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A model of technological breakthrough in the renewable energy sector

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Models with induced technological change in the energy sector often predict a gradual expansion of renewable energies, and a substantial share of fossil fuels remaining in the energy mix through the end of our century. However, there are historical examples where new products or technologies expanded rapidly and achieved a high output in a relatively short period of time. This paper explores the possibility of a 'technological breakthrough' in the renewable energy sector, using a partial equilibrium model of energy generation with endogenous R&D. Our results indicate, that due to increasing returns-to-scale, a multiplicity of equilibria can arise. In the model, two stable states can coexist, one characterized by a lower and one by higher supply of renewable energy. The transition from the low-output to the high-output equilibrium is characterized by a discontinuous rise in R&D activity and capacity investments in the renewable energy sector. The transition can be triggered by a rise in world energy demand, by a drop in the supply of fossil fuels, or by policy intervention. Under market conditions, the transition occurs later than in the social optimum. Hence, we identify a market failure related to path-dependence and technological lock-in, that can justify a strong policy intervention initially. Paradoxically, well-intended energy-saving policies can actually lead to higher emissions, as they reduce the incentives to invest in renewable energies by having a cushioning effect on the energy price. Hence, these policies should be supplemented by other instruments that restore the incentives to invest in renewable energies. Finally, we discuss the influence of monopoly power in the market for innovations. We show that market power can alleviate the problem of technological lock-in, but creates a new market failure that reduces static efficiency.

1. Introduction

Models of induced technological change often predict a gradual expansion of the renewable energy sector.[1] Many authors conclude that even after several decades of capacity investments and R&D in the renewable energy sector, a major share of world energy supply will still rely on fossil fuels.[2] However, looking back in history, there are various examples of rapid expansion of new technologies or products. For instance, who would have thought at the beginning of the 20th century that the automobile might become the primary means of transportation just a few decades later? At the time, cars were slow, dangerous, and expensive. But due to ongoing innovations, they became cheaper and their quality improved, which paved the way for their massive proliferation. Another example is the rapid expansion of the market for mobile phones, that, presumably, few people anticipated at the beginning of the 1980s.

Clearly, there are scale economies involved in examples of rapid expansion. But these are not merely economies of scale in production. It seems more likely that they are related to innovation efforts. R&D is most valuable when it leads to an improvement of a technology that is applied to a large output quantity, which was originally not the case for cars. Only as output started to grow, major investments in R&D became profitable. This led to lower production costs and higher product quality, stimulating further output growth. Self-enforcing processes in the interplay between production and innovation may, thus, help to explain the possibility of a 'technological breakthrough'.

This paper explores the possibility of a technological breakthrough in the renewable energy sector, using a partial equilibrium model of world energy generation with endogenous R&D. Firms in the renewable energy sector invest in capacities for energy generation. The productivity of their investment depends on the amount of knowledge that the firms can apply. The input factor "knowledge" is supplied by an innovation sector for renewable energy technologies. The fossil energy sector is approximated by a linear supply curve. This simplification reflects the idea that the scope for cost-reducing innovations is presumably smaller in this mature sector, and effects of technological progress may be offset by increasing fossil fuel scarcity. Energy demand is also approximated by a linear curve, and a competitive equilibrium concept is applied.

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[1] See e.g. Edenhofer et al. (2005), Gerlagh and Lise (2005), Goulder and Schneider (1999).

[2] Gerlagh (2008), Goulder and Mathai (2000), Nordhaus and Boyer (2000), Nordhaus (2002), Popp (2004, 2006).

In the model, a multiplicity of equilibria can arise. Intuitively, when there is a larger amount of capacity investments, each innovation is applied more often. This increases the incentives to invest in R&D. Although each innovator has monopoly power over its (patented) innovations, substitutability between them implies that a larger supply of knowledge leads to a lower patent fee. This reduces the investment costs in the renewable energy sector, and makes further capacity investments profitable. As a result of this positive feedback effect, two stable states can coexist: one with a higher and one with a lower share of renewables in the energy mix. At the transition from one state to the other, output in the renewable energy sector rises discontinuously. This reflects our notion of a technological breakthrough.[3]

The transition can be triggered by an exogenous rise in world energy demand (due to economic growth), or by a reduction in fossil fuel supply (due to resource depletion). Under market conditions, the location of the transition point depends on when agents *coordinate* to switch from one stable state to another. Hence, it cannot be determined within our model. However, once a stable state is reached, there is no reason why the energy sector would spontaneously switch to another state. Therefore, the transition may occur too late with respect to what would be the social optimum. In other words, we identify a market failure related to path-dependence and technological lock-in, which reflects an inefficient equilibrium selection under a multiplicity of equilibria.

A remark needs to be made on our methodological approach. Despite the dynamical problem context, our analysis is based on a static modeling framework. Namely, output and investment decisions over a certain interval of time are condensed into a single period. This simplification permits a more thorough analysis of the interplay between the renewable energy sector and the innovation sector. A static approach seems justified when the exact transitional dynamics from one steady state to another are not at the core of interest (see e.g. Krysiak, 2008). Our focus is on the problem of *equilibrium selection*. To this end, we conduct comparative statics over a parameter that is related to world energy demand. Since energy demand is likely to increase over the next decades, a variation of this parameter is comparable to a sequential application of our model to different starting dates for the time interval of interest. Hence, by "slowly" changing this parameter, it is as if we were moving along the time axis, and the "dynamics" that one obtains correspond to a succession of static equilibria. At the transition point to the high-output state, our model behaves as if there were a "jump" from one steady state to another. In a dynamic model, a "technological breakthrough" would, instead, be characterized by a discontinuous rise in the *growth rate* of the renewable energy sector.

Apart from the problem of equilibrium selection, two other market failures are embedded in our model. They prevent optimality even when the problem of equilibrium selection has successfully been addressed. One reflects imperfect appropriability of innovations, the other one is related to the non-rivalness property of knowledge. We derive an optimal policy mix to correct these inefficiencies. To this end, we distinguish between policies to alter the equilibrium selection, and policies to correct the remaining inefficiencies in the generation and distribution of information. To address the problem of equilibrium selection, a tax on fossil fuels/a carbon price is a sufficient instrument. The tax shifts the inverse supply curve of fossil fuels upwards and increases the demand for renewable energy. If the carbon price is sufficiently high, a transition to the high-output state is always triggered. Once the high-output state is reached, softer instruments are sufficient to eliminate the remaining market imperfections. Note, that our model neglects environmental externalities. Hence, our results are based on pure *cost-efficiency* aspects.

In practice, effective carbon prices seem difficult to implement for political reasons. Governments have often resorted to *energy-saving* policies (such as promotion of better heat insulation for houses or less fuel-demanding cars) that are sometimes perceived as 'substitute policy tools'. However, we show that such policies can hinder or postpone a transition to the high-output state by having a cushioning effect on the energy price. Paradoxically, energy-saving policies can, thus, actually lead to *higher* emissions, and should be supplemented by other instruments that restore the incentives to invest in renewable energies.

Finally, we discuss the influence of imperfect competition in the market for innovations on our results. To this end, we assume (for simplicity) a monopoly in the innovation sector. Temporary monopoly power may e.g. result from the invention of a new technology that cannot easily be substituted by alternative technologies. We show that by charging a sufficiently low license fee, the monopolist can always trigger capacity investments in the renewable energy sector. This effectively resolves the coordination problem that exists under perfect competition, and the transition to the high-output state of the renewable energy sector occurs earlier than under market conditions. However, in line with the well-known trade-off between 'static' and 'dynamic efficiency', this comes at the price of a new market failure: the monopolist artificially restricts the supply of knowledge in order to achieve a higher mark-up.

1.1. Related Literature

The possibility that under increasing returns, an inferior technology may become locked-in due to path-dependency has been described by Arthur (1989). If an inferior technology initially offers a higher return, adoption of a superior technology (characterized by higher "returns-to-adoption") may not occur. Applied to the energy sector, this problem is sometimes referred to as "carbon lock-in" (Unruh, 2000).

