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Lagom generiC: an agent-based model of growing economies[‡]

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Abstract

Building upon recent work of Gintis, we propose a class of agent-based dynamics for disaggregated growth models. We report the results of simulations obtained in a three-sector representation of the German economy.

Key Words: Agent-Based Model, Economic Growth, German Economy.

JEL Codes: C63, C67, O12, O42, O52

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1 Introduction

As suggested by the case of climate policies, the analysis of economic evolution might sometimes require models which on the one hand have a level of granularity sufficient to single out objects such as the production of renewable energies and on the other hand allow for shifts of regimes triggered by changes in technologies and behaviors.

However the seminal literature on economic growth, from (Ramsey 1928) to (Romer 1990) through (Solow 1956), has mainly dealt with highly aggregated models and focused on "equilibrium" trajectories originating in the intertemporal maximization of a social welfare function by a benevolent planner.

In a more disaggregated setting, the consideration of equilibrium dynamics becomes problematic because of the lack of analytical tractability, the puzzle of equilibrium selection (underlined by the Sonnenschein-Mantel-Debreu Theorem, see (Sonnenschein 1973))), but also because when introducing a refined setting, one aims at a refined description of the economic activity. For instance, one might want to emulate unbalanced growth among sectors, involuntary unemployment, price rigidities, the influence of monetary and fiscal policy, the presence of different time-scales in economic activity (see (Leijonhufvud 2006)), a whole class of phenomena which are hard to account for within, or even discarded by, the equilibrium paradigm.

In search for alternatives, agent-based models might provide a useful experimental field (see (Lebaron and Tesfastion 2008), (Colander 2006)). For example, in two recent contributions, (Gintis 2006) and (Gintis 2007), Gintis has obtained surprising convergence and equilibrium transition properties, in a framework where agents use private prices as conventions in the sense of (Peyton-Young 1993). Still, Gintis acknowledges the limitation of his model "There is no inter-industry trade and there is only one financial asset. Consumers do no life-cycle saving and labour is homogeneous.[...]." Also as pointed out by (Bilancini and Petri 2008) his model lacks capital accumulation. We try to address part of these issues by developing a "Gintis-like" model of a growing economy with an explicit production structure (informed for example by input-output tables, see (Duchin 1998))

The present contribution mainly aims at presenting the structure of this model, which we call *Lagom generiC*. *Lagom* is a Swedish word denoting a sense of balance and harmony (perhaps akin to the chinese "Tao") used as a label for a class of models developed at PIK (see e.g (Haas and Jaeger 2005), (Jaeger 2005)). The term *generic* refers on the one hand to the aim of using the model as a "controlled laboratory setting" [1] in which one could test various micro-economic specifications in order to determine which lead to the emergence of realistic macro-economic properties. On the other hand, we also aim at applying *Lagom generiC* to simulate the economic dynamics of a wide range of countries/regions. A first attempt is presented below: we focus on a three-sector representation of the German economy, in which growth is triggered by the

[1] As (Lebaron and Tesfastion 2008)) put it

increase of labor productivity proportionally to investment (see (Arrow 1962)).

The paper is organized as follows. As preliminaries, we provide in section 2 a tentative mathematical definition for agent-based dynamics in an intertemporal economic framework. Section 3 contains a description of the dynamics we propose in terms of agents' behavioral rules and interactions, as well as a more aggregate view on the structure of those dynamics. In section 4, we present the results of a first round of simulations on the German economy and compare them with emprical data. Finally, section 5 contains concluding remarks and an agenda for further investigation.

2 Mathematical Preliminaries

2.1 A Tentative Definition

The label of agent-based dynamics, agent-based models or multi-agent models has been applied to a large number of computer-based models of social phenomena (in particular in economics, see (Lebaron and Tesfastion 2008) and references therein) without, to our knowledge, a precise mathematical definition. It is not our aim here to set the standard in this respect. However a tentative definition, valid at least for the work on economic growth presented here, might ease the description of the model and the understanding of its relations with more standard approaches.

We shall try to characterize agent-based models as a subclass of discrete random dynamical systems. Let us then consider an arbitrary probability space $(\Omega, \mathcal{F}, P)$, a set X and a discrete random dynamical system on X defined by its transition function

$$\phi : \Omega \times X \to X. \tag{1}$$

Definition 1 *We shall say that (X, Ω, ϕ) exhibits agent-based dynamics if:*

1. *The state space X is a cartesian product of the form $S_1 \times \cdots \times S_{N_A} \times E$, with $N_A \geq 2$.*
2. *ϕ is of the form:*

$$\phi(\omega, s_1, \cdots, s_{N_A}, e) = (\phi_1(\omega, s_1, e), \cdots, \phi_{N_A}(\omega, s_{N_A}, e), \psi(\omega, s_1, \cdots, s_{N_A}, e))$$

Each of the S_n is interpreted as the state space of a particular agent, N_A being the number of agents. The set E, which we shall call the environment, is a container for the variables that are of concern for more than one agent, for example the physical state of the outside world, the communications between agents, a schedule of the actions to be performed.

The main feature of this type of dynamics is that, because the state-space of an agent is shielded away from individual transitions performed by other agents, any transformation which involves the state of more than one agent (e.g a flow between two agents) has to channel through E. In practice, this will allow (or even force) one to identify explicitly the carrier of interactions among the state variables of E.

This definition can be linked to more informal tentatives in the literature such as this of (Franklin and Graesser 1997): "An autonomous agent is a system situated within and a part of an environment that senses that environment and acts on it, over time, in pursuit of its own agenda and so as to effect what it senses in the future."

2.2 Intertemporal Economies

In order to operationalize this definition in the context of economic growth, we shall restrict attention to the following class of intertemporal economic frameworks.

