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How changing sea level extremes and protection measures alter coastal flood damages

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[1] While sea level rise is one of the most likely consequences of climate change, the provoked costs remain highly uncertain. Based on a block-maxima approach, we provide a stochastic framework to estimate the increase of expected damages with sea level rise as well as with meteorological changes and demonstrate the application to two case studies. In addition, the uncertainty of the damage estimations due to the stochastic nature of extreme events is studied. Starting with the probability distribution of extreme flood levels, we calculate the distribution of implied damages in a specific region employing stage-damage functions. Universal relations of the expected damages and their standard deviation, which demonstrate the importance of the shape of the damage function, are provided. We also calculate how flood protection reduces the damages leading to a more complex picture, where the extreme value behavior plays a fundamental role.

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1. Introduction

[2] In the debate about climate change induced sea level rise, land loss is a commonly mentioned consequence [*Nicholls et al.*, 2010; *Devoy*, 2008]. On a closer look, however, it turns out that land loss itself is often a result of extreme events, such as storm surges, which either erode the coastline [*Stive et al.*, 2002] or inundate the considered area so frequently that a repeated restoration might be inefficient and the land is abandoned.

[3] Considering sea levels, one can observe that the mean sea level is superposed by fluctuations, whose magnitudes significantly surpass the expected mean sea level rise. Accordingly, if one wants to investigate consequences of sea level rise, these fluctuations need to be taken into account. Tides and winds are the main influencing factors of these fluctuations [*Woodworth et al.*, 2011] and together with the mean sea level they determine the magnitude of an extreme event. Thus, extreme floods are influenced by climate change in two ways: via sea level rise and via meteorological changes. We study the consequences of these two effects as well as the impact of potential flood protection measures on the expected flood damages, where damages describe the monetary losses in a specific area. Furthermore, the variability of the damages is examined.

[4] We employ extreme value theory for the characterization of flood events [*Katz*, 2010], and damage functions [*Merz et al.*, 2010], in order to obtain the associated damages. Accordingly, the distribution of extreme sea levels is translated via the damage function into the distribution of damages, i.e., the probability that a damage higher than a certain value occurs is related to the probability that the annual maximum flood exceeds a certain level. In particular, sea level rise, which is the main driver for changing extreme value behavior [*Menéndez and Woodworth*, 2010], leads to modified damages. Additionally, climate change could alter meteorological patterns, which induces a change of variability of extreme events [*McInnes et al.*, 2013; *Woth et al.*, 2006] and in turn affects the damage distribution.

[5] We elaborate this setting in a general sense and analytically derive relations for the expectation value of the damages and the standard deviation as a function of the mean sea level and as a function of the variability of annual maximum sea levels. The resulting expressions describe the asymptotic behavior and highlight the importance of the damage function. We complement the results with an analysis of the effect of a protection measure in the form of a dike or a sea wall protecting the area from floods up to a specific maximum sea level. Again, we derive analytical expressions and find that in this case the expressions depend sensitively on the extreme value behavior of sea levels.

[6] All general results are supported by numerical calculations for the city of Copenhagen and a case study area in Kalundborg (Denmark).

[7] The manuscript is organized as follows. In section 2, our approach connecting extreme sea levels and damage functions is introduced. Information on the two case studies is provided in section 3. Changes in the extremes are presented in section 4 and the influence of protection in section 5. In section 6, we draw conclusions and discuss limitations of our findings. Detailed derivations are provided in the Appendix A.

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2. Methodological Approach

[8] We want to estimate the annual, monetary damages caused by coastal floods. For this purpose, we assume there is one extreme flood per year in the considered region and obtain the corresponding damage costs from a macroscopic damage function, providing the typical damage of a coastal flood of specific magnitude (other contributing factors such as inundation duration or flow velocity [*Wind et al.*, 1999; *Thieken et al.*, 2005; *Middelmann-Fernandes*, 2010] are not taken into account).

[9] Thus, we consider the distribution of annual maximum sea levels and derive the distribution of annual damages by using the damage function. Naturally, if the distribution of maxima changes, the distribution of damages is also altered. This is the starting point of our approach. Once the distribution of damages is known, the expectation value and the standard deviation can be calculated. Moreover, protection measures modify the damage function, e.g., sea walls determine the flood level below which no damage occurs.

[10] The approach is sketched in Figure 1. The distribution of maximum sea levels is illustrated in Figure 1a, the damage function in Figure 1b, and the distribution of damages in Figure 1d. An alternative damage function supposing a protection measure is displayed in Figure 1c, leading to a different damage distribution.

2.1. Extreme Value Statistics

[11] The reason why we base our analysis on extreme value statistics is that in general the distribution of sea levels is unknown. Considering a certain case study, one can analyze the gauge sea level values (if available in high resolution and with sufficient statistics), generate a histogram in order to estimate the distribution and make assumptions about its functional form. Next, one can impose a sea level rise and move the distribution in order to estimate the increase of damages. The problem, however, is that the sea level distributions differ from gauge to gauge and the assumption about the functional form is not transferable. Therefore, such an approach is hardly feasible in practice, and typically, extreme value theory (which is well established in flood frequency analysis and mathematically substantiated) is employed. We follow a widely spread extreme value method—namely, the block maxima approach.

