisions of the regional agents. This feedback is widely neglected in existing models. This is due to the increasing returns to scale effect which defies control of classical general equilibrium theory. But if empirical studies suggest a link between positive productivity gains and capital trade (e.g. Lee, 1995; Coe et al., 1997; Eaton and Kortum, 2001), why should agents not take this into account in decision-making and why should foresighted agents not be more proactive in attracting capital exports? In contrast to the majority of literature, which deals with spillovers as a purely external effect, we present an approach where technological spillovers are anticipated by the regional actors, hence, influencing the dynamics of the control variables.

The paper is structured as follows: The trade algorithm is elaborated in combination with a stylized multi-region model, the structure of which is presented in section 2. In order to make this model computable, we carry out a decomposition. The decomposed model and the iterative algorithm that searches for a pareto-optimal solution are presented in section 3. We discuss the equilibrium properties of its solution in section 4. Results from numerical experiments on potential welfare and terms-of-trade implications of technological spillovers are presented in section 5. It turns out that at least for the capital-exporting region benefits from spillovers are sensitive to the type of specialisation and that taking technological spillovers into account could substantially change the optimal trade pattern. We end with some conclusions in section 6.

2 The multi-region model

In this section, we present the multi-region model. It represents a dynamic model of international trade. The decomposition that makes the model computable is given in the next section. We decided against a presentation of a more general model that differs from the computational model in order to avoid redundancy later on. The algorithm that solves the model, however, can be applied to other specifications (in particular of the production functions) as well. Analytical elaboration, provided in the Appendix A, is based on a more compact and general exposition of the model. The model can be classified as an optimal economic growth model. A representative agent is assumed to summarize households' consumption decisions and firms' investment and trade decisions.

The following indices are used throughout the presentation:

t	1,2,...,T	time periods,
i, k, l	1,2,...,n	regions,
j		goods,
r	1,2,...,m	iterations.

With $J = \{G, F\}$ and $j \in J$, the following types of traded goods are distinguished:

G	consumption good,
F	investment good.

Each good is produced in a different sector. Hence, j also represents a sectoral index denoted by a superscript throughout the presentation. Although the model and the equations are implemented in a time-discrete framework, we use the continuous formulation throughout the paper for simplicity. Time derivatives are represented as usual. Each variable actually bears the time and iteration index. For transparency reasons, they are suppressed as often as possible.

The objective of the multi-region model is to maximize the welfare U of n regions:

$$\max\{U\} \qquad U = (U_1, U_2, ..., U_i, ..., U_n), \tag{1}$$

with

$$U_i = \int_{t=1}^{T} f[C_i(t)] \cdot e^{-\rho t} dt. \tag{2}$$

These welfare functions measure the utility of each regions' representative household. Utility is expressed as a function *f* of the consumption path *C* subject to discounting by the pure rate of time preference ρ. As an instance of the function *f*, we apply a common logarithmic or Bernoullian utility function

$$f(C_i) = L_i \cdot ln\frac{C_i}{L_i}, \tag{3}$$

where *L* represents the regions' population which provides the exogenously given production factor labor. The production function Y^G for the consumption goods sector is specified as a Cobb-Douglas function :

$$Y_i^G = A_i \cdot [(1-\theta_i) \cdot K_i]^{\alpha_i} \cdot L_i^{1-\alpha_i}, \tag{4}$$

with

$$0 < \theta_i < 1. \tag{5}$$

Variable *A* denotes the productivity level and variable θ denotes the share of total capital stock which is allocated to the investment goods sector. *K* and *L* represent the capital and labor production factors; α is the capital-output elasticity parameter. We assume that labor is used only in the consumption goods sector. This means that there is a fixed endowment of this production factor which, furthermore, is internationally immobile. This prevents the model from coming up with exaggerated specialization.

Investment goods production is assumed to be a function of capital only. Region *i*'s production function of investment good *F* is given by

$$Y_i^F = \kappa_i \cdot (\theta_i \cdot K_i)^{\phi}. \tag{6}$$

When the elasticity parameter ϕ is equal to 1, this equation becomes a Leontief-type production function and parameter κ can be interpreted as a technological coefficient (investment goods output per unit capital stock). [1]

Capital is allocated from a common pool. Thus, perfect cross-sectoral mobility of capital is implicitly assumed (we neglect the vintage and putty-clay structure of the capital stock). Capital accumulation follows the standard equation of capital stock formation:

$$\dot{K}_i = I_i + \sum_{k=1}^{n} X^F_{k,i} - \delta_i \cdot K_i, \tag{7}$$

where I represents domestic investments, while X^F accounts for investment goods imports. The parameter δ represents the depreciation rate of capital. For simplicity reasons, we assume perfect substitutability between domestic and imported capital goods. The same assumption applies to consumption goods. The output of the consumption goods sector represents the regional gross product net of investments. It is used to meet demands on consumption and exports, while being incremented by imports:

$$Y^G_i = C_i + \sum_{k=1}^{n} (X^G_{i,k} - X^G_{k,i}). \tag{8}$$

Variable $X_{i,k}$ denotes the export from region i to region k. It simultaneously denotes the import of region k from region i. Note that the trade variables represent net export and net import values. The use of separate export and import variables is due to the subsequent modeling of technological spillovers, which imply a price differential between capital exports and imports.

The investment goods sector provides domestic investments and meets foreign demands on investment goods by

$$Y^F_i = I_i + \sum_{k=1}^{n} X^F_{i,k}. \tag{9}$$

The range of regional interactions usually modeled is extended by technological spillovers. Technological spillovers increase the host country's productivity

[1] Following the neoclassical assumption of diminishing marginal productivity, we chose a value for ϕ that is slightly lower than 1. With $\phi < 1$, this production function exhibits decreasing returns to scale.

through capital goods imports. Within the model, an additional change of the total factor productivity *A* in a region i is a function of capital goods exports X^F from region *k* to region *i* and of productivity differences between both regions:

$$\dot{A}_i = \sum_{k=1}^{n} \left[\left(\frac{X^F_{k,i}}{K_i} \right)^{\zeta} \cdot \beta \cdot \max(0, A_k - A_i) \right]. \tag{10}$$

Parameter β represents the spillover intensity, i.e. the actual impact of technological spillovers in the host region, while ζ ($0 < \zeta < 1$) depicts the elasticity of productivity changes on capital goods imports. The capital import variable is divided by the capital stock in order to avoid scaling effects. By choosing the control variable X^F, regions influence the extent of technological spillovers, i.e. technological spillovers are subject of rational expectations.