We consider models with a finite number of goods G (among which labor), countable periods of time indexed by $t \in \mathbb{N}$, a finite number of firms indexed by $j = 1 \cdots N_F$ and a finite number of households indexed by $h = 1 \cdots N_H$.

The production process is one period long so that a basic production plan for firm j has the form $(y_j^t, z_j^{t+1}) \in \mathbb{R}_-^G \times \mathbb{R}_+^G$ where y_j^t is the vector of inputs used in period t and z_j^{t+1} the vector of outputs delivered in period $t+1$.

A consumption plan for household h in period t is represented by a vector $x_h^t \in \mathbb{R}^G$ (where the positive coordinates correspond to the goods consumed and the negative ones to the goods (labor) supplied).

The constraints on technically feasible production plans in period t might depend on the history of economic activity. For sake of generality, we represent those technical possibilities by a correspondence

$$Y_j^t : ((\mathbb{R}^G \times \mathbb{R}^G)^{N_F} \times (\mathbb{R}^G)^{N_H})^t \to \mathbb{R}_-^G \times \mathbb{R}_+^G \tag{2}$$

which associates to the economic history up to time t,[2]

$$\eta^t := ((y_j^\tau, z_j^\tau)_{j=1\cdots N_F}, (x_h^\tau)_{i=1\cdots N_H})^{\tau=0\cdots t-1}, \tag{3}$$

the set of production possibilities of firm j in period t, $Y_j^t(\eta^t)$.

In a similar manner, consumption possibilities and labor supply might depend on the history of economic activity. We therefore represent the "consumption" possibilities (including labor supply) of household h by a correspondence

$$X_h^t : ((\mathbb{R}^G \times \mathbb{R}^G)^{N_F} \times (\mathbb{R}^G)^{N_H})^t \to \mathbb{R}^G \tag{4}$$

which associates to the economic history up to time t, the set of consumption possibilities of household h in period t, $X_h^t(\eta^t)$.

[2] With a slight abuse of notation as "history up to time 0" is not defined. Hence, Y_j^0 and X_h^0 are sets rather than correspondences.

We shall denote by $\mathcal{E}(X, Y)$ the economy defined by the preceding constraints. Those constraints define a notion of feasible paths for the economy: the sequences of production and consumption plans which can be implemented from a vector of initial stocks $(z_j^0)_{j=1\cdots N_F} \in (\mathbb{R}_+^G)^{N_F}$ without further input of goods across time.

Definition 2 *Given initial stocks $(z_j^0)_{j=1\cdots N_F}$, a sequence of production and consumption plans $((y_j^t, z_j^{t+1})_{j=1\cdots N}, (x_h^t)_{i=1\cdots N_H})^{t\in\mathbb{N}}$ is feasible in the economy $\mathcal{E}(X, Y)$ if for all $t \in \mathbb{N}$:*

1. $(y_j^t, z_j^{t+1}) \in Y_j^t(\eta^t)$, *for all* $j = 1 \cdots N_F$
2. $x_h^t \in X_h^t(\eta^t)$, *for all* $i = 1 \cdots N_H$
3. $\sum_{i=1}^M x_h^t + \sum_{j=1}^N y_j^t \leq \sum_{j=1}^N z_j^t$.

2.3 Agent-Based Dynamics in an intertemporal economy

If agent-based dynamics have to be defined on the "physical" framework given by $\mathcal{E}(X, Y)$, it seems natural to require that among the agents considered, there are at least N_F firms and N_H households. Moreover the equivalent of production and consumption plans should be identifiable among the state spaces of these agents and follow, in the course of a simulation, a feasible path.

It is however not necessary that the set of agents be restricted to these two kinds. Indeed, a precise representation of interactions (e.g trading) and the consideration of complementary phenomena (e.g the use of money) might require the introduction of a richer set of agents. Hence, to define a link between an abstract agent-based dynamic (X, Ω, ϕ) and a physical economic framework $\mathcal{E}(X, Y)$, we shall first single out firms and households as agents, second identify the equivalent of production and consumption plans among their respective state space and finally ensure those follow feasible paths (up to time-rescaling). This leads us to the following definition.

Definition 3 *The agent-based dynamics $(S_1 \times \cdots \times S_{N_A} \times E, \phi)$ are said to emulate the growth model $\mathcal{E}(X, Y)$ if $N_A \geq N_F + N_H$ and if (up to a renumbering of the S_n):*

1. *there exist retractions*[3]*:*
   - $\bar{y}_j : S_j \to \mathbb{R}^G$, *for all* $j = 1 \cdots N_F$,
   - $\bar{z}_j : S_j \to \mathbb{R}^G$ *for all* $j = 1 \cdots N_F$,
   - $\bar{x}_h : S_{N_{F+i}} \to \mathbb{R}^G$ *for all* $i = 1 \cdots N_H$,

[3] We recall that $f : A \to B$ is a retraction if there exists $g : B \to A$ such that $f \circ g = Id_B$

2. *for any* $(z_j^0) \in (\mathbb{R}^G)^{N_F}$, $s^0 \in \bar{z}^{-1}(\{z^0\})$, *and any random trajectory* $(\tilde{\omega}) \in (\Omega)^{\mathbb{N}}$ *there exists an increasing sequence* $(\tau_t)_{t\in\mathbb{N}}$ *such that*[4]*:*

   - $y_j^t = \bar{y}_j(\phi^{\tau_t}(\tilde{\omega}, s_0))$,
   - $z_j^{t+1} = \bar{z}_j(\phi^{\tau_{t+1}}(\tilde{\omega}, s_0))$,
   - $x_h^t = \bar{x}_h(\phi^{\tau_t}(\tilde{\omega}, s_0))$

   is a feasible path from (z_j^0) *in the sense of Definition 1.*

3 The Model

3.1 The Economic Framework

Investigating agent-based dynamics in the generic structure introduced in 2.2 might be overambitious for a first attempt. We shall introduce explicit agent-based dynamics in a restricted economic framework, however sufficient to obtain accurate descriptions of the economic activity.