[12] The maxima in blocks of asymptotic length follow the *generalized extreme value* (GEV) distribution [*Coles*, 2001; *Leadbetter et al.*, 1983; *Embrechts et al.*, 1997], characterized by three parameters: μ (location), σ (scale), and ξ (shape). It combines the Gumbel ($\xi = 0$), Fréchet ($\xi > 0$), and Weibull ($\xi < 0$) families and has the following cumulative distribution function:

$$P_{\xi,\sigma,\mu}(x) = \begin{cases} \exp\left[-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-1/\xi}\right] & \text{if } \xi \neq 0, \\ \exp\left[-\exp\left(-\frac{x-\mu}{\sigma}\right)\right] & \text{if } \xi = 0, \end{cases} \tag{1}$$

where the corresponding probability density function is denoted by $p_{\xi,\sigma,\mu}$. In practice, this distribution is used to approximate the distribution of maximum annual water levels [*Hawkes et al.*, 2008], which is in general unknown.

2.2. Damage Functions

[13] Most commonly, a damage function describes the damage to an asset, given a certain inundation depth, either as a percentage damage rate (relative) or as a monetary value (absolute). Several damage models have been proposed in literature, which are typically associated to certain asset types and exhibit different functional forms [*Merz et al.*, 2010].

[14] When one wants to calculate the total damage from a certain flood event, the inundation heights at each asset in the affected region need to be determined (e.g., by hydrodynamic modeling) and the single damages (obtained by small-scale damage functions) are aggregated. We call such a damage function, providing the total damage in a case study region as a function of the sea level, *macroscopic*. The availability of macroscopic damage functions in literature is very scarce and their general form unknown.

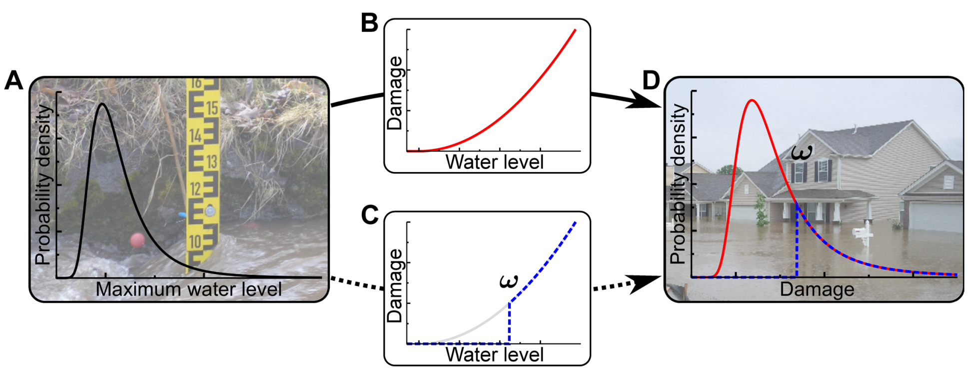

Figure 1. Illustration of the damage model. (a) The distribution of annual maximum water levels transforms via (b), the damage function, into (d), the probability density of damages. (c) An implemented protection measure filters moderate events up to a threshold ω and leads to a modified damage function. In this case, the damage distribution Figure 1d (dashed, blue line) consists of an additional discrete part in 0, which cannot be depicted. (Photographs "Ilmpegel Ilmenau" by M. Sander (2006, http://commons.wikimedia.org/wiki/File:Ilmpegel_Ilmenau.JPG) and "Flooding in Nashville, Tennessee" by E. Hamiter (2010, http://commons.wikimedia.org/wiki/File:Nashville_Flood.jpg) used under a Creative Commons Attribution-ShareAlike license: http://creativecommons.org/licenses/by-sa/3.0/).

Two functions are provided by *Hallegatte et al*. [2011] and *Boettle et al*. [2011], which can be well approximated by power laws, i.e.,

$$F(x) \sim x^{\gamma}, \tag{2}$$

with a damage function exponent $\gamma > 0$ (in order to ensure the existence of the standard deviation of damages, we further need to assume $\gamma < 0.5/\xi$ in the case $\xi > 0$). While in most literature, (stage-)damage functions refer to individual buildings or assets [e.g., *Büchele et al*., 2006; *Merz and Thieken*, 2009; *Dutta et al*., 2003], here they refer to an entire region under consideration. This macroscopic damage function provides an aggregate of the building damages as obtained from microscopic damage functions [*Boettle et al*., 2011], i.e., it gives an estimate of the total monetary damage if case study YZ is affected by a coastal flood of maximum flood level x.