An intertemporal budget constraint *D* based on world market prices p^j has to be met by each region. It is given by

$$D_i(T) = \int_{t=1}^{T} B_i(t)dt = 0, \tag{11}$$

where the budget balance *B* is defined as

$$B_i = \sum_{j \in J} \left(p^j \cdot \sum_{k=1}^{n} [X^j_{i,k} - X^j_{k,i}] \right). \tag{12}$$

This equation serves to level off the trade deficits of each region in the long run. The model is completed by initial conditions:

$$K_i(0) = k_i \tag{13}$$

$$A_i(0) = a_i \tag{14}$$

and non-negativity conditions:

$$C_i, A_i, I_i, K_i, Y^j_i, X^j_{i,k}, X^j_{k,i} \geq 0. \tag{15}$$

3 Decomposed model

Because each region is represented by a distinct and separate utility function, the multi-region model is not operable offhand. Forming a global welfare function is a possible next step towards a solution. The Negishi model represents such a global welfare maximization problem. Negishi (1972) proved the correspondence between the international competitive equilibrium and a welfare optimum [2]

$$\max W(U(C_i)) = \sum_{i=1}^{n} \omega_i \cdot U_i(C_i) \tag{16}$$

for a particular set of strictly positive welfare weights ω_i with $\sum \omega_i = 1$ under usual convexity assumptions. Negishi presented a mapping to derive ω_i. This mapping has a fixed point.

However, the Negishi model fails when technological spillovers have to be considered. Leimbach and Edenhofer (2007) have shown that the Negishi algorithm can not consider spillover effects because it is not able to distinguish between export and import prices and quantities. To overcome this problem, we developed an alternative approach to multi-region modeling based on a decomposition of the original optimization problem into single regional optimization modules and a trade module. While the present decomposition resembles that of Leimbach and Edenhofer (2007), the resulting trade module is a completely different one.

3.1 Region module

In each region module, welfare of the considered region only is maximized:

$$\max_{\theta_i} U_i. \tag{17}$$

This decentralized optimization problem is subject to the constraints (2)-(10) and (13)-(15) from the multi-region model in the previous section. Note that the intertemporal budget constraint is no longer part of the optimization problem. Instead, each region is restricted by import and export bounds $\bar{X}$. These bounds are primarily algorithmic devices generated by the trade module (see below). However, they can be conceived as the net import demand and export offer from other regions that the optimizing region expects to face. Each region can only import what the other regions offer to export to this region:

[2] Negishi (1972) characterized the solution as a maximum point of a social welfare function which is a linear combination of utility functions of individual consumers, with the weights in the combination in inverse proportion to the marginal utilities of income.

$$X^j_{k,i,r} = \bar{X}^j_{k,i,r-1}. \tag{18}$$

Analogously, each region has to meet imports that the other regions demand, yielding the export constraint

$$X^j_{i,k,r} = \bar{X}^j_{i,k,r-1}. \tag{19}$$

Despite the fact that $\bar{X}$ represents the right hand side of an equation, we keep to refer to it as a bound.

3.2 Trade module

The purpose of the trade module is to determine the trade flow boundaries $\bar{X}$. To this end, the trade module is formulated as a single multi-region model. Equally to the Negishi approach, a global objective function combines the welfare functions of all regions by means of welfare weights:

$$\max_{\theta_i, \bar{X}^j_{i,k}} W = \sum_{i=1}^{n} w_i \cdot U_i. \tag{20}$$

The welfare function is optimized with respect to the factor allocation θ and the export/import quantities $\bar{X}$. The optimization problem includes constraints (2)-(10) and (13)-(15) for each region with the only difference that the trade flow variables X are replaced by $\bar{X}$. In order to avoid artificial investment goods exports in anticipation of spillover gains from re-exports, an additional constraint states that each region can only be a net exporter or importer of investment goods:

$$\int_{t=1}^{T} \sum_{k=1}^{n} \bar{X}^F_{i,k}(t)dt \cdot \int_{t=1}^{T} \sum_{k=1}^{n} \bar{X}^F_{k,i}(t)dt = 0. \tag{21}$$

Indeterminacy could cause simultaneous goods export and import in a single region. This is prevented by the constraint

$$\sum_{k=1}^{n} \bar{X}^G_{i,k}(t) \cdot \sum_{k=1}^{n} \bar{X}^G_{k,i}(t) = 0. \tag{22}$$

The trade module represents the problem as a Social Planner problem, generating a solution that assumes the decentralized actors to behave socially optimal.

3.3 Iterative trade algorithm

The decomposed model is solved iteratively by the following steps:

1. Fixing the trade structure.
2. Solve region modules (decentralized model).
3. Extract prices for intertemporal budget constraints.
4. Adjust welfare weights.
5. Solve trade module (Social Planner model) and derive export/import bounds.

In each iteration, the welfare weights of the objective function (20) are updated based on the deviations of the intertemporal trade balances (intertemporal budget constraints) by

$$w_{i,r+1} = w_{i,r} \cdot (1 + h(D_{i,r})). \tag{23}$$

The particular implementation of the function

$$h(D_{i,r}) = \gamma \left[\frac{(ln(r) + 2)}{\sum_{k=1}^{n} V_k + V_i} \right] \cdot D_{i,r}(T) \tag{24}$$

follows Leimbach and Toth (2003, p.163), where

$$V_i = \int_{t=1}^{T} \left[p^G(t) \cdot C_i(t) + \sum_{k=1}^{n} p^j(t) \cdot (X_{i,k}^j(t) - X_{k,i}^j(t)) \right] dt. \tag{25}$$

Weighting factor *V* can be interpreted as the economic power of each region; γ is a parameter that facilitates the convergence process. The iterative procedure of adjusting the welfare weights assures that the intertemporal budget constraints are met when the algorithm converges.