We consider, following the statistical practice, an economy divided in C sectors, each producing a particular kind of output. These outputs can be turned into fixed capital, used for consumption or as circulating capital, stored as inventory. In order to be consistent with the *Arrow-Debreu like* framework introduced in the preceding section, we have to distinguish $G = 2C+1$ goods in every period corresponding to labor, C different kinds of old capital stock and C different kinds of output.

The goods' transformation process handled by firms has five components:

- transfer of output from period to period at depreciation rates $\delta_i \in [0,1]^C$,
- transfer of fixed capital from period to period at depreciation rates $\delta_c \in [0,1]^C$,
- instantaneous transformation of output in fixed capital,
- production of new output from labor, fixed capital and output used as circulating capital, according to a firm-specific production function which describes efficient technologies:

$$f_j : \mathbb{R}_+ \times \mathbb{R}_+^C \times \mathbb{R}_+^C \to \mathbb{R}_+^C \tag{5}$$

- free-disposability of goods.

[4]We use implicitly the following recursive notation $\phi^1(\tilde{\omega}, s) = \phi(\tilde{\omega}_1, s)$ and for all $\tau \geq 1$ $\phi^{\tau+1}(\tilde{\omega}, s) = \phi(\tilde{\omega}_{\tau+1}, \phi^\tau(\tilde{\omega}, s))$

The sets of *efficient* period to period production possibilities are given by[5]:

$$\begin{gathered}\partial Y_j^t = \{(-l, -k', -q),\\ (0, (1-\delta_k) \otimes (k+k'), f_j(l, c, k+k') + (1-\delta_i) \otimes (q-c-k))\\ \mid q \geq c+k\}\end{gathered} \tag{6}$$

where $l \in \mathbb{R}_+$ is the workforce used in the production process, $k' \in \mathbb{R}_+^C$ the stock of fixed capital initially held, $q \in \mathbb{R}_+^C$ the stock of output initially held, $k \in \mathbb{R}_+^C$ the quantity of output turned into fixed capital and $c \in \mathbb{R}_+^C$ the quantity of output used as circulating capital.

Adding free-disposability, the actual sets of production possibilities are:

$$Y_j^t = \partial Y_j^t - (\mathbb{R}_+ \times \mathbb{R}_+^C \times \mathbb{R}_+^C) \times (\mathbb{R}_+ \times \mathbb{R}_+^C \times \mathbb{R}_+^C) \tag{7}$$

One can remark that production possibilities hence defined are constant across time. Technological change will however be introduced in the dynamic by letting firms discover progressively their production possibilities.

Another potential source of technological change, which is also the only possible source of economic growth in our framework, is the evolution of the labor supply. Although, we consider only one dimension for labor, the evolution of the upper bound $l_h^t \in \mathbb{R}_+$ on the labor capacity of household h can be seen as a proxy for the evolution of its human capital or for the growth of the population. This evolution might be triggered by learning by doing, external effects related to investment or imitation of co-workers. For sake of generality, we shall consider that the labor capacity of household h is a function $g_h^t(\eta^t)$ of the economic history up to time t, η^t defined above. As the household only consumes positive quantities of outputs (and no fixed capital), its consumption possibilities are then given by:

$$X_h^t(\eta^t) = [-g_h^t(\eta^t), 0] \times \{0\} \times \mathbb{R}_+^C \tag{8}$$

Feasible paths associated to initial inventories $(i_j^0)_{j=1\cdots N_F}$ and fixed capital stocks $(k_j^0)_{j=1\cdots N_F}$, are then defined by the following dynamic inequations [6].

$$\sum_{j=1}^{N_F} i_j^{t+1} + c_j^{t+1} + (k_j^{t+1} - (1-\delta_c)k_j^t)^+ + \sum_{i=1}^{N_H} x_h^t \leq \sum_{j=1}^{N_F} (1-\delta_i) i_j^t + f_j(c_j^t, k_j^t, l_j^t). \tag{9}$$

$$\sum_{j=1}^{N_F} l_j^t \leq \sum_{i=1}^{N_H} g_h^t(\eta^t), \tag{10}$$

where $c_j^t \in \mathbb{R}_+^C$ is the circulating capital and $l_j^t \in \mathbb{R}_+$ the workforce used by firm j in period t, $k_j^t \in \mathbb{R}_+^C$ the fixed capital stock and $i_j^t \in \mathbb{R}_+^C$ the inventory of firm j in period t, $l_h^t \in \mathbb{R}_+$ is the labor capacity and $x_h^t \in \mathbb{R}_+^C$ the consumption of household h in period t.

[5] The symbol $\otimes$ denotes multiplication coordinatewise.

[6] x^+ denotes the vector whose ith coordinate is given by $max(x_i, 0)$

To conclude this presentation of the economic framework, let us point out that when one only considers a single consumer as well as a single sector with a single firm, the framework specializes to the standard objects of growth theory. The form of the growth function for the labor capacity and of the production function determine the corresponding type of growth model.

Solow Growth Model First, let us consider the case where:

- the firm uses as only inputs labor and fixed capital, that is has a production function of the form $f : \mathbb{R}_+^2 \to \mathbb{R}_+$, associating to a level of fixed capital $k \in \mathbb{R}_+$ and to a labor input $l \in \mathbb{R}_+$, the level of output $f(k, l)$,
- the depreciation rate is zero,
- the Labor capacity grows at an exogenously given rate $n \in \mathbb{R}_+$, that is one has $g(l^t) = (1 + n)l^t$.