2.3. Computational Calculations

[15] Combining the damage function F with the probability density function $p_{\xi,\sigma,\mu}$ of the GEV distribution, for the expected annual damage holds $\mathrm{E}_D := \int_{-\infty}^{\infty} F(x) p_{\xi,\sigma,\mu}(x)\mathrm{d}x$ and for its standard deviation $\mathrm{STD}_D := \left(\int_{-\infty}^{\infty} (\mathrm{E}_D - F(x))^2 p_{\xi,\sigma,\mu}(x)\mathrm{d}x\right)^{1/2}$. For the numerical calculations, the integrals need to be discretized and the range of integration to be bounded. Here the interval $[x_{\min}, x_{\max}]$ was partitioned by equidistant steps of width Δx, where $x_{\min}$ is the highest water level for which no damage occurs and $x_{\max}$ represents the 100,000 year event (or the highest possible sea level in the Weibull case) at the current conditions. For varying protection heights, $x_{\max}$ was adjusted correspondingly. Naturally, due to computational limitations, a tradeoff between increment size and the highest considered annuality has to be made. We found that taking into account return levels of up to 100,000 years, both sources of error could be kept negligible. The equidistant spacing of the interval $[x_{\min}, x_{\max}]$ with midpoints $x_1, \ldots, x_N$ yields the discrete approximations

$$\mathrm{E}_D \approx \Delta x \sum_{i=1}^{N} F(x_i) p_{\xi,\sigma,\mu}(x_i) \quad \text{and} \tag{3}$$

$$\mathrm{STD}_D \approx \left(\mathrm{E}_D^2 P_{\xi,\sigma,\mu}(x_{\min}) + \Delta x \sum_{i=1}^{N} (\mathrm{E}_D - F(x_i))^2 p_{\xi,\sigma,\mu}(x_i) \right)^{1/2}. \tag{4}$$

Using these equations, the expected annual damage and its variability are calculated for changing parameters in sections 4 and 5.

3. Case Studies

[16] In order to support our theoretical findings, we consider two case studies in Denmark: Copenhagen and Kalundborg. While Copenhagen, the capital, has more than 500,000 inhabitants, Kalundborg is much smaller, and the case study refers to a threatened small population area in the south of the city of Kalundborg. For both regions, a macroscopic damage function is available. In the Copenhagen case, *Hallegatte et al*. [2011] elaborated a curve, which provides the direct losses to buildings and their contents in several sectors as well as infrastructural damages in the absence of protection as a function of flood level. The objective of this study was to assess the economic impacts of climate change and possible benefits of adaptation. For Kalundborg, *Boettle et al*. [2011] derived a damage function, comprising monetary damages to residential buildings in the area and investigated the macroscopic damage function for different elevation model qualities and small-scale damage functions.

[17] The case studies have been chosen because two essentials are available: (i) a record of annual maximum sea levels and (ii) a macroscopic damage function. Any other case study providing both would be equally applicable.

3.1. GEV Parameter Estimation

[18] In order to obtain an estimation of the extreme value parameters for the Copenhagen gauge, a time series of maximum water levels at the gauge in the harbor of Copenhagen between 1890 and 2007 was analyzed. The data set consists of 95 values, which represent the maximum water levels within a hydrological year (October–September). The estimation of extreme value parameters requires the assumption of constant parameters. Although this is not given in practice, removing the mean sea level trend should legitimate the assumption of a stationary location parameter. After adding a linear trend of 0.45 mm per year (derived from mean sea level data, available at http://www.psmsl.org), the GEV parameters $\mu \approx 87.50$, $\sigma \approx 18.98$, and $\xi \approx -0.19$ were obtained as maximum likelihood estimators for censored sample data [*Phien and Fang*, 1989]. This implies a Weibull distribution of annual peak values (in agreement with *Hallegatte et al*. [2011]) with a maximum water level of approximately 187 cm.

[19] The analogous analysis of 32 maximum water levels at the gauge in Kalundborg between 1971 and 2006 combined with mean sea level data from the Korsør gauge close to Kalundborg provided estimates for the GEV parameters, again using a maximum likelihood estimation for censored sample data [*Phien and Fang*, 1989]. The values $\mu \approx 91.30$ (location), $\sigma \approx 16.96$ (scale), and $\xi \approx 0.00$ (shape) were obtained, implying a Gumbel distribution, which is unbounded on both sides. Due to the small sample size, these estimates cannot be considered as reliable. Nevertheless, we will proceed with these estimates since more extensive data were not available and the exact parameters are not crucial for our purpose.

3.2. Extrapolation of Damage Functions

[20] The damage functions of both case studies are macroscopic damage functions providing monetary damages for flood levels of variable magnitude. In case of strongly altered GEV parameters, future flood heights might exceed the maximum level covered by the damage functions. Accordingly, in order to evaluate our results, we need to extend the damage functions to higher levels. The shape of such a continuation is in general unknown. Although a saturation of damages for sea levels above a certain magnitude seems plausible, at which point this saturation is reached is not clear and even an increased steepness cannot be ruled out in case further areas become affected by extreme sea levels. Therefore, we extrapolate the curves with the same

behavior as they expose for lower sea levels—namely by a power law (see below).