Most crucially, computing *B* and *V* in Eq. (24) demands for world market prices p^j. These prices are computed as weighted averages of the regional import prices *pi* and export prices *pe*:

$$p^j = \left[\frac{\sum_{i=1}^{n} \sum_{k=1}^{n} (pi_{i,k}^j \cdot X_{i,k}^j + pe_{i,k}^j \cdot X_{i,k}^j)}{\sum_{i=1}^{n} \sum_{k=1}^{n} X_{i,k}^j} \right] /2 \,, \qquad i \neq k. \tag{26}$$

Both prices *pi* and *pe* represent shadow prices of constraints (18) and (19), which can be described in the form of partial derivatives

$$pi^j_{k,i} = \frac{\partial U^*_i}{\partial \bar{X}^j_{k,i}}, \tag{27}$$

$$pe^j_{i,k} = -\frac{\partial U^*_i}{\partial \bar{X}^j_{i,k}}, \tag{28}$$

with U^* as maximum welfare in iteration *r*. Import and export prices are specific for each time period and possibly differ from each other. The following relations hold:

- in the non-spillover case (in general)

$$pe^j_{i,k} \neq pi^j_{i,k} \ , \quad i \neq k$$

$$pe^j_{i,k} \neq pe^j_{k,i} \ , \quad i \neq k$$

$$pi^j_{i,k} \neq pi^j_{k,i} \ , \quad i \neq k$$

$$pe^j_{i,k} = pi^j_{k,i} \ , \quad i \neq k$$

$$\forall i: \quad pe^j_{i,l} = pe^j_{i,k} \ , \quad l \neq k$$

$$\forall i: \quad pi^j_{l,i} = pi^j_{k,i} \ , \quad l \neq k$$

- in deviation from above for the spillover case (in general)

$$\exists i,k: \quad pe^F_{i,k} \neq pi^F_{k,i} \ , \quad i \neq k$$

$$\exists i: \quad pi^F_{l,i} \neq pi^F_{k,i} \ , \quad l \neq k.$$

In the presence of spillovers, export prices of investment goods do not, in general, correspond to import prices, and the latter may also differ depending on the region from which the capital good is imported. While regional prices are not necessarily expected to converge, convergence, however, can be monitored for the willingness to pay and the willingness to accept (see next section for a detailed justification).

The algorithm proposed in this paper couples the trade module and the regional modules to compute a pareto-optimal solution. The trade module can be conceived as a virtual coordinator that has to compute an optimal allocation of all traded goods. Whereas the Walrasian auctioneer monitors excess of demand over supply (or vice versa) and adjusts prices based on this information, here the coordinator knows the willingness of the trading partners to pay for another unit of import or to accept another unit of export. Based on this, the allocation of exports and imports is adjusted. For the single region, this allocation appears as the foreign demand for and supply of trading goods.

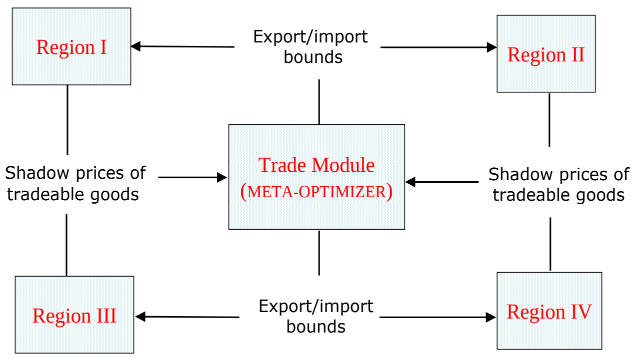

Figure 1: Data flow between modules

Capturing interactions between the regions by means of the virtual coordinator is an iterative process. Within each iteration, first the region modules, confronted with new export/import bounds, are solved. The region modules provide shadow prices for export and import goods from each region, which are used by the trade module to update welfare weights and export/import bounds. Figure 1 shows the data flow between the modules. This iterative adjustment process ends (converging to the fixed point of the decomposed model) when the trade structure does not change anymore, i.e. for all export quantities and prices it holds that

$$\forall j : |\bar{X}_r^j - \bar{X}_{r-1}^j| \leq \epsilon, |p_r^j - p_{r-1}^j| \leq \epsilon. \tag{29}$$

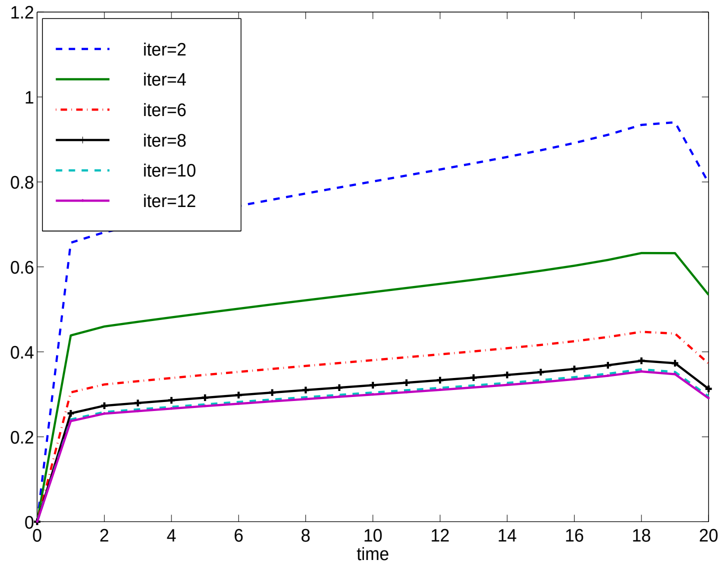

Figure 2: Convergence of consumption good export bound

Furthermore, convergence is given by

$$\forall i : |w_{i,r} - w_{i,r-1}| \leq \epsilon \tag{30}$$

and

$$\forall i : |\bar{C}_i - C_i| \leq \epsilon, |\bar{\theta}_i - \theta_i| \leq \epsilon, \tag{31}$$

where $\bar{C}, \bar{\theta}$ denote consumption and capital allocation computed by the trade module, while the analogue quantities C, θ are computed in the regional modules.

It should be noted that the present algorithm operates in an intertemporal model setting which includes transitional dynamics. Figure 2 demonstrates the convergence process for the time trajectory of the consumption good export variable. It results from a two-region setting as analyzed within the model experiments presented in section 5.