Then, if the whole labor capacity is supplied inelastically and that a constant fraction $s \in [0, 1]$ of output is invested, one obtains a discrete version of the Solow growth model (Solow 1956) of capital accumulation:

$$k^{t+1} - k^t = sf(k^t, (1 + n)l^t) \tag{11}$$

AK Model Second, let us consider the case where:

- the production function is of the Leontieff type, that is one has : $f(k, l) = \min(\frac{k}{\kappa}, \frac{l}{\lambda})$ with $\kappa, \lambda > 0$,
- the depreciation rate for fixed capital is $\delta \in (0, 1)$,
- The labor capactiy increases proportionally to net investment [7] , that is one has: $l^{t+1} = l^t \frac{k^t}{k^{t-1}}$.

Then, if the whole labor capacity is supplied inelastically and that a constant fraction $s \in [0, 1]$ of output is invested one obtains, after elimination of labor in the equations, the AK model of capital accumulation:

$$k^{t+1} = Ak^t \tag{12}$$

where $A = \frac{s}{\kappa} - \delta$

[7] E.g. through learning by doing as in (Arrow 1962)

3.2 A Proposal For Agent-Based Dynamics

3.2.1 Structure of the dynamics

We explore, in the economic framework described above, the dynamics generated by the interactions between a population of agents consisting in a set of firms and households, a government and a financial system. That is, in accordance with definition 1, we consider a random dynamical system on a set X with the following characteristics [8].

- The set X is defined as the cartesian product of the state spaces S_F of N_F firms, S_H of N_H households, S_G of a government, S_{FS} of a financial system and of an environment E, that is :

$$X = S_F^{N_F} \times S_H^{N_H} \times S_G \times S_{FS} \times E \tag{13}$$

- The dynamics are defined on the basis of a discrete schedule of events, sch, stored in the environment. It consists in a list[9] of pairs of action and identity. An identity is a natural number n in $\{1, \cdots, N_F + N_H + 2\}$ to which is associated unambiguously an agent in the population whose type we denote by $\mathcal{T}(n) \in \{S, H, FS, G\}$. An action a is an element of a finite set A (e.g of names or of natural numbers) which indexes a family of transition functions on the state spaces of agents. That is for every $a \in A$ and every $\mathcal{T} \in \{S, H, FS, G\}$ is defined a random transition function

$$f_{a,\mathcal{T}} : \Omega \times S_{\mathcal{T}} \times E \to S_{\mathcal{T}} \times E \tag{14}$$

  which corresponds to the performance of action a by an agent of type $\mathcal{T}$.

  The computation launches sequentially individual state transitions on the basis of the state of the schedule. Namely, when the first element of the schedule is the pair (a, n), an element ω is drawn randomly in $(\Omega, \mathcal{F}, P)$ and the transition $f_{a,\mathcal{T}(n)}(\omega, \cdot)$ is applied to the state of agent n.

  Denoting by s_n the current state of agent n, one can then write symbolically the complete dynamics on X using the following algorithm[10]:

  repeat

$$\begin{aligned} (a, n) &:= \mathit{head}\ \mathit{sch} \\ \omega &:= \mathit{randomize}\ (\Omega, P) \\ (s_n, e) &:= f_{a,\mathcal{T}(n)}(\omega, s_n, e) \end{aligned} \tag{15}$$

  until $sch = []$

[8] The associated probability space $(\Omega, \mathcal{F}, P)$ is left unspecified, we shall denote by ω a generic element of Ω

[9] Definition and Notation: a list of elements of a set X is a finite sequence of elements of X. we denote by $[X]$ the set of such lists and by $[x]$ a generic element of $[X]$. The symbol *head* $[x]$ denotes the first element of the list $[x]$, the symbol $[]$ denotes the empty list, i.e a list which contains no element.

[10] $randomize(\Omega, P)$ stands for the random drawing of an element in Ω according to the probability distribution P. The condition $sch = []$ states that the schedule sch is empty, i.e that there is nor more action to be performed.

Such dynamics fit into the framework of definition 1 by setting $\phi_n(\cdots, (a, n))$ equal to the projection on S_n of $f_{a,\mathcal{T}(n)}$ and $\psi(\cdots, (a, n))$ equal to the projection of $f_{a,\mathcal{T}(n)}$ on E, while $\phi_\nu(\cdots, (a, n))$ is equal to the identity on S_ν for $\nu \neq n$.

Remark 1 *There is a certain degree of arbitrariness in the mathematical exposition of a computer program. The actual implementation of the model is much more parallelized than what the above description suggests, although it requires certain actions to take place sequentially. The above description has been chosen for sake of uniformity.*

3.2.2 Structure of the economic process

At the aggregate level, the schedule structures the dynamics in steps of distinct periodicities. This corresponds to a partition of the economic activity in processes of different natures evolving along different time-scales. In the current version of the model, we consider:

- a core economic cycle consisting in production, consumption, trading, accounting and beliefs' updating,
- a labor market step,
- a financial updating step,
- a genetic evolution step.