[21] The damage function for the city of Copenhagen was published by *Hallegatte et al*. [2011] and was obtained from Stéphane Hallegatte. It provides the direct losses for water levels between 0 m and 4 m above current mean sea level. Figure 2 depicts the curve and the extrapolation as a power function.

[22] In *Boettle et al*. [2011], the Kalundborg case study was treated in detail, including the elaboration of a macroscopic damage function for water levels between 0 m and 4 m (Figure 3). As in the Copenhagen case study the following extrapolation technique was used.

[23] The given damage functions were extrapolated by fitting the power law, equation (2), with an additional proportionality constant to the available curve. Only water levels for which the curve shows power law behavior, i.e., above a certain threshold, were used for the fitting. The thresholds were 20 cm in Copenhagen and 140 cm in Kalundborg. Minimizing the mean-squared error, the exponents $\gamma \approx 1.57$ (with 95% confidence interval [1.56, 1.59]) and $\gamma \approx 4.06$ (with 95% confidence interval [4.00, 4.12]) were estimated for Copenhagen and Kalundborg, respectively.

4. Changes in the Extremes

[24] We investigate how damages are influenced by changes in the water level extremes. Considering sea level rise, we assume that it is solely reflected in an increasing location parameter μ of the flood level GEV distribution [*Kauker and Langenberg*, 2000], which corresponds to a shift of all flood levels by the corresponding sea level rise. In contrast, an alteration of the scale parameter σ could be attributed to altering meteorological conditions.

[25] We derive the distribution of damages as caused by the distribution of block maxima via a general damage function of the form of equation (2) and determine the expectation value E_D and the standard deviation STD_D of the damages. Finally, we provide the dependencies of E_D and STD_D on μ and σ.

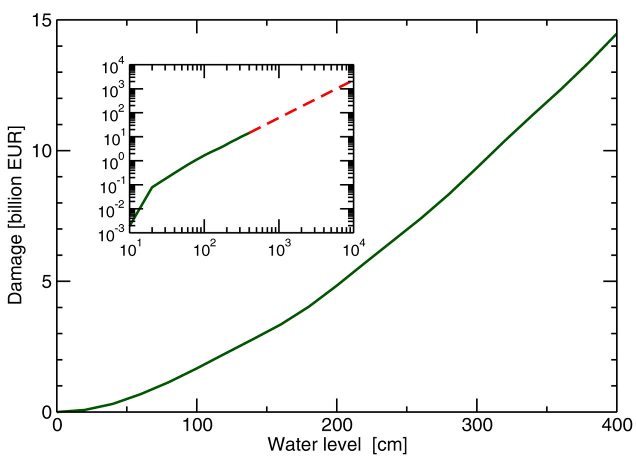

Figure 2. Damage function (green) for the case study Copenhagen obtained from *Hallegatte et al*. [2011]. The inset additionally shows the extrapolation as a power law with exponent $\gamma \approx 1.57$ (dashed red) in double logarithmic scale.

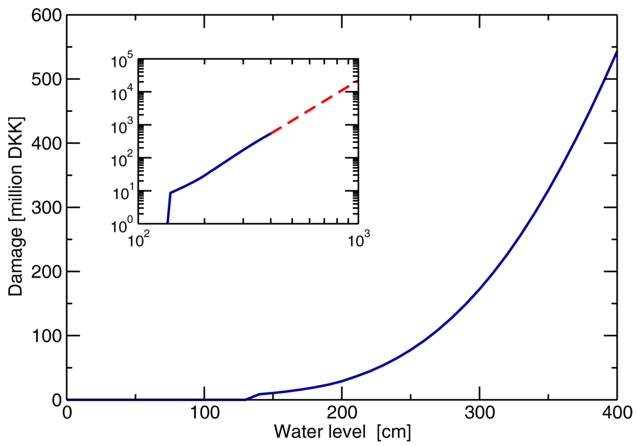

Figure 3. Damage function (blue) in the case study area Kalundborg using quadratic building damage functions and a flood fill algorithm via four nearest neighbors [*Boettle et al*., 2011]. The inset additionally shows the extrapolation as a power law with exponent $\gamma \approx 4.1$ (dashed red) in double logarithmic scale.

4.1. Influence of the Location Parameter

[26] First, we investigate a systematic alteration of the location parameter μ. This describes a simple shift of today's extreme events toward higher water levels. As derived analytically in Appendix A, we find that the expected annual damage, equation (3), increases asymptotically for high values of μ with the damage function exponent γ, i.e.,

$$E_D(\mu) \sim \mu^{\gamma}. \tag{5}$$

Since damage functions are typically steeper than linear [e.g., *Hallegatte et al*., 2011; *Boettle et al*., 2011], the damage increases faster than the sea level. Presuming a certain case study where the damage increases cubically with the flood level (i.e., $\gamma = 3$, which is between the values of both case studies) and assuming that the sea level rises quadratically over time at the corresponding coast (as suggested by *Rahmstorf et al*. [2012]), equation (5) implies that the expected annual damages increase with exponent $3 \times 2 = 6$ over time.