Based on his concept of a 'techno-institutional complex', Unruh (2000) argues that institutions and the political system itself may contribute to a lock-in effect.[4] For instance, in the electricity sector, a techno-institutional complex comprises the grid, other capital, knowledge, individuals and institutions involved in the provision of electricity. Once established, a techno-institutional complex tends to perpetuate itself, as agents acquire specific knowledge and capital that is valuable only within the complex. Hence, they try to prevent discontinuous technological change that threatens the complex, and socially desirable policy measures may be blocked. The evidence is compelling: economists have argued for decades that greenhouse gas emissions should have a price (Nordhaus, 1992), and while there remains disagreement about the optimal level, there is a wide consensus that it should be *positive*. However, few countries have so far implemented effective carbon prices, and many countries still subsidize the use of fossil fuels—a paradox that can be explained by techno-institutional lock-in.

Problems of path-dependence and technological lock-in are often overlooked in formal modeling, although the market failure they create can be severe. But there exists a sizable strand of literature that explains inertia in the adoption of new technologies as a result of evolutionary learning processes. Carillo-Hermosilla (2006), e.g., highlights the idea that policies should aim directly at the *process* of technological change, to complement more conventional equilibrium-oriented policies.[5]

[3] An alternative explanation for rapid technological progress is the arrival of a "general purpose technology", triggering innovation in a variety of sectors (see Helpman, 1998). Our focus is on markets with an existing mature technology, and an alternative technology with a larger potential for cost reduction.

[4] See also Barrett (2006), Unruh (2002), and Unruh and Carrillo-Hermosilla (2006).

[5] For an overview over the evolutionary approaches, see Metcalfe (1994), Janssen and de Vries (1998), van den Bergh and Gowdy (2000), and the references cited therein.

We develop a model that more closely resembles the neoclassical framework. It is a first step towards an integration of market failures related to path-dependence and technological lock-in into partial or general equilibrium models. Technically, our model is related to Gerlagh and Lise (2005). Following their approach, we also assume that innovations are produced by an external R&D sector. However, they treat energy from fossil and renewable sources as imperfect substitutes. This assures that even without policy intervention, the renewable energy sector always supplies a positive amount of energy. In our view, this assumption of imperfect substitutability may not be well justified, especially as some of the major technical issues concerning energy storage and conversion may be resolved during the next decades. Furthermore, Gerlagh and Lise (2005) assume that knowledge enters the production function for renewable energy in a multiplicative way, along with the current capital stock and labor. This implies that new knowledge makes old capacities (from previous periods) more productive, which is clearly not the case for wind or solar energy. We adopt an assumption made by Edenhofer et al. (2005), namely that knowledge affects the productivity of *investments* in capacity.

Our discussion of an optimal policy mix is in the spirit of Fischer and Newell (2008). While we focus on a 'technological breakthrough', these authors point out that their model is *not* suitable to analyze this type of technological change. Furthermore, they use a technical simplification, namely the assumption of a 'representative firm'. While this can be justified under certain conditions, in the presence of increasing returns it seems important to verify that the assumption of perfect competition is actually consistent with the modeling choices. Hence, for the purpose of our analysis, a representative firm approach seems inappropriate.

Previous studies demonstrated that if the number of available policy instruments is limited, the carbon price may be raised above the Pigouvian level to boost emissions-saving technology investments (Hart, 2008; Gerlagh et al., 2009). Our results point in the same direction, but for a different reason. Hart (2008), e.g., argues that if a carbon tax is used to correct multiple market failures (environmental externalities and problems of knowledge spillovers), a fairly high tax may be required to reach a "second-best solution". We argue that a high carbon price (or some other instrument) may be required *initially* to 'jump-start' the renewable energy sector. Once this is achieved, self-sustained growth will prevail, and a lower carbon price is in order.

The remainder of this paper is organized as follows. Section 2 introduces a social planner version of the model, and characterizes the social optimum. The market version is analyzed in Section 3, and Section 4 derives an optimal policy mix. Section 5 discusses possible effects of monopoly power in the R&D sector. Section 6 concludes.

2. Social Planner Version

The capacity for energy generation in the renewable energy sector is denoted by K. To install a capacity of K, the planner uses two input factors: an investment good I, and knowledge a. A higher technological standard – formalized as a larger choice of a – implies that a given investment spending I yields more capacity. More specifically, we assume:

$$K = \kappa(a)I \equiv a^{\eta}I, \tag{1}$$

where $\kappa(a)$ is an increasing function that reflects the productivity of capacity investments. Throughout the paper, we use the specification $\kappa(a) = a^{\eta}$, where $\eta \in (0,1)$ is the elasticity of the productivity of knowledge (assumed constant).

By assumption, the installed capacity in the renewable energy sector is fully used for energy generation (there is no idle capacity). This reflects the idea that variable costs of energy generation are negligible, once the capacity is installed (think, e.g., of solar panels).

Knowledge is generated via R&D investments, denoted by $r \geq 0$. We do not assume any scale or saturation effects in the generation of knowledge. Therefore, a linear relation obtains:

$$a = a_0 + \rho r \tag{2}$$

ρ is the productivity of R&D investments, and a_0 is the initial amount of knowledge. The amount of new knowledge generated via R&D is $a - a_0 \geq 0$.

The costs of renewable energy are the sum of the investment costs in capacity I, and R&D investments r. Using Eqs. (1) and (2), they can be written as a function of K and a:

$$C^{\text{ren}}(K, a) = a^{-\eta}K + \frac{a - a_0}{\rho} \tag{3}$$

The planner's problem can be divided into two steps: 1. determine the optimal amount of capacity K in the renewable energy sector, and 2. determine the cost-minimizing combination of capacity investments I and knowledge a that yields this capacity. Let us proceed by backwards induction, and minimize the cost function $C^{\text{ren}}(K,a)$ over a first. Intuitively, Eq. (1) defines a set of isoquants (combinations of I and a that yield a fixed capacity K). The planner, thus, computes the cost-minimizing location on the isoquant that corresponds to the given target capacity K. The first-order condition yields for the optimal amount of knowledge for a given K (using Eq. (3)): $a = (\rho\eta K)^{\frac{1}{1+\eta}}$. This holds if K is sufficiently large so that $a \geq a_0$. Otherwise, the planner sets $a = a_0$. Hence:

$$a(K) = \begin{cases} (\rho\eta K)^{\frac{1}{1+\eta}}, & \text{if } K \geq a_0^{1+\eta}/\rho\eta \\ a_0, & \text{otherwise} \end{cases} \tag{4}$$

Together with Eq. (1), Eq. (4) defines the optimal location on the isoquant in the $I-a$-space. Substituting for a in Eq. (3) (using Eq. (4)), we obtain the following continuous cost function:

$$C^{\text{ren}}(K) = \begin{cases} (1+\eta)(\rho\eta)^{-\frac{\eta}{1+n}} K^{\frac{1}{1+\eta}} - \frac{a_0}{\rho}, & \text{if } K \geq a_0^{1+\eta}/\rho\eta \\ a_0^{-\eta}K\ , & \text{otherwise} \end{cases} \tag{5}$$

The cost function of the renewable energy sector is, thus, *linear* in K if K is small. In this case, the planner uses only the existing stock of knowledge a_0, and does not invest in R&D. The cost function, then, reflects constant returns-to-scale (K is linear in I, see Eq. (1)). However, when the target capacity becomes sufficiently large, cost-reducing R&D becomes profitable, and the cost function becomes *concave* (as can be confirmed using Eq. (5)). This reflects *increasing* returns-to-scale, resulting from cost savings achieved via R&D activities.