These steps have increasing periodicity, corresponding to the different time-scales of evolution of stocks, prices, labor contracts, interest rates and technologies. Each can be seen as a random transition of the form $\Omega \times X \to X$ where $(\Omega, \mathcal{F}, P)$ is a well-chosen probability space. If one then considers the space of random trajectories $\tilde{\Omega} = \Omega^{\mathbb{N}}$, the complete dynamics can be seen as a stochastic process $\Gamma : \tilde{\Omega} \times \mathbb{N} \to X$ which satisfies the symbolic equation:

$$\Gamma_{\tilde{\omega}}^{t+T} = \mathcal{G}_{\tilde{\omega}} \circ (\mathcal{F}_{\tilde{\omega}} \circ (\mathcal{L}_{\tilde{\omega}} \circ (\mathcal{B}_{\tilde{\omega}})^{T_{\mathcal{L}}})^{T_{\mathcal{F}/\mathcal{L}}})^{T_{\mathcal{G}/\mathcal{F}}} \left[\Gamma_{\tilde{\omega}}^{t}\right] \tag{16}$$

where

- $\mathcal{B}_{\tilde{\omega}}$ is the Core economic cycle
- $\mathcal{L}_{\tilde{\omega}}$ is the labor market step and $T_{\mathcal{L}}$ its periodicity with regards to the core cycle,
- $\mathcal{F}_{\tilde{\omega}}$ is the financial updating step and $T_{\mathcal{F}/\mathcal{L}}$ its periodicity with regards to the labor step,
- $\mathcal{G}_{\tilde{\omega}}$ is the genetic evolution step and $T_{\mathcal{G}/\mathcal{F}}$ its periodicity with regards to the financial step.
- $T = T_{\mathcal{L}} \times T_{\mathcal{F}/\mathcal{L}} \times T_{\mathcal{G}/\mathcal{F}}$ is the periodicity of a complete dynamic cycle.

3.2.3 Agents' description

In a nutshell, firms are coarse profit maximizers and monopolistic price setters, households are wage earners and, also coarse, utility maximizers, the financial system sets the interest rate, collects savings from households and lends money to firms, the government is responsible for providing an unemployment insurance.

Let us point out a priori that the state space of firms contain, among others, stocks of fixed capital, inventories, circulating capital and workforce while this of households contain labor capacity and quantities consumed. This ensures the first part of definition 3 is satisfied. Together with equations (17) to (21),(24) and (37) defined below, this will imply that the agent-based dynamics introduced below are, in the sense of definition 3, compatible with the economic framework introduced in 3.1.

Core Economic Cycle The core economic cycle consists in good's production, consumption and trading, accounting and beliefs' updating operations:

- *Production:* each firm[11] produces the maximal possible quantity given its stock of fixed capital $k \in \mathbb{R}_+^C$, its stock of circulating capital $c \in \mathbb{R}_+^C$, and its workforce $l \in \mathbb{R}_+$. Production takes place according to the current technology of the firm (see the genetic step for its evolution) specified by input coefficients $\kappa \in \mathbb{R}_+^C$ for fixed capital, $\gamma \in \mathbb{R}_+^C$ for circulating capital and $\lambda \in \mathbb{R}_+$ for labor, so that the actual production $q \in \mathbb{R}_+$ satisfies:

  $$q = min(\frac{k}{\kappa}, \frac{c}{\gamma}, \frac{l}{\lambda}) \leq f(k, c, l) \tag{17}$$

  Moreover, during the production process, circulating capital is consumed while fixed capital (resp. inventory) is depreciated at rate δ_k (resp. δ_i), so that the variation $\Delta c \in \mathbb{R}^C$ of circulating capital, $\Delta k \in \mathbb{R}^C$ of fixed capital and $\Delta i \in \mathbb{R}$ of the inventory during the production process are given by

  $$\Delta k = -\delta \otimes \kappa q \tag{18}$$

  $$\Delta c = -\gamma q \tag{19}$$

  $$\Delta i = q - \delta_i \otimes i \tag{20}$$

  where q is given by equation 17.

- *Consumption:* each household consumes its whole stock of goods. That is the variation $\Delta x \in \mathbb{R}^C$ of the stock of goods $x \in \mathbb{R}^C$ is given by

  $$\Delta x = -x \tag{21}$$

[11] The subscript j is omitted in the following

- *Trading:* in a random order each firm and household observes the stocks and prices of a random sample of suppliers, determines its demand, addresses it to the suppliers it has observed starting with the cheapest, and is delivered according to availability.

  - The demand of a firm is entirely determined by its target production $\bar{q} \in \mathbb{R}_+$ (see below), its current stock of circulating and fixed capital, and its production technology:

$$d_{firm} = (\bar{q}\kappa - k)^+ + (\bar{q}\gamma - c)^+ \tag{22}$$

  - The demand of an household is a function of its money holdings $m \in \mathbb{R}_+$, its current consumption technology $\chi \in \mathbb{R}_+^C$ and of average prices observed $p \in \mathbb{R}_+^C$:

$$d_{household} = \frac{m}{p \cdot \chi}\chi \tag{23}$$

  An important feature of the trading step is conservation of quantities:

$$\Delta i = \Delta x + \Delta k + \Delta c \tag{24}$$

  where Δi (resp. Δx, Δk, Δc,) is the variation of inventory (resp. households' stocks, fixed capital, circulating capital).

  Although each trade is accompanied by the corresponding money transfer, it is not true that the total quantity of money is also conserved as firms may run deficit (if they are still in deficit at the end of the trading step, they will have to subscribe a debt towards the financial system during the accounting operations (see below)).