[27] The damage variability, which emerges from the stochasticity of extreme events, can be characterized by the standard deviation STD_D as a measure of uncertainty. For large μ, we obtain asymptotically

$$STD_D(\mu) \sim \mu^{\gamma-1}, \tag{6}$$

as derived in Appendix A. This expression comprises only the aleatory uncertainty, i.e., the inherent variability due to stochasticity. Further (epistemic) uncertainties [*Thieken et al*., 2005], for instance, caused by the vague stage-damage relation, are not considered in this context. A quantification of these uncertainties would require very detailed additional information about the building damage functions and the entire inundation process (e.g., possible flow velocities, contaminations, etc.). Including these uncertainties, significantly higher values for the standard deviations could be expected [*Merz et al*., 2004].

[28] In case of rising sea levels at a specific site with $\gamma > 1$ (which in general can be presumed) both, the expected damages and their standard deviation, increase. However, the standard deviation grows less steep, which leads to decreasing relative errors of our estimates $\mathrm{STD}_D/\mathrm{E}_D$. This reduced relative variability of annual damages can be perceived as a better predictability of flood damages, since the relative deviation of the real damages from our estimates becomes smaller. We would like to note that both relations are independent of the extreme value type, i.e., independent of ξ.

[29] These general asymptotic findings shall be compared with calculations from the case studies. Using the Copenhagen damage function with a power law extrapolation for water levels above 4 m and the determined GEV parameters, Figure 4 exhibits the numerically calculated expectation values and standard deviations of damages as a function of the location parameter. It can be seen that a rising sea level and the corresponding shift of μ leads to an increase of damages, approaching the asymptotic relation expressed by equation (5). The same holds for the standard deviation and equation (6).

[30] At this point, we are also interested on how the expected damages evolve in time. From the Dynamical Interactive Vulnerability Assessment (DIVA) tool [*Hinkel and Klein*, 2003; *Vafeidis et al*., 2008], mean sea level projections for the city of Copenhagen have been extracted for several socioeconomic scenarios [*Intergovernmental Panel on Climate Change (IPCC)*, 2000]. Assuming that changes in mean sea levels take place in the form of a shift of extreme events, we add sea level changes to the location parameter μ of the GEV distribution. Sea level projections

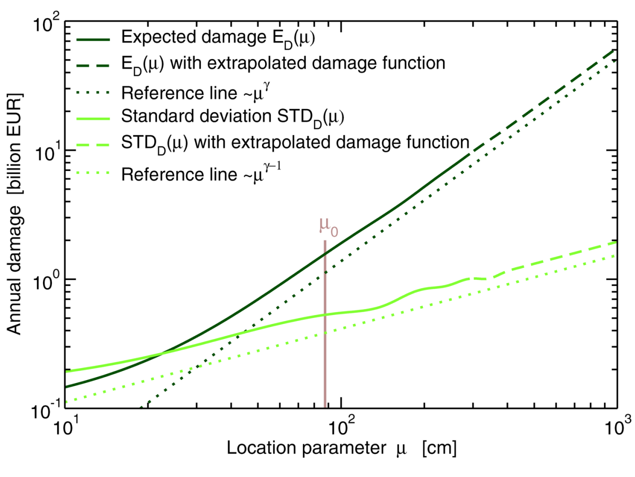

Figure 4. Expected annual damage (dark green) and standard deviation (light green) in Copenhagen as a function of the location parameter μ of the GEV distribution. The solid lines were numerically calculated with the available damage function using equations (3) and (4), the dashed continuations used an extrapolation of the damage function as a power law with exponent $\gamma \approx 1.57$ (see Figure 2). The dotted lines show the asymptotic relations equations (5) and (6). The current value of the location parameter $\mu_0 \approx 88$ cm is displayed as a brown vertical line.

are shown in Figure 5a for two socioeconomic scenarios: (i) the ecologically friendly and globally homogeneous scenario B1 supposing medium climate sensitivity and (ii) a rapid economically growing world A1B with a balanced emphasis on all energy sources, supposing high climate sensitivity. As can be seen in Figure 5b, the corresponding expected damages are steeper than the rise itself. Finally, an increase of flood risk by the factors 1.48 (B1, medium climate sensitivity) and 2.37 (A1B, high climate sensitivity) for a mean sea level rise of 28 cm (B1) and 74 cm (A1B) by 2100 was found.