Using Eq. (5), we obtain the following marginal cost function for the renewable energy sector:

$$MC^{\text{ren}}(K) = \begin{cases} (\rho\eta K)^{-\frac{\eta}{1+\eta}}\ , & \text{if } K \geq a^{1+\eta}/\rho\eta \\ a_0^{-\eta}\ , & \text{otherwise} \end{cases} \tag{6}$$

In a standard model without R&D, the marginal cost is the additional cost of the next unit, while the production costs of the other units remain unchanged. Here, the optimal amount of knowledge a is *embedded* in the cost function. When K is marginally raised, the optimal a increases. Therefore, when an additional unit of capacity is produced, all other units become cheaper as well, due to the rise in a (unless $K < a_0^{1+\eta}/\rho\eta$: no R&D takes place).

To close the model, we need to define the supply of fossil energy and the world energy demand. Fossil energy supply is approximated by a linear supply curve:

$$S^{fos}(p) = A + \alpha p, \tag{7}$$

where p is the energy price, and A and α are parameters. The linear supply curve corresponds to decreasing returns-to-scale in the technology of fossil energy generation. World energy demand is also approximated by a linear curve:

$$D(p) = B - \beta p \tag{8}$$

Market clearing on the world energy market requires that (using Eqs. (7) and (8)):

$$B - \beta p = A + \alpha p + K \tag{9}$$

By appropriately rescaling the units of energy, we can normalize $a+\beta$ to 1. Furthermore, let $Z \equiv B - A$ and assume $Z>0$. The energy demand parameter Z reflects excess demand of energy when $K=0$, at an energy price of zero.[6] Under these assumptions, Eq. (9) simplifies to:

$$p = Z - K \tag{10}$$

Eq. (10) defines an inverse demand curve $p(K)$ for renewable energy.

We are now ready to maximize welfare, that we define as consumer surplus minus total costs of energy generation. Consumer surplus is the area under the inverse demand curve. The costs of fossil energy are the area under the inverse fossil energy supply curve, and the costs of renewable energy are given by Eq. (5). Assuming that the inverse demand curve correctly reflects the marginal benefit of energy usage (as is standard in a partial model), and the inverse fossil energy supply curve reflects the marginal costs of fossil energy generation (this holds under perfect competition in this sector), welfare maximization leads to the well-known optimality condition "price equals marginal cost" (here: $p = MC^{ren}$). Using Eqs. (6) and (10), this yields for $K < a_0^{1+\eta}/\rho\eta$:

$$K = Z - a_0^{-\eta} \tag{11}$$

This is the optimal capacity choice when no R&D investments are undertaken, given that the non-negativity constraint $K \geq 0$ is fulfilled (otherwise, the optimal capacity is zero). For the case $K \geq a_0^{1+\eta}/\rho\eta$, we obtain the following condition:

$$(\rho\eta K)^{-\frac{\eta}{1+\eta}} = Z - K \tag{12}$$

This is a non-linear equation in K. A closed-form solution can not generally be obtained, but the solutions can be derived numerically. Eq. (12) has at most two real-valued solutions.

Lemma 1. Whenever Eq. (12) has two real-valued solutions, the solution with the larger value of K is a local maximum of the welfare function. The other solution is a local minimum and, hence, never welfare maximizing.[7]

To understand the intuition behind these results, it is useful to visualize the marginal cost function of the renewable energy sector (6).

The intersection points of the marginal cost curve $MC^{ren}(K)$ and the price schedule $p(K)$ are candidate solutions to the planner's maximization problem. Note, that $p(K)$ shifts upwards as Z increases. For low values of Z, no intersection point exists, and marginal costs in the renewable energy sector always exceed the price $p(K)$. Hence, in the optimum, the planner does not invest in renewable energy. If Z increases, an intersection point of $p(K)$ and the horizontal segment of $MC^{ren}(K)$ emerges (see Fig. 1). This corresponds to solution (11), with $r=0$ (no R&D). In the following, we refer to a solution with $r=0$ as "lower state" (or low-output state of the renewable energy sector). When Z rises further, two other intersection points emerge. These are the solutions to Eq. (12). The intermediate intersection point is a *minimum*, because a rise in K leads to a situation where $p(K) > MC^{ren}(K)$, which implies that welfare increases in K (similarly for a reduction in K). The other intersection point is a maximum, and we refer to this as the "upper state". It corresponds to an outcome where the planner invests both in capacities and in R&D for renewable energy generation. Note, that for some parameter values, the lower and the upper state *coexist*.

The following proposition summarizes the planner's outcome for varying values of Z, that can be interpreted as changes in world energy demand (due to economic growth), or as changes in the supply of fossil fuels (due to resource depletion).

Proposition 1. *If Z is small, the planner satisfies the energy demand with fossil fuels only. If Z is in an intermediate range, the planner invests in capacities for renewable energy generation, but not in R&D.*[8] *If Z is sufficiently large, the planner invests in capacities and R&D for renewable energies. As long as the initial stock of knowledge a_0 does not exceed a certain limit, a discontinuous rise in renewable energy supply, and a drop in fossil energy supply occurs, when R&D in the renewable energy sector sets in.*

The technical details are in Appendix A. Here, we give a qualitative description of the planner's solution. Fig. 2 shows the optimal amount of R&D investments r in the renewable energy sector, for varying values of Z.

The arrows in Fig. 2 illustrate which type of solution the planner chooses for a given value of Z. Note, that the figure may be interpreted quasi-dynamically as the system's evolution under a (slowly) growing energy demand. The dotted curve shows the location of the local welfare minimum. If Z is sufficiently small (so that the upper solution does not exist), the planner chooses the lower solution with $r=0$. If Z is sufficiently high, the planner always chooses the upper solution. In the intermediate range where the upper and the lower solution coexist, the planner compares welfare in these two states. In the interior of this range, a critical value for Z exists, where welfare is *identical* in both states (see Appendix A). At this point, investments in the renewable energy sector discontinuously rise to a higher level as Z increases. The vertical arrows in Fig. 2 indicate the location of the discontinuity point.[9]

Fig. 3 illustrates the transition of a fossil energy based economy to a fossil *and* renewable based one. If Z is sufficiently small (region A), the planner fulfills the entire energy demand with fossil fuels only, because the marginal costs are initially low in this sector. However, they increase with the output level. In region B, they become as high as in the renewable energy sector, that operates with a constant returns-to-scale technology when no R&D investments are undertaken. R&D becomes profitable when the capacity investments in this sector are sufficiently large. When R&D sets in, K rises *discontinuously*. This leads to a drop in the supply of fossil energy, visible at the transition from regions B to C in Fig. 3. If Z rises further, the output of the fossil energy sector declines, as renewable energy becomes increasingly cheap.

[6] Due to the linearization, fossil energy supply can be positive when the energy price is zero. This is an artifact of the model, but the main results do not depend on this. Note, that in equilibrium, the price never actually becomes zero. The linearization is for mathematical tractability.

[7] As the graphical intuition is obvious (see Fig. 1, and the subsequent discussion), a formal proof is omitted.

[8] For some parameter values, this intermediate range does not exist. The planner then switches directly from the lower state with $K=r=0$ to the upper state with $K>0$ and $r>0$ (see Appendix A).

[9] The location of the discontinuity, as well as the solutions to Eq. (12) shown in Fig. 2, have been computed numerically.

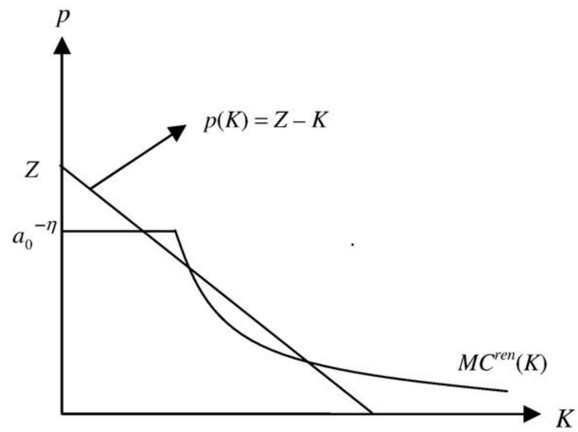

Fig. 1. Marginal cost function of renewable energy sector.