4 Equilibrium solution

While the trade algorithm is developed to search for a pareto-optimal solution in the presence of technological spillovers, as a first benchmark we consider the non-spillover case. In Appendix A, we provide a concise analytical deduction of the equilibrium properties of the trade algorithm and show that it computes an international competitive equilibrium in the non-spillover case, if it converges[3]. Therefor we formulate the basic model, the optimal solution of which represents a competitive equilibrium, consisting of Eq. (17) s.t. (3-15), (21), (22). In this model, the intertemporal budget constraint has to be considered explicitly. This provides new costate variables π_i which appear to evolve according to $\pi_i(t) = \pi_i(0)e^{\rho t}$. In the basic model, capital allocation and trade is chosen such that

$$\pi_i p^G = \frac{1}{C_i} \tag{32}$$

and

$$\pi_i p^F = \hat{\lambda}_i, \tag{33}$$

where $\hat{\lambda}_i$ denotes the shadow prices of capital in the basic model. This means that prices equal marginal utility, corrected by a factor representing the impact of the budget constraint. When selecting $\pi_i(0) = w_i^{-1}$, it can be shown (see Appendix A) that this implies the same values for the control variables of the basic model as the trade algorithm computes. The last equality is an intertemporal analogue to the Negishi approach, where the inverse of Negishi weights is equal to the marginal utilities of income.

We call $\pi_i pi^j_{k,i}$ the importers' i willingness to pay (WTP), and $\pi_k pe^j_{k,i}$ the exporters' k willingness to accept (WTA) good j. The equilibrium solution includes convergence of the importers WTP and convergence of the exporters WTA, giving the notion of world market prices a clear meaning. Figure 3 shows an example of convergence. Results are based on numerical experiments with a two-region setting as analyzed in the next section.

However, the conclusion on achieving an equilibrium solution is only reliable for the non-spillover case. Spillover externalities (Eq. 10) of the analyzed type generate increasing returns to scale associated with non-convexities and thus causing multiple local optima. In particular, the asymmetry of spillover effects, which occurs if a region imports investment goods from a region with higher level of productivity, prevents convergence of the willingness to accept and the willingness to pay.

To represent this formally, spillover effects lead to an additional shadow price μ_i, which sums future utility gains of improved productivity due to imported capital

[3]A proof of convergence is beyond the scope of this paper.

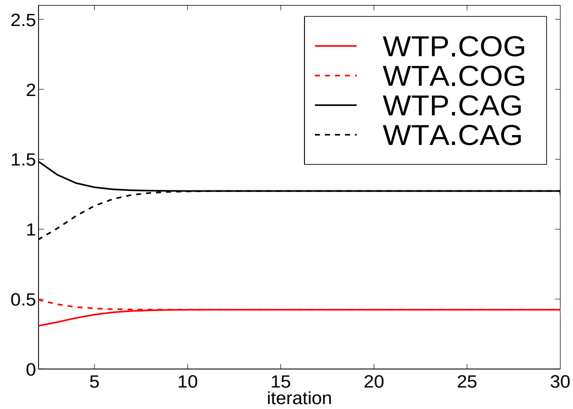

Figure 3: Convergence of the willingness to pay (WTP) and the willingness to accept (WTA) for a selected time period (COG: consumption good, CAG: capital good)

goods. Then, the resulting prices are

$$pe_{i,k}^{F} = \lambda_i \tag{34}$$

and

$$pi_{k,i}^{F} = \lambda_i + \mu_i \frac{\partial \dot{A}_i}{\partial X_{k,i}^{F}}, \tag{35}$$

where $\mu_i > 0$ if region i imports from region k with a higher productivity level. $\dot{A}_i$ represents the productivity change (cf. Eq. 10). As long as the productivity gradient remains and $\mu_i > 0$, the utility gains from import (and therefore the WTP) are always higher than from exports (and therefore the WTA). If the world market price is between the WTA and the WTP, we come up with a paradox situation where the region has to pay less than it is willing to pay for demands, but gets more than wanted for sales. It would thus aim at selling all capital it has, and at the same time imports all capital that is available on the market. If there is a second region in this situation, it is unclear how a market mechanism can resolve the conflict of allocating all available capital. It is, moreover, impossible to speak of an equilibrium price if regions want to sell and buy for different prices. In our model, this problem is avoided by constraint (21) that requires each region to choose between being an exporter or an importer.

When the externality is internalized within the decision-making process of the decomposed model (Social Planner mode), the equivalence between the solution of the decentralized basic model and the Social Planner solution disappears. The decentralized agents do not behave socially optimal. A region which imports capital and receives the technological spillover under the trade algorithm, may use the high price for capital goods to export capital goods by its own and forgoing the technological spillovers in the basic model solution. This could increase its welfare, but it implies a trade structure that is certainly not optimal from the other regions' point of view.

As suggested by economic theory, a tax or subsidy is needed to achieve the social pareto-optimum in a decentralized model. We implement this, first, by defining a market price $\tilde{p}^F$ as the weighted average of the export prices pe^F only (compare Eq. 26):

$$\tilde{p}^F = \frac{\sum_{i=1}^n \sum_{k=1}^n pe_{i,k}^F \cdot X_{i,k}^F}{\sum_{i=1}^n \sum_{k=1}^n X_{i,k}^F}\ , \qquad i \neq k. \tag{36}$$

In addition, we define a subsidy $\sigma_{i,k}$ that represents a mark-up on the price of capital goods exported by the technologically advanced regions. For the importing regions, this represents a tax. The intertemporal budget constraint that the decentralized agents have to meet changes to

$$\int_{t=1}^{T} \left[p^G \cdot \sum_{k=1}^{n} (X_{i,k}^G - X_{k,i}^G) + (\tilde{p}^F + \sigma_{i,k}) \cdot \sum_{k=1}^{n} (X_{i,k}^F - X_{k,i}^F) \right] dt = 0. \tag{37}$$

It is beyond the scope of this paper to find a general closed form expression for the optimal subsidy. For a two-region setting (for which we made numerical experiments), however, we can provide

$$\sigma_{i,k} = \begin{cases} p^F - \tilde{p}^F & : \quad A_i < A_k \\ 0 & : \quad A_i \geq A_k. \end{cases} \tag{38}$$

as an optimal subsidy.

To verify whether the approximated fixed point indeed represents a pareto-optimum as well as market equilibrium, a numerical test can be performed. After the trade algorithm has finished, for each region the basic model is solved numerically with the prices p^j in the intertemporal budget constraint as given by the trade algorithm. Then it is compared whether the optimal import and export quantities reproduce the bounds of the trade algorithm. With the subsidy/tax approach the spillover experiments pass this numerical test.

5 Model experiments

In this section, we apply the model introduced in section 3 and the developed trade algorithm in order to analyze the impact of technological spillovers on economic growth and the trade structure. We run the model within a stylized conventional 2 goods x 2 factors x 2 regions setting[4]. All results can be interpreted in a qualitative sense only.