[31] Figure 6 is the analogue of Figure 4 for the Kalundborg case study. The expected damages and standard deviations for varying parameter μ show good agreement with the asymptotic results already for moderate parameter values. The asymptotic behaviors in equations (5) and (6) therefore provide good estimations.

4.2. Influence of the Scale Parameter

[32] Climate change can also affect the scale parameter σ of the water level distribution [*Mudersbach and Jensen*, 2010], which reflects changes in the variability, e.g., through evolving meteorological patterns. For varying σ, we obtain asymptotically (again, independent of ξ):

$$\mathrm{E}_D(\sigma) \sim \sigma^{\gamma} \quad \text{and} \quad \mathrm{STD}_D(\sigma) \sim \sigma^{\gamma}, \tag{7}$$

as analytically derived in Appendix A. Accordingly, the expected annual damage increases with the width of the distribution of maximum sea levels following the same degree as the damage function. Contrary to equation (6), this also holds for the standard deviation. Hence, the relative uncertainty $\mathrm{STD}_D/\mathrm{E}_D$ is (asymptotically) constant being basically unaffected by changes in the width of the extreme value distribution.

[33] For the Copenhagen case study, the asymptotic predictions from equation (7) for a varying shape parameter σ are confirmed in Figure 7. However, a less steep increase for σ close to the present value σ_0 is found. The analogous results for Kalundborg are shown in Figure 8. In this case, the curves show good agreement with the asymptotic results already for parameter values around the current value σ_0. In summary, the asymptotic findings show the same sensitivity of damages for a changing variability as for changing sea levels. However, both case studies suggest a less steep increase in the near future.

5. Influence of Protection Measures

[34] Finally, we investigate how the expected damage and the uncertainty depend on the height of hypothetical protection measures. For this purpose, we follow the same approach as in section 4 but take protection measures, such as a dike or a sea wall, into account by censoring small floods, i.e., setting the damage function to zero below the corresponding threshold value ω. Please note that our model excludes protection failures such as dike breaches. The parameters μ and σ are kept constant. Then, we study the damage distribution and extract the expectation value and the standard deviation of the damages as functions of the protection height ω.

[35] In contrast to the previous results, the expected damages as a function of the protection height depend

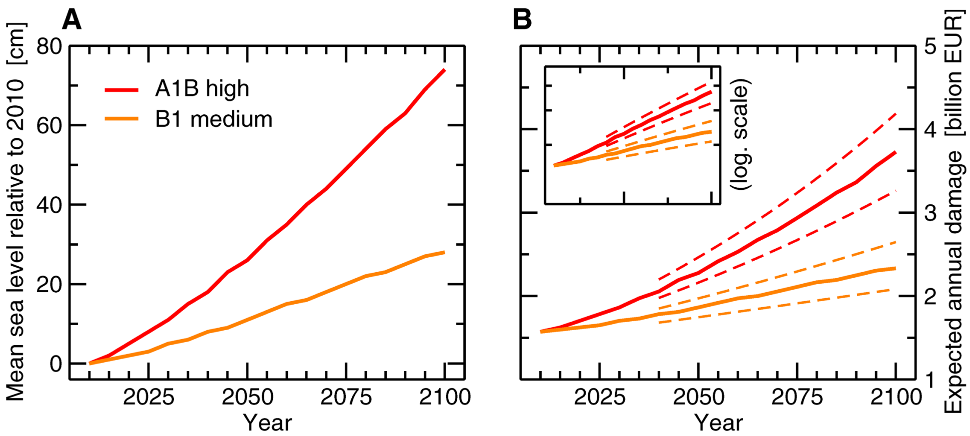

Figure 5. (a) Mean sea level projections for Copenhagen provided by the DIVA tool [*Hinkel and Klein*, 2003; *Vafeidis et al.*, 2008] for the SRES scenarios A1B (high) and B1 (medium). (b) The expected annual damage as a function of time, based on the two scenarios. The dashed lines are reference curves according to $f(\text{year}) = (\text{year} - k)^{\tau}$ with $k = 1650$ and $\tau = 4.5, 3.5, 2.5, 1.5$ (from top to bottom). The inset shows the same curves in a semilogarithmic scale.

fundamentally on the GEV type. The asymptotic relations are analytically derived in Appendix A.

[36] 1. *Gumbel case*. We find in the Gumbel case ($\xi = 0$) the asymptotic relationship

$$\mathrm{E}_D(\omega) \sim \omega^{\gamma} \mathrm{e}^{-\omega/\sigma}, \tag{8}$$

for large ω. The decay is independent from μ and dominated by an exponential component. It is noteworthy, that the range of ω, for which the expression in equation (8) increases, is not relevant for the asymptotic behavior.

[37] 2. *Fréchet case*. For asymptotically large ω, we find a power law decay in the Fréchet case ($\xi > 0$):

$$\mathrm{E}_D(\omega) \sim \omega^{\gamma - 1/\xi}, \tag{9}$$

which is independent from the scale parameter σ.