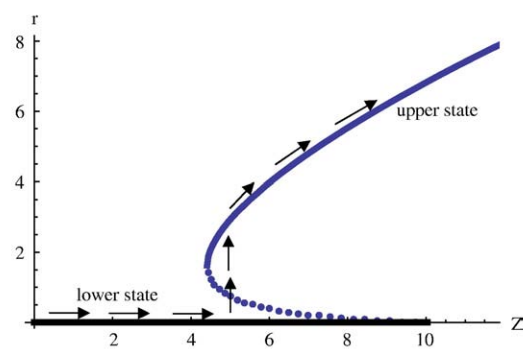

Fig. 2. Optimal R&D effort r, plotted for $\rho = 0.05$, $\eta = 0.5$, $a_0 = 0.01$.

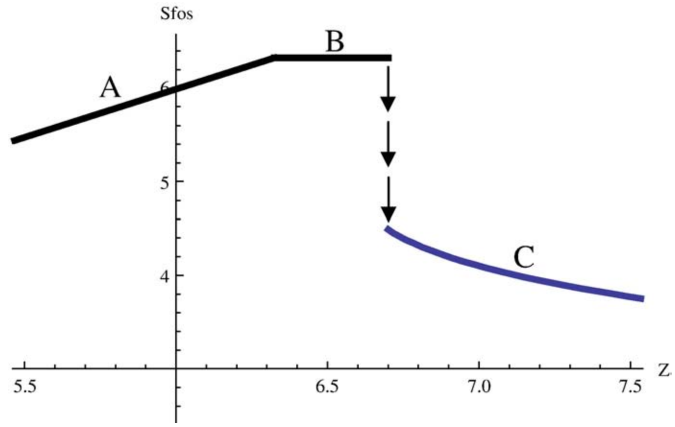

Fig. 3. Optimal output of fossil energy sector S^{fos}, plotted for $\rho = 0.01$, $\eta = 0.5$, $a_0 = 0.025$, $A = 0$, $\alpha = 1$.

3. Market Solution

In the previous section we have seen that the welfare function can have several local maxima, of which the social planner chooses the *global* maximum. In a market economy, the existence of more than one stable state can lead to inefficiency. Agents will not generally be able to coordinate on the welfare maximizing state. Once a stable state is reached, there is no reason why the energy sector would spontaneously switch to another state. Hence, there is a potential coordination problem. Furthermore, even if the 'correct' state is chosen, there are other sources of market failure that can prevent optimality.

Let us introduce a market version of the model, and analyze its properties. Energy demand and fossil energy supply are modeled as in Section 2. The focus is on the interaction between the renewable energy sector and the innovation sector.

3.1. Renewable Energy Sector

Firms in this sector are indexed by j. I_j is firm j's investment in capacity for energy generation, and K_j the resulting capacity.[10] The productivity of firm j's investment, $\kappa(a_j)$, depends on the amount of knowledge a_j applied by firm j:

$$K_j = \kappa(a_j) I_j = a_j^{\eta} I_j \tag{13}$$

Let a_j^{priv} be the number of licenses for patented innovations purchased by firm j. The license fees are, by assumption, linear in the investment spending (think e.g. of software where a license must be purchased for each computer). Under these assumptions, firm j's profit equals: $\pi_j = pK_j - I_j - \theta a_j^{priv} I_j$, where θ denotes the price per license. Using Eq. (13), we obtain:

$$\pi_j = (p - a_j^{-\eta}(1 + \theta a_j^{priv}))K_j \tag{14}$$

Let a be the total stock of knowledge. It is the sum of public knowledge a^{pub} and private knowledge a^{priv}. Public knowledge reflects expired patents, publicly funded R&D, and spillovers. Since it is free, all firms use the entire stock ($\alpha_j^{pub} = \alpha^{pub}\ \forall_j$). When determining the demand for private knowledge a_j^{priv}, firms take the license fee θ as given. Maximizing π_j over a_j^{priv}, we obtain the following first-order condition:

$$(1 + \theta a_j^{priv})\eta \le \theta a_j, \text{ with equality if } a_j > a^{pub} \tag{15}$$

Using Eq. (15), we can derive firm j's total demand for knowledge:

$$a_j(\theta | a^{pub}) = \begin{cases} \dfrac{\eta}{1-\eta}\left(\dfrac{1}{\theta} - a^{pub}\right), & \text{if } \theta \le \eta / a^{pub} \\ a^{pub}, & \text{otherwise} \end{cases} \tag{16}$$

By Eq. (16), all firms use the *same* amount of knowledge, independent of their target capacity K_j. Furthermore, Eq. (16) reveals that (for $\theta \le \eta / a^{pub}$), the total demand for knowledge a_j *decreases* in the amount of public knowledge a^{pub}.[11]

Knowledge that is obsolete is not produced by the R&D sector. Therefore, in equilibrium, the total amount of private information matches its demand:

$$a_j^{priv} = a^{priv}, \text{ and } a_j = a\ \forall j \tag{17}$$

Firms, thus, purchase licenses for *all* existing private innovations.

In the maximization of π_j over K_j (see Eq. (14)), we obtain the following equilibrium condition:

$$p \le a^{-\eta}(1 + \theta a^{priv}), \text{ with equality if } K > 0 \tag{18}$$

Finally note, that the assumption of *perfect competition* in the renewable energy sector is consistent with the increasing returns-to-scale technology (see Eq. (13)), because the price of the factor "knowledge" is not just linear in the number of licenses purchased (a_j^{priv}), but also depends on the investment I_j for which these innovations are actually used. The model, thus, diverts from the

[10] Existing capacities from previous periods are zero. This is a useful approximation, given the currently small share of renewable energies in the world energy mix.

[11] Using $a_j = a_j^{priv} + a^{pub}$, firm j's costs are: $C_j(I_j, a_j) = (1 - \theta a^{pub})I_j + \theta a_j I_j$. Hence, a^{pub} has similar effects as a *subsidy* on the investment I_j, and distorts the outcome towards a lower demand for knowledge.

standard neoclassical approach, where each commodity can be sold only once, and prices are linear in quantity. This divergence reflects the *non-rivalness property* of knowledge: the usage of an innovation by one firm does not exclude the usage by another firm – hence, each innovation can be sold many times. It can be shown that, as a result of this, the sector is *not* a natural monopoly (despite increasing returns-to-scale).[12]

3.2. R&D Sector for Renewable Energies

Firms in this subsector (indexed by i) produce innovations that improve the productivity of investments in capacity for renewable energy generation. Let r_i be firm i's R&D expenditure, and let a_i^{priv} be the number of patents held by firm i. As in Section 2, we assume that knowledge accumulates in a linear way. Therefore, Eq. (2) remains valid.[13] However, we assume that private innovators cannot fully appropriate their innovations. Let σ be the rate of appropriability.[14] Thus, we obtain for firm i: $a_i^{\text{priv}}=\sigma\rho r_i$. Aggregation over i yields:

$$a^{\text{priv}} = \sigma\rho r \tag{19}$$

Given that all firms in the production sector purchase firm i's licenses (see Section 3.1), firm i's profits are: $\pi_i=\theta I a_i^{\text{priv}}-r_i$, where $I=\sum_j I_j$ is the aggregate investment in capacity. Using $a_i^{\text{priv}}=\sigma\rho r_i$, this becomes:

$$\pi_i = (\sigma\rho\theta I-1)r_i \tag{20}$$

The maximization over r_i (given θ and I) yields the following equilibrium condition:

$$\sigma\rho\theta I \leq 1, \text{ with equality if } r_i > 0 \tag{21}$$

$\sigma\rho\theta I$ is the marginal revenue of R&D, and the marginal cost is 1.