- *Accounting:*

  - Each firm pays $w \in \mathbb{R}_+$ in wages to its workers, and interests $\rho d \in \mathbb{R}_+$ on its debt $d \in \mathbb{R}_+$ to the financial system at the prevailing interest rate $\rho \in [0, 1]$. It then updates its profit[12]

$$\Delta\pi = -w - \rho d \tag{25}$$

    It then allocates its profits between a share σ_{div} towards distribution of a dividend, a share σ_{debt} towards the reimbursement of the principal of its debt and a share σ_{money} towards the increase of its money holdings $m \in \mathbb{R}_+$. This yields:

$$div = \sigma_{div} \times \pi \tag{26}$$

[12] Which also takes into account the trading's operations

$$\Delta d = -\sigma_{debt} \times \pi \tag{27}$$

$$\Delta m = \sigma_{money} \times \pi \tag{28}$$

where $\sigma_{money} + \sigma_{debt} + \sigma_{div} = 1$

If it is in deficit (i.e its money holdings $m \in \mathbb{R}_+$ are negative) , it subscribes a debt towards the financial system in order to balance its budget:

$$\Delta d = -m \qquad \text{if } m < 0 \tag{29}$$

– Each Household receives its income $r \in \mathbb{R}_+$:

$$r = w + div + \rho s \tag{30}$$

where $w \in \mathbb{R}_+$ are wages (or unemployment insurance), $div \in \mathbb{R}_+$ dividends from the firms it owns , and $\rho s \in \mathbb{R}_+$ interests on its savings $s \in \mathbb{R}_+$ at the prevailing interest rate $\rho \in [0, 1]$.

– Part of this income is taxed by the government at the rate necessary to pay for unemployment insurance:

$$\tau = \frac{\upsilon\alpha \cdot \tilde{w}}{R} \tag{31}$$

where $\upsilon \in [0, 1]$ is the unemployment rate, $\tilde{w} \in \mathbb{R}_+$ the average wage, $\alpha \in [0, 1]$ the rate of unemployment insurance and $R \in \mathbb{R}_+$ the aggregate income.

– Each household then determines the amount it intends to spend on consumption using Deaton rule of thumb: it saves (at the rate $\sigma \in [0, 1],$) or unsaves part of the difference between its actual income and its expected one $\hat{r} \in \mathbb{R}$. This yields the following variations $\Delta m \in \mathbb{R}$ of its money holdings and $\Delta s \in \mathbb{R}$ of its savings.

$$\Delta m = \begin{cases} \min(\hat{r}, r + s) \text{ , if } r < \hat{r} \\ \hat{r} + (1 - \sigma)(r - \hat{r}) \text{ , if } r > \hat{r} \end{cases} \tag{32}$$

$$\Delta s = r - \Delta m \tag{33}$$

- *Beliefs' Updating*:

– Each firm determines its target production for the next period taking into consideration the demand it has faced $q^d \in \mathbb{R}_+$, its inventory, its stock of fixed capital, and the profit it has made:

$$\Delta \bar{q} = dq(\pi, i, k, q^d, \bar{q}) \tag{34}$$

where dq is a step function increasing with respect to all the variables (but the inventory). The price is updated in a similar manner.

- The household updates its expected income and its reservation wage $w^* \in \mathbb{R}_+$ (see labor market step below) as functions of its current income (resp wage) and of the inflation rate $\iota \in \mathbb{R}_+$.

$$\Delta \hat{r} = (1+\iota)(\mu r + (1-\mu)\hat{r}) - \hat{r} \tag{35}$$

$$\Delta w_h^* = (1+\iota)(\mu w + (1-\mu)w^*) - w^* \tag{36}$$

  where $\mu \in [0,1]$ is the rate of belief's evolution.

- Finally, the labor capacity l of an household evolves according to the economic history up to period t, represented by η^t

$$l^t = g(\eta^t) \tag{37}$$

Labor Market The employment relations are recontracted on the middle-term, during the Labor Market Step. The negotiation of employment relations are based on work contracts, which specify a unit wage and a quantity of labor. The labor market operates according to the following algorithm (households acting first, firms second, the sequence of operations being otherwise random):

- Each household has a reservation wage w^*. It first check if this fallback is higher than its current wage, in which case it quits his job. It then browses a random sample of firms and switches jobs if it finds a firm which is looking for employees and offers a wage higher than its actual one (and than its fallback).

- Each firm has a target employment $l^* \in \mathbb{R}_+$ which corresponds to the workforce it can actually put to work given its capital stock:

$$l^* = \lambda \frac{k}{\kappa} \tag{38}$$

  If its actual workforce is greater than this target employment, the firm fires the corresponding number of workers starting with the less productive.

  Otherwise, it repeats until it has reached its target employment the following operations:

  - Propose a working contract to a sample of unemployed workers (the offer is accepted if the wage offered is higher than the worker's fallback)
  - Propose a working contract to a sample of employed workers (the offer is accepted if the wage offered is higher than the current wage plus a switching cost)
  - Increase the offered wage.

Updating of the interest rate On the middle-term also, the financial system updates the interest rate according to the Tayor rule. It observes the inflation rate $\iota \in [0,1]$ and the unemployment rate $\upsilon \in [0,1]$ and sets the interest rate $\rho \in [0,1]$ according to:

$$r = \rho^* + \iota^* + \phi_\iota(\iota - \iota^*) + \phi_\upsilon(\upsilon^* - \upsilon) \tag{39}$$

where $\rho^* \in [0,1]$ is the "natural interest rate," $\iota^* \in [0,1]$ the target inflation, $\upsilon^* \in [0,1]$ the target unemployment rate, ϕ_ι and ϕ_υ adjustment coefficients for inflation and unemployment respectively.