[38] 3. *Weibull case*. In the Weibull case ($\xi < 0$), the possible water levels are bounded from above, which implies that the water level cannot exceed a certain maximum value $x_{\max}$. Hence, within our model, a protection height of $x_{\max}$ guarantees full flood safety and the expected damage becomes 0. Therefore, we investigate the behavior for a

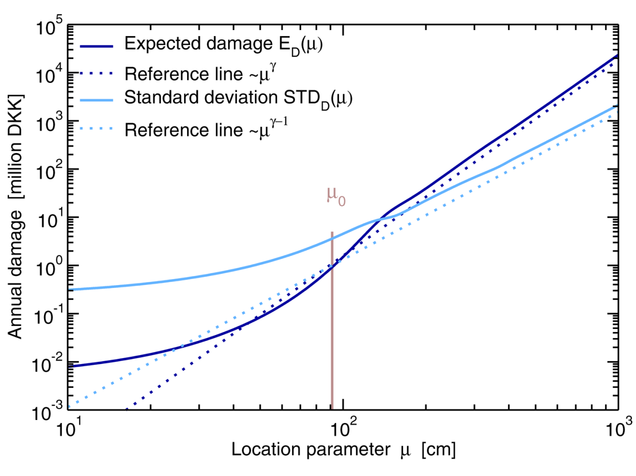

Figure 6. Expected annual damage (dark blue) and standard deviation (light blue) in the case study area Kalundborg as a function of the location parameter μ of the GEV distribution (analogous to Figure 4). The solid lines were calculated with the available damage function and its extrapolation as a power law with exponent $\gamma \approx 4.1$. The dotted line represents the asymptotic relations equations (5) and (6). The current value of the location parameter $\mu_0 \approx 91.3$ cm is displayed as a brown vertical line.

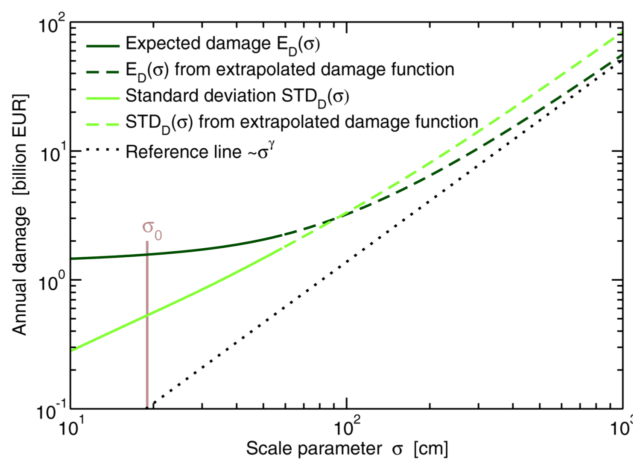

Figure 7. Expected annual damage (dark green) and standard deviation (light green) in Copenhagen as a function of the scale parameter σ of the GEV distribution. The dotted line shows the theoretical asymptotic results from equation (7). The current value of the scale parameter $\sigma_0 \approx$ 19 cm is displayed as a brown vertical line.

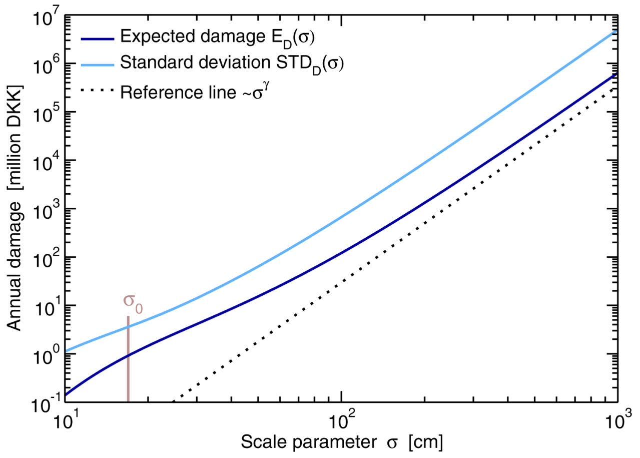

Figure 8. Expected annual damage (dark blue) and standard deviation (light blue) in the case study area Kalundborg as a function of the scale parameter σ of the GEV distribution (analogous to Figure 7). The dotted lines represent the theoretical asymptotic results from equation (7). The current value of the scale parameter $\sigma_0 \approx 17$ cm is displayed as a brown vertical line.

protection height approaching the maximum water level from below and obtain asymptotically for $\omega \to x_{\max}$:

$$E_D(\omega) \sim (x_{\max} - \omega)^{-1/\xi}. \quad (10)$$

Remarkably, this expression is independent from the power of the damage function γ—in contrast to the other cases. Still, due to its range of validity, this result is rather of theoretical interest.