Technological spillovers are subject of rational expectations. The focus is on spillovers that increase the productivity in the consumption goods sector. First, we analyze a setting where both regions differ in their productivity level A only.

From an economic point of view, the question arises whether there are welfare gains for both regions and what changes result in the trade structure. Figure 4 illustrates some of the impacts [5]. For presentation reasons, the two regions are labeled by IR and DR in the Figures. Under a traditional free trade scenario, region IR, which has higher productivity in the production of the consumption good ($a_{IR} = 2.0$, $a_{DR} = 1.2$), will export the consumption good and extend its production by capital good imports. This picture changes when technological spillovers are taken into account. According to the optimal trade structure, region IR exports the investment good and region DR exports the consumption good. The dynamics of the current account is also reversed. Whereas DR starts with a trade surplus in the non-spillover case, a trade balance deficit comes with the optimal solution within the spillover scenario. The consumption path in Figure 4 indicates gains in consumption for both regions.

Consumption gains in both regions increase with time. There are yet significant differences in the patterns of gains. DR gains in all periods. IR, in contrast, loses in initial periods. Consumption and welfare gains of DR are directly linked to productivity increases caused by technological spillovers. Positive feedbacks to

[4]All modules are programmed in GAMS (www.gams.com) and numerically solved with the non-linear programming solver CONOPT3. The programs are available from the authors upon request. For the default parameters and initial values see Appendix B.

[5]The current account is defined as intertemporal trade balance $D_i(t)$ based on Eq. (11).

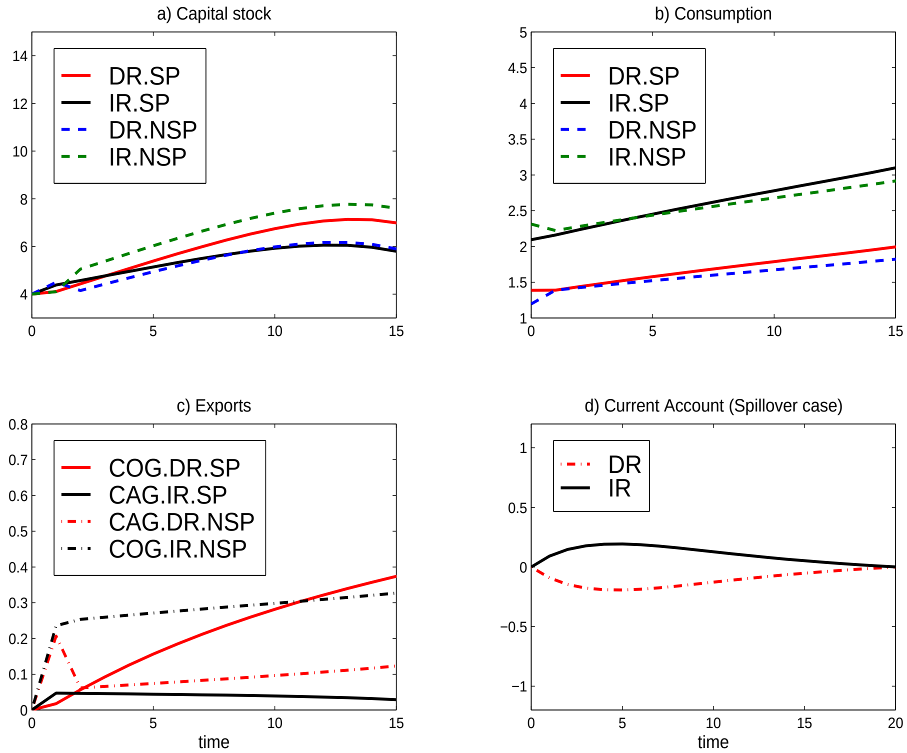

Figure 4: Impact of technological spillovers (SP: spillover case; NSP: non-spillover case; COG: consumption goods; CAG: capital goods; IR,DR: regions)

IR, while on a moderate level only, are mainly due to higher prices of investment goods. Figure 5 demonstrates the price differences between the spillover case and the non-spillover case. Differences exist for the investment goods only. Higher prices of investment goods occur due to the anticipation of the spillover effect and the willingness of DR to pay a higher price. In order to meet the intertemporal trade balance, DR compensates the expansion of investment goods imports (compared to the non-spillover case) increasingly by consumption goods exports.

Due to the discounting effect, consumption gains of region IR do not manifest in an equal increase in welfare. While IR increases its welfare in the spillover case compared to the autarky case by 0.5%, it loses compared to the non-spillover case by 0.6% (DR increases welfare by 17.0%). IR faces weakened terms of trade. As IR is exporter of consumption goods in the non-spillover case (see Figure 4), it first suffers from a decrease of relative prices of consumption goods. Becoming

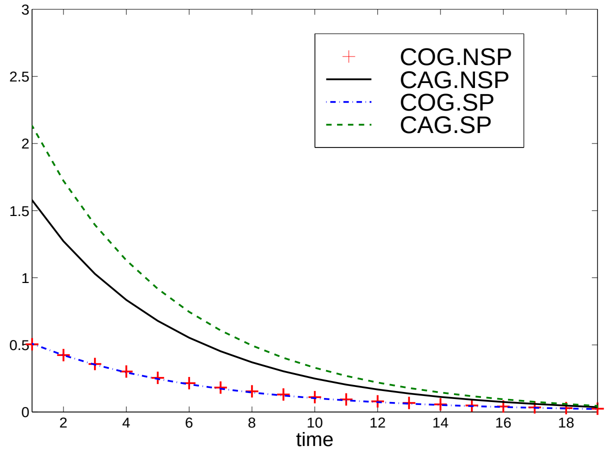

Figure 5: Price levels of tradeable goods (SP: spillover case, NSP: non-spillover case, COG: consumption goods, CAG: capital goods).

an exporter of investment goods and benefiting from the price increase can not completely compensate this terms-of-trade effect.

However, the level of positive feedbacks from the spillover effects on the capital-exporting region heavily depends on the existing trade structure and diversity of regional characteristics. Within alternative scenarios where IR is not only distinguished by higher productivity in the consumption goods sector but also by either

- higher productivity in the investment good sector ,
- higher capital intensity,
- and lower pure rate of time preference,

IR increases its welfare compared to the non-spillover scenario (e.g. in the scenario with higher capital intensity by 0.7%). Higher capital intensity is implemented as a reduction of exogenous labor supply to 60%. Higher productivity is implemented by increasing parameter κ_{IR} from 0.16 to 0.18. For these scenarios, the change of consumption gains is demonstrated in Figure 6. In general, benefits occur for all cases where IR is an exporter of investment goods already in the non-spillover scenario. While this is accompanied by a higher growth rate of percentage consumption gains, it does not change the impact pattern of technological spillovers on consumption.