Genetic Evolution On the long term, technologies and prices evolve genetically according to the profitability of firms:

- *Entry and exit:* in every sector where average profit is negative, the less profitable firms are shut down. Meanwhile, in every sector where average profit is positive, new firms are activated. Those new firms are initialized with the characteristics of the most profitable firms of the sector and are endowed with financial capitals provided by the savings of some of the richest households who then become the owners of the firms.
- *Imitation:* the less profitable firms of a sector copy the operating characteristics (technology, wages, price) of the most profitable ones.
- *Mutation:* Operating characteristics of the firms randomly mutate:
  - Price and Wages variate over a fixed range:

    $$\Delta w = \mu_w(\omega) dw \tag{40}$$

    $$\Delta p = \mu_p(\omega) dp \tag{41}$$

    where ω represents symbolically a random drawing, $\epsilon > 0$ is the mutation rate, dw (resp dp) are variation ranges for the wages (resp. price) and μ_p and μ_w are random variables such that $P(\mu = 1) = P(\mu = -1) = \epsilon$ and $P(\mu = 0) = 1 - 2\epsilon$.
  - The production technique variates randomly along an isoline of the production function:

    $$\Delta(\gamma, \kappa, \lambda) = (\mu_\gamma(\omega), \mu_\kappa(\omega), \mu_\lambda(\omega)) \tag{42}$$

    $$\text{with} \quad f(\gamma, \kappa, \lambda) = f(\gamma + \mu_\gamma(\omega), \kappa + \mu_\kappa(\omega), \lambda + \mu_\lambda(\omega)), \tag{43}$$

    where given a mutation rate $\epsilon > 0$ and a variation range for technologies $d\theta \in \mathbb{R}_+$, the $(\mu_\gamma, \mu_\kappa, \mu_\lambda)$ are random variables such that $P(\mu_\gamma(\omega), \mu_\kappa(\omega), \mu_\lambda(\omega) = 0) = 1 - \epsilon$ and $\|\mu_\gamma(\omega), \mu_\kappa(\omega), \mu_\lambda(\omega))\| \leq d\theta$ (ω represents symbolically a random drawing).

4 Results of simulations on the German economy

In a first round of simulations, we have tested these agent-based dynamics on a three-sector partition (industry, services, energy) of the German economy with two hundred firms and a thousand households. The main characteristic of the economic framework (cf section 3.1) is that the labor capacity increases at the same rate as net investment (as in (Arrow 1962)[13]). This external effect of investment on labour is the source of the economic growth apparent in our experiment. It is also worth noting that we assume the production functions (cf equation (5)) are linear.

Part of the model has been initialized (at the base year 1978) using data provided by the German Statistical Institute:

- Input-output tables, data on the state of the capital stocks and on the workers' distribution allow us to infer the initial technology of firms (cf equation (17)).
- Capital stocks depreciation rates are used as such (cf equation (18)),
- Total production determines the initial target production of individual firms (cf equation (34)),
- Final consumption determines initial consumption coefficients (cf equation (23)).
- Workers' distribution among sectors and wages determine initial employment relations (see the labor market step).
- Monetary holdings, savings and debts are allocated among firms and households.

The tuning parameters of the model are on the one hand those related to the genetic step (e.g mutation rates and ranges, imitation rates). In particular, a bias has been introduced in the mutation of technologies towards substitution of industrial inputs by services. On the other hand, parameters related to the agents' decision rules:

- For the financial system, the parameters of the Taylor rule (cf equation (39)).
- For the firms, the target production function (cf equation (34)), the profit allocation rule (cf equations (27), (28) and (3.2.3)).
- For the households, the rate of belief's evolution (cf equations (35) and (36)) and the saving rate (cf equation (32)).

[13]See also references therein

Finally to introduce some heterogeneity among agents, an initial randomization takes place at the individual level.

The following figures present time-series for the main economic variables generated by a typical simulation in front of their real counterpart. Though not quantitatively sound[14], our results remain of the right order of magnitudes and have roughly the same qualitative properties as empirical phenomena. One also recognizes the characteristic non-smoothness present in real data.

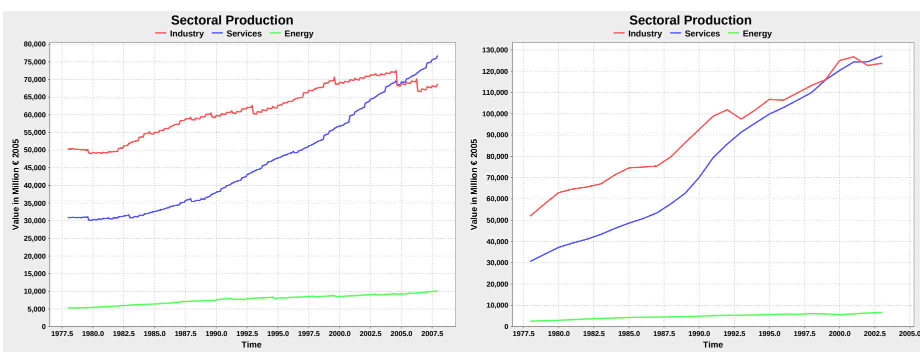
Figure (1) shows the monthly production among sectors. One observes, in both the simulation and empirical data, a shift from industry to services. The actual growth rate is however higher than the one obtained in the simulations.

Figure (2) shows the evolution of the unemployment rate. One might distinguish in the simulations the kind of cyclical behavior real data show. Values are fairly similar in both cases.

Figure (3) represents the evolution of money holdings for the simulation and the actual evolution of M1. Both have the same exponential dynamics with a growth rate of approximately five percents.

Figure (4) shows the evolution of prices. It is clear that the actual evolution of energy prices has been driven by exogenous factors. For industry and services, both the simulation and the data show approximately constant relative prices. The inflation rate is however much too low in the simulations.

Finally, figure (5) shows the dynamics of wages. The overall growth rate is similar in both cases whereas the qualitative properties of the dynamics seem quite different in both cases.