By Eq. (21), there is an *inverse* relation between the license fee and the aggregate investment I (when $r_i>0$). Intuitively, when I is large, each innovation is used to build a large amount of new capacity. This increases the incentives to innovate, and (due to the substitutability of innovations) drives down the license fee θ. This triggers additional capacity investments, as the investment costs are reduced. This positive feedback effect gives rise to the possibility of multiple equilibria.

3.3. Characterization of the Market Solution

As in the social planner's case, we distinguish between two types of solutions: an upper solution with positive capacity and R&D investments in the renewable energy sector, and a lower solution with $r=0$, and $K>0$ or $K=0$ (depending on the parameter values). As we show in Appendix A, the market solution has qualitatively the same properties as the solution to the planner's problem.[15] However, whereas the planner compares welfare in the upper and lower state when both states coexist, equilibrium selection becomes a matter of *coordination* under market conditions. Hence, the exact location of the transition point from the lower to the upper state cannot be determined within our model. Once a stable state is reached, there is no reason why the energy sector would spontaneously switch to the other state. Reflecting the idea of *path-dependence*, it, therefore, seems plausible to assume that a transition to the upper state only occurs near the point where the lower solution ceases to exist. The following proposition summarizes the properties of the market solution:

Proposition 2. *Along different values for the energy demand parameter Z, the behavior of the market solution is qualitatively the same as in the planner's case (see Proposition 1). However, the transition from the lower to the upper state is a matter of coordination, and the location of the transition point cannot be determined within this model. Furthermore, market imperfections related to the generation and distribution of knowledge prevent full optimality even when the upper state is reached.*

The renewable energy sector is still in its infancy (relatively low output and R&D), and most of the developments in this sector seem to be policy-driven. There does not seem to be self-sustained growth yet. Assuming a large potential for further cost reductions, it does not seem implausible that a high-output state may already coexist. Hence, the renewable energy sector may currently be locked into in a 'low-investment trap'. Without further policy intervention, the upper state may only be reached when world energy demand becomes so high or fossil energy supply so low that the lower state (almost) ceases to exist. However, the social optimum clearly requires an earlier transition.[16] Therefore, there may be a serious market failure related to path-dependence and technological lock-in, that can justify a strong initial policy intervention to "jump-start" the renewable energy sector.

4. Optimal Policy Mix

There are three market failures in the model, and – following the "Tinbergen rule" – we will introduce three different policy instruments to correct them. The first market failure (and the most crucial one for the argument of this paper) consists of the inefficient equilibrium selection when multiple equilibria coexist. Two further market failures are related to inefficiencies in the generation and distribution of information. One is the well-known problem of imperfect appropriability, captured by the parameter σ. The other one is related to non-rivalness in the use of knowledge, which allows R&D firms to sell (appropriated) innovations to *all* firms in the renewable energy sector. This deviates from the standard neoclassical framework, where each good is sold only once, and prices are just linear in quantity. Therefore, the standard welfare theorems do not apply.

4.1. Policies to Alter the Equilibrium Selection

As pointed out in Section 3, the location of the transition point to the upper solution cannot be determined within this model, as it depends on when private agents coordinate to switch from one state to another. However, any policy that pushes the energy sector beyond the point where the lower solution ceases to exist, is *always* sufficient to implement the upper solution.

Let us introduce a fossil energy tax τ into the model. Eq. (7) now reads:

$$S^{\text{fos}}(p) = A + \alpha(p-\tau) \tag{22}$$

Using $\alpha+\beta\equiv 1$ and $Z\equiv B-A$ as in Section 2, Eq. (10) becomes:

$$p = (Z + \tau)-K \tag{23}$$

Eq. (23) defines an inverse demand function $p(K)$ for renewable energy. As the equation shows, a rise in τ induces upwards pressure on the energy price. It corresponds to a motion along the Z-axis to the right (see

[12] To see this formally, note that firm j's marginal cost is: $a_j^{-\eta}(1+\theta a_j^{priv})$ (see (14)), and (by (16)) independent of K_j. Therefore, firm j's profit is linear in K_j, and a horizontal supply curve $K_j(p|\theta,a^{pub})$ is obtained, as in a standard neoclassical model with a *constant* returns-to-scale technology.

[13] For simplicity, we assume that all existing knowledge a_0 is public.

[14] Unappropriated innovations leak to the pool of public knowledge.

[15] We also show in Appendix A that the model of Section 3 can be rewritten in a reduced form that corresponds to a standard learning-by-doing problem. This is a useful insight, because stable relations between cumulated capacity and costs have been identified empirically (IEA 2000).

[16] Liquidity constraints may explain why an individual firm cannot bridge the unstable region on its own and transfer the renewable energy sector to the high-output state.

Fig. 2). This can trigger additional investments in the renewable energy sector. If τ is sufficiently high, the point is reached where the lower state ceases to exist. The following (sufficient) condition for the existence of the lower solution is derived in Appendix A: $Z < a_0^{-\eta} + \frac{a_0^{1+\eta}}{\varphi\rho}$ (see Eq. (29)). From this, one immediately obtains:

Proposition 3. *A tax on fossil energy is a sufficient policy instrument to trigger a transition to the upper state.*

Note, that other policy instruments may achieve the same goal. Publicly funded R&D programs, e.g., may increase the initial amount of public knowledge a_0, and reduce the investment costs in renewable energies. Similarly, an output subsidy for renewable energies may trigger a transition to the upper state.

The implementation of fossil energy or carbon taxes seems difficult for political reasons. Energy-saving policies tend to face less opposition and are sometimes perceived as 'substitute policy tools'. However, policies that reduce the demand for energy have a cushioning effect on the energy price. Paradoxically, they can even lead to *higher emissions*. Formally, an energy-saving policy corresponds to a *reduction* in the parameter Z (contrary to a tax on fossil energy). This leads to the following important result:

Corollary 3. *Energy-saving policies can hinder or postpone a transition to the high-output state.*

Such policies should, therefore, be supplemented by other instruments that restore the incentives to invest in renewable energies.

4.2. Policies to Eliminate Inefficiencies in Knowledge Generation

Suppose, the upper state is socially optimal (given the parameter values of the model), and the inefficiency related to path-dependence has already been resolved.[17] In this case, two sources of inefficiency remain (outlined above), requiring – in general – two independent policy tools to reach full optimality. Although we cannot derive closed-form expressions for the market solution, we can derive an optimal policy mix by bringing the equations that determine the market solution into the same form as the ones that define the social optimum.

Let s_I be an investment subsidy for renewable energies, covering a share s_I of firm j's capacity investment costs. Furthermore, let s_r be an R&D subsidy, covering a share s_r of an innovator's R&D expenditure. We show in Appendix A that the market version of the model can be rewritten in a reduced form that corresponds to the standard formulation of a learning-by-doing problem. To this end, we express the knowledge a and the profit of an individual firm j as functions of the aggregate capacity K. A closed-form solution for $a(K)$ can only be derived for the special case $a_0 = 0$. Introducing the two policy parameters into the derivation, firm j's marginal cost becomes: $c(K) = \left(\frac{1-s_I}{1-\sigma\eta}\right)^{\frac{1}{1+\eta}}\left(\frac{\sigma\eta\rho}{1-s_r}K\right)^{-\frac{\eta}{1+\eta}}$ (see Eq. (32)). Perfect competition drives the energy price down to a level where firms do not earn profits. Hence, in equilibrium we have: $p = c(K)$. Eliminate p using Eq. (10) to obtain:

$$\left(\frac{1-s_I}{1-\sigma\eta}\right)^{\frac{1}{1+\eta}}\left(\frac{\sigma\eta\rho}{1-s_r}K\right)^{-\frac{\eta}{1+\eta}} = Z-K \tag{24}$$