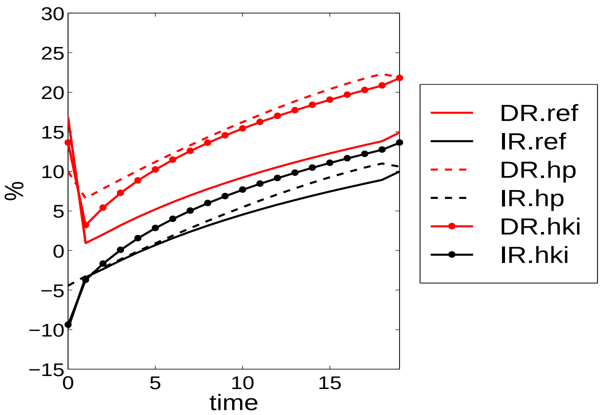

Figure 6: Per capita consumption gains from technological spillovers (ref - Spillover reference scenario, hp - higher productivity of IR in investment goods sector , hki - higher capital intensity in IR)

Besides the regional differences, the level of gains mainly depends on the specification of parameter β (spillover intensity). There is no empirical foundation for β so far. We carried out sensitivity analyses with respect to the external effect. The spillover coefficient β was varied within the interval [0, 1.6]. Smooth changes of welfare and per capita consumption over a wide range (see Figure 7 and Figure 8) demonstrate robustness of the approach again .

6 Conclusions

We presented an alternative approach to multi-region modeling based on a novel trade algorithm. This approach is applicable in an intertemporal optimization framework. On the one hand, it shares some basic features with the approach of Leimbach and Edenhofer (2007), on the other hand, it includes a completely different trade module. Due to its similarity with the Negishi approach, this new approach is more transparent and convincing in the way it derives a pareto-optimal solution. This is further supported by an in-depth analysis of the analytical properties of the trade algorithm.

The algorithm presented in this paper is distinguished by its ability to deal with spillover externalities numerically. It is applicable to other types of externalities

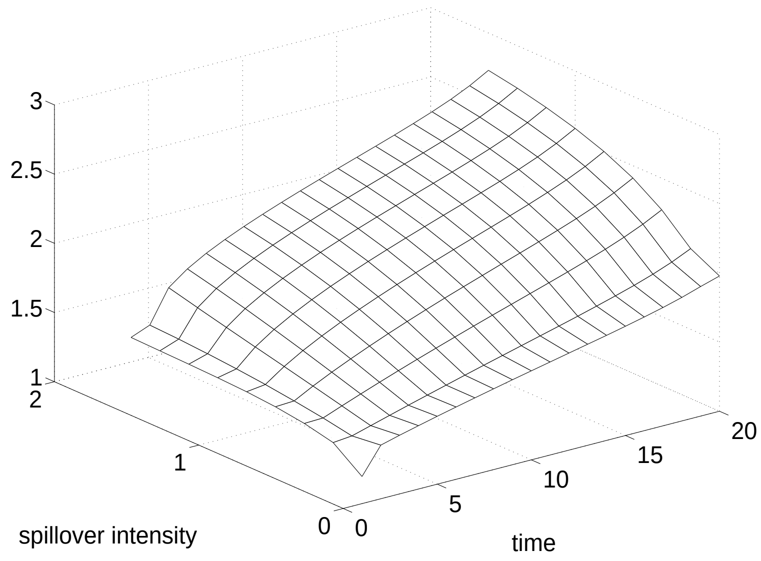

Figure 7: Per capita consumption sensitivity on spillover intensity in DR

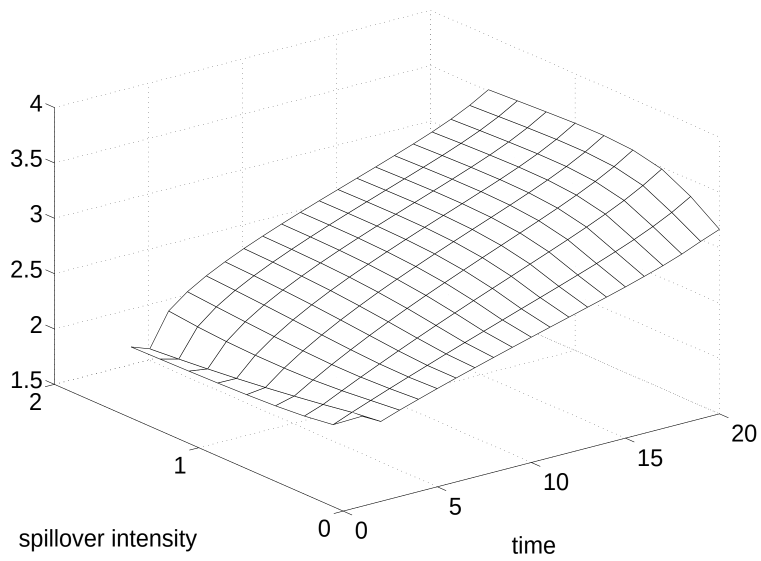

Figure 8: Per capita consumption sensitivity on spillover intensity in IR

as well. First experiences exist in analyzing external effects from greenhouse gas emissions. Non-convexities induced by the external effects may prevent the algorithm from finding the global optimum. Nevertheless, the convergence process in finding the local optimum was demonstrated to be quite robust. Further research is needed to proof general validity of the algorithm for cases with externalities.

Numerical experiments show that in the presence of technological spillovers, the optimal trade structure may reverse. The most significant spillover effect is the primary productivity-increasing effect for the capital importing region. Secondary price and terms of trade effects will affect the capital-exporting region as well. Benefits for that region are the higher the more it would export investment goods already in the absence of technological spillovers. This would guarantee that this region benefits completely from higher prices of the investment goods that occur in the presence of technological spillovers. Higher capital stock per capita, higher productivity in the investment goods sector and a lower pure rate of time preference increase the benefits of the spillover effect for the capital-exporting region.