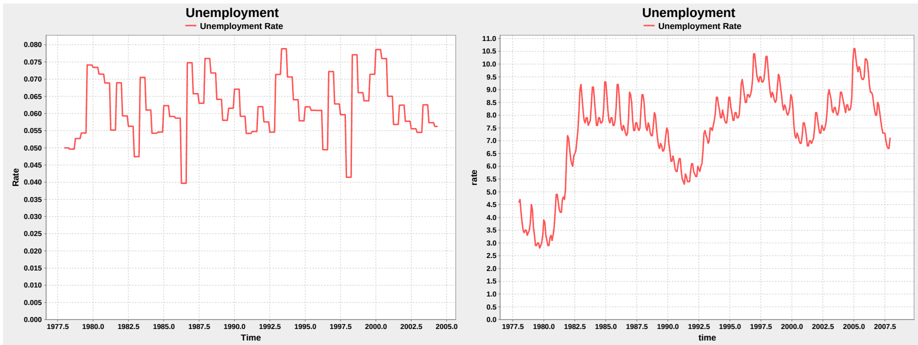


(a) Simulation (monthly values)

(b) Real data (yearly values)

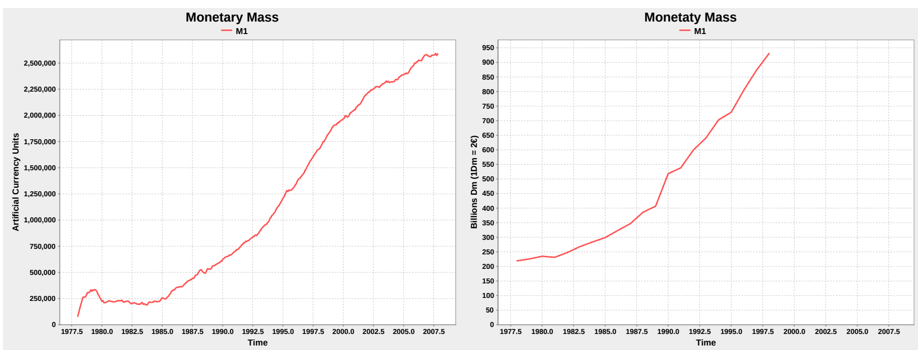
Figure 1: Sectoral Production

[14]Something we could not except in a model which has neither foreign trade nor governmental policy and without huge calibration efforts



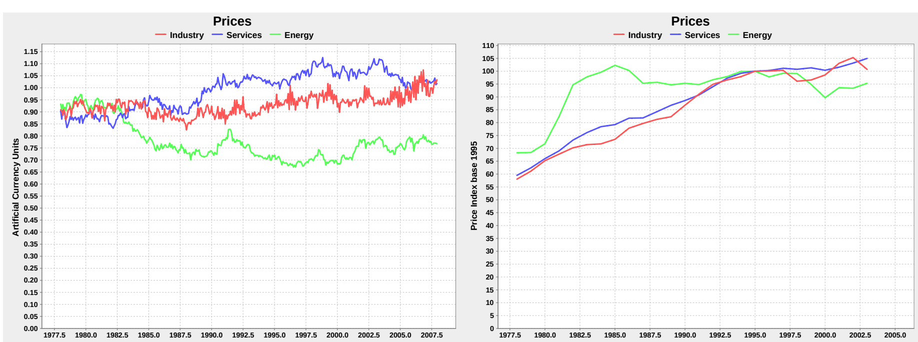
(a) Simulation (b) Real data

Figure 2: Unemployment Rate



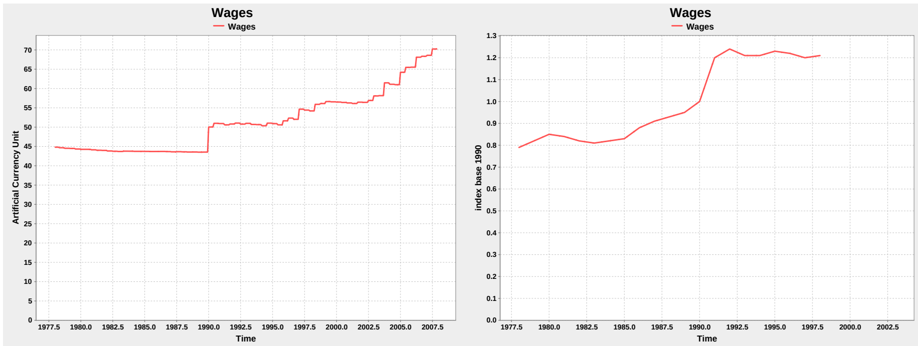
(a) Simulation (b) Real data

Figure 3: Monetary Aggregate



(a) Simulation (b) Real data

Figure 4: Prices



(a) Simulation (average wage)

(b) Real data (wage Index)

Figure 5: Income

5 Concluding Remarks

The present contribution aims at illustrating a few facts on the use of agent-based models in economics. First, agent-based dynamics could be rigorously defined within the seminal framework of analysis of economic systems devised by Arrow and Debreu in the 1950s (see (Debreu 1959)). They might then help explore unknown areas of this map, typically out-of-equilibrium paths. This has been achieved by Gintis in (Gintis 2006) and (Gintis 2007) on the issue of equilibrium selection in relatively simple cases. We hope our work might prove useful to obtain similar insights in more complex frameworks such as (Benhabib and al. 2000).

Second, agent-based dynamics might prove a way to reproduce naively, but accurately, empirical facts: sound choices for the representation of the behavior of micro-economic entities might lead to the emergence of accurate macro-economic properties. The present tentative is in this respect much incomplete, further insight has to be gained on the representation of agent's decision rules, additional agents (e.g banks) have to be introduced, other phenomena (e.g imports and exports, environmental feedbacks) have to be encompassed. We are currently working in this direction. However, in this respect, agent-based simulations should not be seen as models of economic reality, but as tools that might be used to derive models or to test policy scenarios. The corresponding mathematics might well be different than those of Arrow and Debreu, and more akin to those of (Peyton-Young 1993) and (Freidlin and Wentzell 1984).

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