This non-linear equation implicitly defines the aggregate capacity K in a market solution. It is of the same form as Eq. (12), that defines K in the planner's case. To reach optimality, adjust s_I and s_r such that the left-hand side of Eq. (24) becomes as in Eq. (12). This holds for $s_r = 1-\sigma$ and $s_I = \sigma\eta$. Note, that the R&D subsidy s_r compensates the lack of appropriability of innovations. The other instrument is used to correct for the market failure due to the non-rivalness property of knowledge.[18]

Using these instruments, the same capacity is installed under market conditions as in the planner's solution. However, to ensure that we actually reach the social optimum, we must verify whether this policy mix also fulfills Eq. (4): $a(K) = (\rho\eta K)^{\frac{1}{1+\eta}}$. This assures that the target capacity K is installed with the right combination of capacity investments I and knowledge a. Introduce the policy parameters into the derivation of Eq. (31) to obtain for the market case: $a(K) = \left(\frac{\sigma\eta\rho(1-s_I)}{(1-\sigma\eta)(1-s_r)}K\right)^{\frac{1}{1+\eta}}$. Now insert $s_r = 1-\sigma$ and $s_I = \sigma\eta$ to find that this becomes identical to Eq. (4) (as required). Hence, the above policy mix implements the social optimum.

The situation is slightly more complicated when $a_0 > 0$. In this case, a closed-form solution for the optimal amount of knowledge $a(K)$ cannot be derived. By eliminating variables, it is still possible to derive a non-linear equation in a single variable (K), that characterizes the market solution. However, this condition is mathematically *not* of the same form as Eq. (12), because the existence of public knowledge a^{pub} distorts the total use of the factor knowledge (see Eq. (16)). To eliminate this distortion, suppose that the regulator can charge the same fee θ for the use of the initial public knowledge a_0 as firms in the private R&D sector (this is not implausible if a_0 reflects publicly funded R&D), and the regulator also faces the same lack of appropriability (captured by σ). Firm j's total expenditure on license fees now becomes: $\theta a^{\text{priv}} I_j + \theta\sigma a_0 I_j$, which simplifies to $\theta\sigma a I_j$. Following the same steps as before, it can be shown that one obtains the same expressions for $a(K)$ and for $c(K)$ as for $a_0 = 0$. Hence, the same optimal policy mix applies. The following proposition summarizes:

Proposition 4. *When the upper solution is socially optimal and the problem of equilibrium selection has successfully been addressed, an optimal policy mix consists of an R&D subsidy* $s_r = 1-\sigma$ *and an investment subsidy* $s_I = \sigma\eta$*, and (if* $a_0 > 0$*) of a fee for employing initial public knowledge* a_0 *equal to the one for private knowledge (*θ*).*

In sum: a strong policy intervention may be required initially to jump-start the renewable energy sector and overcome the problem of equilibrium selection. Once the high-output state is reached, softer instruments are sufficient to eliminate the remaining sources of market failure. In our model, the carbon tax then becomes obsolete, as we excluded environmental damages from the analysis (our results are entirely driven by cost-efficiency aspects).

5. Monopoly Power in the Innovation Sector

The model in Section 3 relies on the assumption of perfect competition in the innovation sector. Although innovators obtain patents for their (appropriated) innovations, they cannot earn positive profits because they compete with other innovators whose innovations are *perfect substitutes* for their own innovations. In practice, markets for innovations are sometimes characterized by imperfect competition. For instance, the inventor of a new technology may enjoy temporary monopoly power if the innovation cannot easily be substituted by alternative technologies.

In this section, we analyze how the presence of monopoly power in the market for innovations affects our results. For simplicity, we assume that the R&D sector is dominated by a single monopolist. This is an extreme assumption, but it reveals effects that can arise under imperfect competition. Furthermore, we assume that the initial amount of public knowledge a_0 is zero (this simplifies the exposition).

[17] When the lower state is socially optimal, no policy intervention is required, as in this case, the market outcome coincides with the planner's solution.

[18] Intuitively, when a firm in the renewable energy sector increases its investment spending, this has the positive externality upon other firms of a reduction in the license fee θ. But this externality is neglected by the firm, so the investment subsidy is needed to correct for this inefficiency.

Using $a^{\text{priv}} = \sigma a$, the monopolist's profit is given by: $\pi^m = \theta l \sigma a - r$. Using the optimality conditions (16) and (18), the market clearing conditions (23), and (1), this can be rewritten as follows:

$$\pi^m(K) = \sigma\eta K(Z-K) - \rho^{-1}((1-\sigma\eta)(Z-K))^{-1/\eta} \qquad (25)$$

Hence, the monopolist's revenue is a share of $\sigma\eta$ of the *total* revenue of the renewable energy sector. When there is no lack of appropriability ($\sigma = 1$) and η is close to 1, the monopolist captures almost the entire revenue. If η is small (hence, the marginal value of knowledge declines rapidly), the monopolist captures only a smaller share of the revenue.

By comparing the shape of the revenue and the cost function (see (25)), it is straight-forward to show that there exists a *unique* solution to the profit maximization problem. However, only if the resulting profit is non-negative, a positive amount of knowledge is supplied. The following proposition summarizes the properties of the monopoly solution:

Proposition 5. *If Z is small, the monopolist does not invest in R&D, and energy demand is covered by fossil fuels only (K = 0). If Z reaches a critical level, there is a discontinuous rise in the monopolist's R&D activity and in capacity investments in the renewable energy sector. If Z rises further, the R&D activity declines, as the license fee for innovations increases.*

The finding that the license fee for innovations increases in Z, and hence, the R&D activity declines, is surprising. Intuitively, one would expect that a higher demand for energy leads to higher capacity investments, and – in turn – to additional investments in knowledge (as in Section 3). However, the monopolist faces a trade-off: a reduction in the license fee triggers more investments in capacity, but the margin is lower. Hence, if the market size is relatively small (lower values of Z), the monopolist charges a low license fee in order to *trigger* investments in capacity. When Z is large, and hence, the investments are fairly high, the monopolist prefers to charge a higher mark-up and, thus, reduces the supply of knowledge.

A central finding of Section 3 was that under market conditions, equilibrium selection can be inefficient due to a coordination problem among competitive firms. When the R&D sector is a monopoly, this coordination problem disappears. Since the monopolist is not a price-taker, he can always stimulate investments in the renewable energy sector by charging a sufficiently low license fee. Hence, the presence of monopoly power can alleviate the problem of path-dependence and technological lock-in. However, this comes at the cost of a new market failure: for larger values of Z, the monopolist artificially reduces the supply of knowledge.

The following figure compares the capacity in the renewable energy sector for all three cases (social planner, market solution, R&D monopoly).

Fig. 4 illustrates that the social optimum requires a higher capacity in the renewable energy sector than under market conditions or in the case with an R&D monopoly. The R&D monopoly leads to the lowest capacity in the upper state, but the transition to the upper state occurs *earlier* than under market conditions, because the monopolist can trigger capacity investments by charging a low license fee. Note, that in the market case, the location of the transition point cannot be determined within our model, but it may be located near the point where the lower state ceases to exist. In Fig. 4, this is indicated by the curly brace.

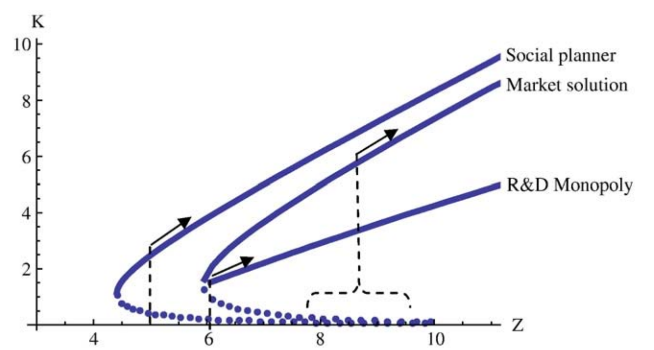

Fig. 4. Capacity in the renewable energy sector, $\rho = 0.05$, $\eta = 0.5$, $\sigma = 0.5$, $a_0 = 0.01$.