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A Analytical properties of the trade algorithm

It is shown in this appendix that the trade algorithm, if it converges in the non-spillover case, produces a competitive equilibrium which is given by the optimal solution of the basic model. The composition of the basic model is defined in section 4. The strategy is as follows. At first we derive some analytical features of the regional module (for given import- and export bounds) and of the trade module (for given welfare weights), assuming an interior solution. When the algorithm converges, both modules compute coherent results. We show that these results respect the usual necessary conditions for an optimal solution of the basic model. For clarity, all variables of the trade module are indicated by a bar $\bar{\cdot}$, those of the basic model by a hat $\hat{\cdot}$, while variables of the regional module have no particular accents.

A.1 Regional module for the case without spillovers

We first derive properties of the optimal paths of the regional modules where import and export bounds $\bar{X}_i^j$ are prescribed by the trade module. Therefore, the bounds are parameters in the optimization problems of the regional modules. The derivative of optimal utility with respect to these parameters is used by the trade module for the next iteration (cf. Eqs. 27,28). Since we concentrate on interior solutions in the sense that $I_i > 0$, the current-value Hamiltonian L_i for the regional module reads

$$L_i = \ln(C_i) + \lambda_i(Y_i^F - \Delta\bar{X}_i^F - \delta_i K_i), \tag{39}$$

where, for convenience, net exports of region i are denoted by

$$\Delta\bar{X}_i^j = \sum_k (\bar{X}_{i,k}^j - \bar{X}_{k,i}^j). \tag{40}$$

Note that for the condition $C_i \geq 0$ no Kuhn-Tucker parameter is needed due to the logarithmic form of the utility function. The costate variable λ_i represents the shadow price for capital. The associated equation of motion for the shadow price is

$$\dot{\lambda}_i = \lambda_i(\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{1}{C_i}\frac{\partial Y_i^G}{\partial K_i}, \tag{41}$$

together with the transversality condition $\lambda(T) = 0$.

By the envelope theorem, the optimal utility U_i^* resulting from an optimal allocation path θ_i depends on the parameters $\bar{X}_{i,k}$ by $\partial U_i^*/\partial \bar{X}_{i,k} = \partial L_i^*/\partial \bar{X}_{i,k}$, if the latter is evaluated on the optimal production path. Therefore,

$$pe_{i,k}^G = -\frac{\partial U_i^*}{\partial \bar{X}_{i,k}^G} = -\frac{1}{C_i}, \tag{42}$$

$$pe_{i,k}^F = -\frac{\partial U_i^*}{\partial \bar{X}_{i,k}^F} = -\lambda_i. \tag{43}$$

A.2 Trade module

For the trade module, the Hamiltonian is expanded to

$$L = \sum_i w_i \ln(\bar{C}_i) + \sum_i \bar{\lambda}_i (Y_i^F - \Delta \bar{X}_i^F - \delta_i \bar{K}_i). \tag{44}$$

Based on the costate equation

$$\dot{\bar{\lambda}}_i = \bar{\lambda}_i (\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{w_i}{\bar{C}_i} \frac{\partial Y_i^G}{\partial K_i}, \tag{45}$$

with the transversality condition $\bar{\lambda}_i(T) = 0$, the trade bounds are computed from $\max_{\bar{X}_{k,i}^j, \bar{X}_{i,k}^j, \bar{\theta}_i} L$. Hence, the optimal allocation $\bar{\theta}_i$ is determined from

$$-\frac{\partial Y_i^G}{\partial \theta_i} \frac{w_i}{\bar{C}_i} = \bar{\lambda}_i \frac{\partial Y_i^F}{\partial \theta_i}. \tag{46}$$

The derivatives of the Hamiltonian with respect to export and import bounds are

$$\frac{\partial L}{\partial \bar{X}_{i,k}^G} = -\frac{\partial L}{\partial \bar{X}_{k,i}^G} = \frac{w_k}{\bar{C}_k} - \frac{w_i}{\bar{C}_i}, \tag{47}$$

and

$$\frac{\partial L}{\partial \bar{X}_{i,k}^F} = -\frac{\partial L}{\partial \bar{X}_{k,i}^F} = \bar{\lambda}_k - \bar{\lambda}_i. \tag{48}$$

Setting Eq. (47) to zero describes an equation system for all net exports of the consumption good since $\bar{C}_i = Y_i^G - \Delta \bar{X}_i^G$ by definition. From its solution, import and export quantities are uniquely determined by the constraint (22). We can also derive the equation

$$\forall i, k = 1, \ldots, n : \frac{\bar{C}_i}{\bar{C}_k} = \frac{w_i}{w_k}. \tag{49}$$

For the trade of investment goods the situation is more complicated, since Eq. (48) does not depend directly on $\bar{X}^F_{i,k}$. In the steady-state solution capital changes $\dot{\bar{K}}_i$ are chosen such that

$$\forall i, k = 1, \ldots, n : \bar{\lambda}_i \equiv \bar{\lambda}_k. \tag{50}$$

In the following, we concentrate on that case, justifying to introduce the variable $\bar{\lambda} = \bar{\lambda}_i = \bar{\lambda}_k$ for the solution of the costate equation.

We now put together the results for the regional and trade module in the fixed point of the algorithm. There, $\theta_i = \bar{\theta}_i, C_i = \bar{C}_i$, and the intertemporal budget is balanced. We now prove the important equation

$$\forall i = 1, \ldots, n : \bar{\lambda} \equiv w_i \lambda_i. \tag{51}$$

Define $\bar{\lambda} = w_i \lambda_i$. Then, by Eq. (41)

$$\dot{\bar{\lambda}}_i = w_i \dot{\lambda}_i = w_i \lambda_i (\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{w_i}{C_i} \frac{\partial Y_i^G}{\partial K_i}, \tag{52}$$

which in the fixed point equals (by definition of $\bar{\lambda}$)

$$\bar{\lambda}(\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{w_i}{\bar{C}_i} \frac{\partial Y_i^G}{\partial K_i}, \tag{53}$$

which obviously solves Eq. (45). Trivially, $\bar{\lambda}(T) = w_i \lambda_i(T) = 0$. Therefore, $\bar{\lambda}$ is indeed the shadow price computed by the trade module. This shows that the shadow prices for capital in the regions only differ by a constant coefficient which is equivalent to their welfare weights.

A.3 Basic model

We will now show that the trade algorithm computes an international competitive equilibrium. This is the case when the optimal trade flows of the basic model reproduce the results of the trade algorithm. We demonstrate this by validating that the dynamics induced by the solution of the algorithm, namely

$$\hat{\theta}_i = \theta_i, \quad \hat{X}_i^j = \bar{X}_i^j, \tag{54}$$

and consequently $\hat{C}_i = C_i$, is consistent with the necessary optimality conditions of the basic model. Note that the Pontryagin maximum principle states that the evolution of state and costate variables, brought about by an optimal control path, defines an optimization problem for every time step, which is solved by the control variables. We verify this for the control variables computed by the trade module. Again, we concentrate on interior solutions.

The Hamiltonian of the basic model is

$$L_i = \ln(\hat{C}_i) + \hat{\lambda}_i(Y_i^F - \Delta\hat{X}_i^F - \delta\hat{K}_i) + \hat{\pi}_i(p^F\Delta\hat{X}_i^F + p^G\Delta\hat{X}_i^G). \tag{55}$$

The new costate variable $\hat{\pi}_i$ corresponds to the budget balance equation (cf. Eq. 11)

$$\dot{\hat{D}}_i = p^F(t)\Delta\hat{X}_i^F(t) + p^G(t)\Delta\hat{X}_k^G(t), \tag{56}$$

and $\hat{D}_i(T) = 0$. Note that the latter is always satisfied if quantities and prices are determined from the convergent trade module. We assume that world market prices are determined by $p^F(t) = e^{-\rho t}\frac{1}{n}\sum_k w_k\lambda_k(t)$ and $p^G(t) = e^{-\rho t}\frac{1}{n}\sum_k \frac{w_k}{C_k}$ in the fixed point of the trade algorithm. The associated costate equation is solved by

$$\hat{\pi}_i(t) = \hat{\pi}_i(0)e^{\rho t}, \tag{57}$$

where $\hat{\pi}_i(0)$ has to be chosen appropriately. The costate equation for $\hat{\lambda}_i$ evolves according to

$$\dot{\hat{\lambda}}_i = \hat{\lambda}_i(\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{1}{\hat{C}_i}\frac{\partial Y_i^G}{\partial K_i}, \tag{58}$$

and $\hat{\lambda}_i(T) = 0$, which is identical to the costate equation for the regional module (see Eq. 41). Note that, due to convergence, also $\hat{C}_i = C_i$, $\hat{\theta}_i = \theta_i$ (cf. Eq. 31) and $\hat{K}_i = K_i$, such that we conclude

$$\hat{\lambda}_i \equiv \lambda_i. \tag{59}$$

So far we have only determined how the state and costate equations of the basic model evolve if the control variables are chosen as in the result of the trade algorithm. Now it has to be verified that the control variables also maximize the Hamiltonian of the basic model. We claim that choosing

$$\hat{\pi}_i(0) = w_i^{-1} \tag{60}$$

is appropriate. This is an intertemporal analogue to the Negishi approach, where the inverse of Negishi weights is equal to the marginal utilities of income.

By differentiating the Hamiltonian with respect to the control variable $\hat{\theta}_i$, we obtain the first order condition

$$0 = \hat{\lambda}_i \frac{\partial Y_i^F}{\partial \theta_i} - \frac{\partial Y_i^G}{\partial \theta_i} \frac{1}{\hat{C}_i} = \lambda_i \frac{\partial Y_i^F}{\partial \theta_i} - \frac{\partial Y_i^G}{\partial \theta_i} \frac{1}{C_i}, \tag{61}$$

where the second equation is due to Eq. (59) and $\hat{C}_i = C_i$. This equation is obviously the same as Eq. (46), such that $\hat{\theta}_i$ indeed maximizes L_i. For import and export quantities, the first order conditions are

$$\frac{\partial L}{\partial \hat{X}_{i,k}^G} = -\frac{\partial L}{\partial \hat{X}_{k,i}^G} = \hat{\pi}_i p^G - \frac{1}{\hat{C}_i} = 0, \tag{62}$$

and

$$\frac{\partial L}{\partial \hat{X}_{i,k}^F} = -\frac{\partial L}{\partial \hat{X}_{k,i}^F} = \hat{\pi}_i p^F - \hat{\lambda}_i = 0. \tag{63}$$

Due to Eq. (49),

$$\hat{\pi}_i p^F - \hat{\lambda}_i = \hat{\pi}_i e^{-\rho t} \frac{1}{n} \sum_k \frac{w_k}{C_k} - \frac{1}{C_i} = \hat{\pi}_i(0) \frac{w_i}{C_i} - \frac{1}{C_i}. \tag{64}$$

Since $\hat{\pi}_i(0) w_i = 1$, the first order condition for trade with the consumption good is satisfied. By using Eq. (59) we also obtain

$$\hat{\pi}_i p^F - \hat{\lambda}_i = \hat{\pi}_i e^{-\rho t} \frac{1}{n} \sum_j w_j \lambda_j - \lambda_i \tag{65}$$

for the capital good. This is, by Eq. (51), equivalent to

$$\hat{\pi}_i(0) \frac{1}{n} \sum_j w_j \frac{\bar{\lambda}}{w_j} - \frac{\bar{\lambda}}{w_i} = \hat{\pi}_i(0) \bar{\lambda} - \frac{\bar{\lambda}}{w_i}, \tag{66}$$

and vanishes again due to $\hat{\pi}_i(0) = w_i^{-1}$. We can thus conclude that the dynamics of the basic model induced by the trade structure of the trade algorithm is consistent with an optimal selection of control variables in the decentralized case.

As a by-product we have derived an interpretation for the welfare weights. It is one corollary that the regional shadow prices for capital λ_i differ from the world market price only by the ratio $\hat{\pi}_i$, which is proportional to w_i^{-1}.

B Default parameters and initial values

ρ	:	0.03
δ	:	0.08
β	:	0.4
ζ	:	0.4
α_{IR}	:	0.33
α_{DR}	:	0.33
κ_{IR}	:	0.16
κ_{DR}	:	0.16
ϕ	:	0.9
k_{IR}	:	4
k_{DR}	:	4
a_{IR}	:	2.0
a_{DR}	:	1.2
$L_{IR}(0)$	:	1.0 (constant)
$L_{DR}(0)$	:	1.0 (constant)
First iteration:		
$\bar{X}^j_{IR}$	:	0.0
$\bar{X}^j_{DR}$	:	0.0
w_{IR}	:	1.0
w_{DR}	:	1.0

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