6. Conclusion

Using a simple conceptual model, we explored the possibility of a technological breakthrough in the renewable energy sector. It has been shown that – due to increasing returns-to-scale – a multiplicity of stable states can arise. A state with lower output in the renewable energy sector can coexist with a high-output state. In the low-output state, no R&D takes place in the renewable energy sector, and the share of fossil fuels in the world energy mix is high. The high-output state is characterized by positive R&D efforts and a higher share of renewables in the energy mix. At the transition from the low-output to the high-output state, a discontinuous rise in the supply of renewable energy, and a drop in the supply of fossil energy occur. The transition can be triggered by increasing world energy demand, by a reduction in the supply of fossil fuels, or by policy intervention. Under market conditions, it is plausible to assume that the transition takes place near the point where the low-output state ceases to exist, reflecting problems of path-dependence and technological lock-in. However, the social optimum requires an earlier transition. Hence, we identified a market failure that reflects an inefficient equilibrium selection when multiple stable equilibria coexist. Paradoxically, well-intended energy-saving policies can be *harmful* to the climate, as they have the potential to postpone the transition to the high-output state by having a cushioning effect on the energy price. They should be supplemented by other policies to restore the incentives to invest in renewable energies.

A positive relationship between energy prices and innovative activity in the energy sector has been identified empirically by Popp (2002). However, the identified relationship is surprisingly weak: "Even during the peak of the energy crisis, energy prices result in just a 3.14-percent increase in patents." This finding is, however, *not* a contradiction to the results of this paper. Our model predicts that – as long as the renewable energy sector is in the low-output state – a rise in energy prices should have *no* effect upon innovative activities.

It does not seem implausible to assume that the renewable energy sector is currently in a low-output state, as most of the observed progress appears to be *policy-driven*. There may not be self-sustained growth yet. While the subsidies for renewable energies and other policies clearly have a measurable effect, the current stimulation may be too weak to trigger a rapid expansion of the renewable energy sector in the near future. The results of our analysis suggest that on pure cost-efficiency grounds, a stronger policy intervention may be justified.

Appendix A.

Characterization of the social optimum (Proposition 1)

We define three characteristic points along the Z-axis that are useful in the description of the planner's solution. Let $\underline{Z}$ be the critical value of Z, above which Eq. (12) has two solutions. To compute $\underline{Z}$, note that the tangency point of $MC^{\text{ren}}(K)$ and $p(K)$ (see Fig. 1) is defined by $\frac{dp(K)}{dK} = \frac{dMC^{\text{ren}}(K)}{dK}$, which yields (using Eq. (6)): $K = \frac{1}{\rho\eta}\left(\frac{\rho\eta^2}{1+\eta}\right)^{\frac{1+\eta}{1+2\eta}}$. Insert this into Eq. (12) to get: $\underline{Z} = \frac{1+2\eta}{1+\eta}\left(\frac{1+\eta}{\rho\eta^2}\right)^{\frac{\eta}{1+2\eta}}$. Let $\overline{Z}$ be the critical value of Z, above which the lower solution *and* the local minimum cease to exist. Eq. (11) is valid if $K < a_0^{1+\eta}/\rho\eta$ holds (when $Z > \overline{Z}$, the condition is violated). Therefore, to compute $\overline{Z}$, equalize $K = Z - \alpha_0^{-n}$ with $a_0^{1+\eta}/\rho\eta$ to obtain: $\overline{Z} = a_0^{-\eta} + \frac{a_0^{1+\eta}}{\rho\eta}$.

Another critical value is $a_0^{-\eta}$. When $Z < a_0^{-\eta}$, solution (11) is negative and, thus, violates the non-negativity constraint $K \geq 0$. In this case, a corner solution with $K = 0$ obtains. Let us visualize Eqs. (11) and (12), with the help of these critical points.

The dotted curve in Fig. 5 is the local minimum. The figure illustrates that when $Z < a_0^{-\eta}$, the lower solution yields $K = 0$. When $Z \in (\underline{Z}, \overline{Z})$, there are two candidate solutions: the lower solution with no R&D and $K \geq 0$ (depending on $a_0^{-\eta}$ relative to $\underline{Z}$), and the upper solution with $r > 0$ and $K > 0$. In the interval $(\underline{Z}, \overline{Z})$, there is a critical value for Z, denoted by Z^{disc} (for "discontinuity"), where welfare in the upper solution is as high as in the lower solution. If $Z > Z^{\text{disc}}$ ($Z < Z^{\text{disc}}$), the planner chooses the upper (lower) solution. When Z reaches Z^{disc}, K and r rise *discontinuously*, and the fossil energy supply S^{fos} falls. An analytical expression for Z^{disc} cannot be derived unless a closed-form solution to Eq. (12) exists. However, we can show that Z^{disc} lies in the *interior* of the interval $(\underline{Z}, \overline{Z})$. To see this, remember that the solution to Eq. (12) with the lower value of K is a minimum. At $Z = \underline{Z}$, the local minimum *coincides* with the upper solution. Hence, welfare *decreases* in K and is, thus, unambiguously higher in the lower solution. Similarly, at $Z = \overline{Z}$, the local minimum coincides with the lower solution. Hence, welfare is unambiguously higher in the upper solution. Note, that the interval $(\underline{Z}, \overline{Z})$ *can* be the empty set. This occurs if a_0 is sufficiently large. In that case, the technological standard of the renewable energy sector is fairly high to begin with. The discontinuity, then, disappears, and K is continuous in Z.

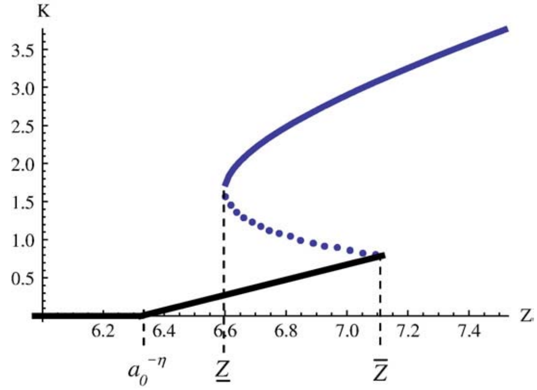

Fig. 5. Optimal capacity in the renewable energy sector, for $\rho = 0.01$, $\eta = 0.5$, $a_0 = 0.025$.

Characterization of the market solution (Proposition 2)

To reduce the number of variables, solve Eq. (2) for r and insert into Eq. (19) to obtain: $a^{\text{priv}} = \sigma(a - a_0)$. Use Eq. (1), that is valid also in the market case (by aggregation of Eq. (13)) in Eq. (23) to get: $I = (Z - p)a^{-\eta}$. For notational convenience, let $\varphi \equiv \sigma\eta$. φ, thus, replaces σ, that (below) will often appear as $\sigma\eta$. Finally, use Eqs. (15), (17), (18), and (21) to obtain a reduced set of conditions that determines the variables of the model:

$$pa^{\eta} \leq 1 + \theta\sigma(a - a_0), \text{ with equality if } K > 0 \tag{26}$$

$$\theta \geq \frac{\eta}{(1-\varphi)a + \varphi a_0}, \text{ with equality if } a > a_0 \tag{27}$$

$$\sigma\rho\theta(Z - p) \leq a^{\eta}, \text{ with equality if } r > 0 \tag{28}$$

Technically, the upper solution is an interior solution. It requires that Eqs. (26)–(28) hold with equality. The lower solution is a corner solution with $r = 0$, so Eqs. (27) and (28) are not binding.

Let us first characterize the *lower solution*. When $K > 0$, Eq. (26) holds with equality. Using $a = a_0$, we obtain: $p = a_0^{-\eta}$. The equilibrium capacity K is obtained using $I = (Z - p)a^{-\eta}$ in Eq. (1): $K = Z - a_0^{-\eta}$ (as in the planner's case). The patent fee θ is not fully determined, but it must be sufficiently large so that Eq. (27) is fulfilled. Using $a = a_0$, we find that: $\theta > \eta / a_0$. However, θ must not be too large so that Eq. (28) remains fulfilled (otherwise, R&D investments are triggered). Using $p = a_0^{-\eta}$, we obtain: $\theta < \frac{a_0^{\eta}}{\sigma\rho(Z - a_0^{-\eta})}$. Combining these two inequalities, we obtain a sufficient condition for the existence of a lower solution with $r = 0$:

$$Z < a_0^{-\eta} + \frac{a_0^{1+\eta}}{\varphi\rho} \equiv \overline{Z} \tag{29}$$

By $K = Z - a_0^{-\eta}$, the non-negativity constraint is only fulfilled if: $Z \geq a_0^{-\eta}$. Otherwise, a corner solution with $r = 0$ and $K = 0$ is obtained.

Let us turn to the characterization of the *upper solution*. Eqs. (26)–(28), thus, hold with equality. It is convenient to eliminate θ, and to discuss the resulting equations in a and p: [19]

$$p = \frac{a^{1-\eta}}{(1-\varphi)a + \varphi a_0} \equiv p(a), \; a = \varphi\rho p(Z - p) \equiv a(p) \tag{30}$$

Eq. (30) defines two curves in the $a - p$-space ("$a(p)$" and "$p(a)$"). The intersection points are (candidate) interior solutions. $a(p)$ is a quadratic function (plotted against the vertical p-axis). $p(a)$ is quasi-concave and hump-shaped (see Fig. 6, below). There are at most three intersection points with positive values of a and p. However, the intersection point with the lowest value of a violates the non-negativity constraint $r \geq 0$. It is, thus, *not* an equilibrium. To see this, compute the first derivative of $p(a)$ using Eq. (30) to find that the unique maximum is located at: $a = \frac{(1-\eta)\varphi a_0}{(1-\varphi)\eta} \equiv a_{max}$. To show that $r < 0$ holds at the left intersection point, it suffices to show that $r(a_{\max}) < 0$. Insert $a_{\max}$ into Eq. (2) to obtain: $r(a_{\max}) = -\frac{(1-\sigma)a_0}{(1-\varphi)\rho} < 0$. Hence, two candidates for an interior solution remain. However, as we show below, the intermediate intersection point of $a(p)$ and $p(a)$ (see Fig. 6) is *unstable*.

As in the planner's case, critical points for Z can be defined to characterize the market solution. $\overline{Z}$ is given by Eq. (29). However, due to the increased complexity, a closed-form solution for $\underline{Z}$ cannot be obtained. Nevertheless, we could derive the following condition for the coexistence of two stable states: $a_0 < \left(\frac{\varphi - \eta - 1}{\varphi\rho(\varphi - \eta)}\right)^{-\frac{1}{1+2\eta}}$.[20]

Stability analysis of market solutions

Suppose, a firm in the production sector unilaterally deviates from its equilibrium capacity investment I_j. Hence, the aggregate investment I differs from its equilibrium value. Suppose, all other variables (in particular the demand and supply of innovations) adjust optimally to the new value of I.[21] *Stability* requires that the deviation is unprofitable. The same must hold true for a unilateral deviation in r_i. Otherwise, the equilibrium is unstable.

Stability of interior solutions (solutions to (30)):[22] Consider the equilibrium with the lower value of a (hence, the intermediate intersection point of $a(p)$ and $p(a)$—see Fig. 6). It can be shown that an increase in I corresponds to a downward motion *along* the $a(p)$-curve, away from the $p(a)$-curve. As Fig. 6 shows, the price p, thus,

[19] This appears to be the simplest way of expressing the equilibrium conditions. The functions $a(p)$ and $p(a)$ can be used to do comparative statics, and are convenient to discuss the stability of the solutions (see below).

[20] For the sake of brevity, the technical details are suppressed. This condition assures that for the given parameter values, $\underline{Z} < \overline{Z}$ holds. For sufficiency, Z must lie in the interval from $\underline{Z}$ to $\overline{Z}$.

[21] This is for tractability. The difficulty in the stability analysis for the market case is that analytical expressions for the interior solutions cannot be obtained. We circumvent this problem, using the functions $a(p)$ and $p(a)$.

[22] The stability analysis of the corner solutions (with $r = 0$) is technically simpler and is suppressed for the sake of brevity. It can be shown that corner solutions are stable whenever they exist. A complete version of the stability analysis can be obtained from the authors upon request.

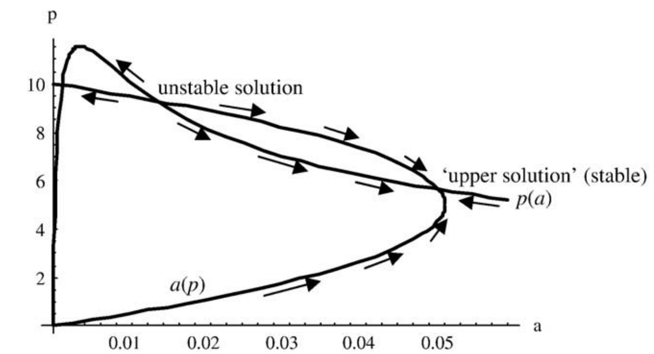

Fig. 6. Deviations in I and r from the interior solutions.

becomes greater than the one that leads to a profit of zero in the renewable energy sector (this condition defines the $p(a)$-curve), as in this region, $a(p)$ lies above $p(a)$. Hence, the profit in the renewable energy sector becomes positive, which implies that the increase in I is profitable. The equilibrium is, thus, *unstable*. Now consider the equilibrium with the larger value of a (the "upper solution"). An increase in I implies that p is below the price that leads to $\pi_j = 0$, as the $a(p)$-curve lies below the $p(a)$-curve in this range. The deviation is unprofitable. Using similar arguments, it can be shown that also a deviation in r is unprofitable (an increase in r corresponds to a motion along the $p(a)$-curve to the right). Hence, the equilibrium is *stable*.

Relation between our model and learning-by-doing approach

The market version of the model introduced in this paper (Section 3) can be rewritten in a reduced form that corresponds to a standard learning-by-doing problem (this holds for the upper solution with $r>0$). To this end, use (1) in Eq. (21) (with equality), to obtain: $\theta = \frac{a^{\eta}}{\sigma\rho K}$. Eliminate θ using Eq. (27) (with equality), to obtain a non-linear equation in a and K. The goal is to derive an expression for a as a function of the aggregate capacity K. To obtain a closed-form solution, we must assume $a_0 = 0$.[23] After rearranging, we find:

$$a(K) = \left(\frac{\varphi}{1-\varphi}\rho K\right)^{\frac{1}{1+\eta}} \tag{31}$$

Use Eq. (31), $\theta = \frac{a^{\eta}}{\sigma\rho K}$, and $a^{\text{priv}} = \sigma a$ (since $a_0 = 0$) in Eq. (14) to obtain after rearranging: [24]

$$\pi_j(K_j|K) = pK_j - c(K)K_j, \tag{32}$$

where the marginal cost $c(K) \equiv (1-\varphi)^{-\frac{1}{1+\eta}}(\varphi\rho K)^{-\frac{\eta}{1+\eta}}$ *decreases* in the aggregate capacity K. This corresponds to the standard formulation of a learning-by-doing problem.

[23] When $a_0>0$, the correspondence to the learning curve formulation is still valid, as the equation obtained after eliminating θ *implicitly* defines a function $a(K)$. This is all that is required.

[24] This is firm j's profit as a function of its own capacity choice, for a *given* aggregate capacity. Note, that K implicitly determines the optimal amount of knowledge $a(K)$.

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