[39] The corresponding standard deviations are given in Table 1. They differ by a factor 0.5 in the exponents from the expressions for the expectation value. For the Gumbel case, we find an exponential and for the Fréchet case a power law decay. Therefore, in the latter case the damages decrease much slower with enhanced flood defense. However, in both cases, there is always a residual risk, which vanishes in the Weibull case for large enough ω. Accordingly, although protection suggests safety, it cannot avoid very extreme events in the Gumbel and Fréchet cases. This is also reflected in the increasing relative uncertainty, STD_D/E_D, which indicates a higher contribution of "low-probability high-impact" events to the total damage [*Merz et al.*, 2009].

Table 1. Summary of the Asymptotic Behavior of Expected Damages and the Standard Deviations as a Function of Involved Parameters μ, σ, and ω[a]

	Location μ	Scale σ	Protection Height ω
E_D	$\sim\mu^\gamma$	$\sim\sigma^\gamma$	$\sim\omega^\gamma e^{-\omega/\sigma}$ if $\xi = 0$ $\sim(x_{\max} - \omega)^{-1/\xi}$ if $\xi < 0$ $\sim\omega^{\gamma-1/\xi}$ if $\xi > 0$
STD_D	$\sim\mu^{\gamma-1}$	$\sim\sigma^\gamma$	$\sim\omega^\gamma e^{-0.5\omega/\sigma}$ if $\xi = 0$ $\sim(x_{\max} - \omega)^{-0.5/\xi}$ if $\xi < 0$ $\sim\omega^{\gamma-0.5/\xi}$ if $\xi > 0$

[a]The asymptotics hold for $\omega \to x_{\max}$ in the case $\xi < 0$ and for large parameter values otherwise.

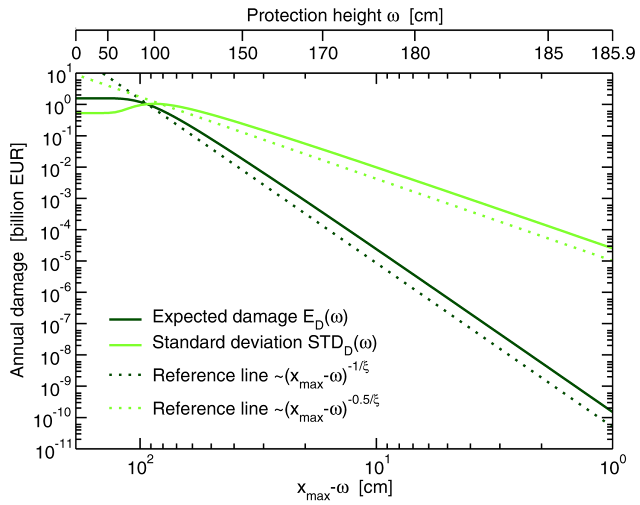

Figure 9. Expected annual damage (dark green) and standard deviation (light green) in Copenhagen as a function of the difference between the protection level ω and the maximum possible water level $x_{\max} \approx 186.9$ cm. The dotted lines follow a power law with exponent $-1/\xi$ (dark green) and $-0.5/\xi$ (light green) as expressed by equations (10) and (A15), respectively, where $\xi \approx -0.19$. Please note that the abscissa has been inverted in order to illustrate that the quantities decrease with increasing protection level ω (on top, corresponding protection heights are displayed).

[40] The expressions were calculated for the case studies and are displayed in Figures 9 and 10. Since the extreme values of the Copenhagen case follow the Weibull distribution, we use equation (10) and find good agreement between the numerically calculated damages and the predictions for protection levels above 100 cm. Also for Kalundborg (Gumbel distribution) a good approximation of

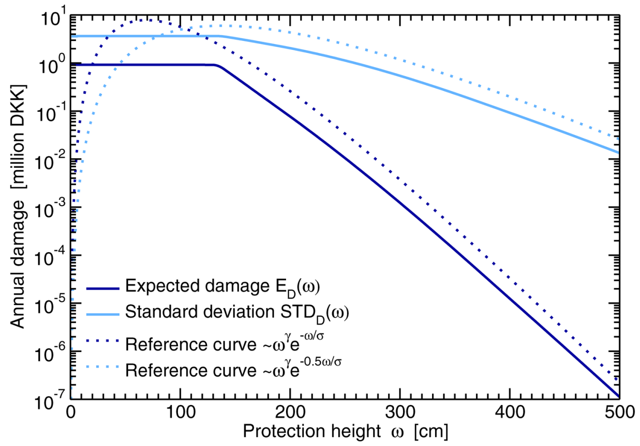

Figure 10. Expected annual damage (dark blue) and standard deviation (light blue) in the case study area Kalundborg as a function of the protection level ω. The dotted lines follow $\omega^\gamma e^{-\omega/\sigma}$ and $\omega^\gamma e^{-0.5\omega/\sigma}$ as expressed by equations (8) and (A19).

the numerical calculation by equation (8) is found for protection heights above 140 cm. For lower protection levels, there is no visible effect on the damages, which implies that the maximum annual sea level exceeds the height in most years and makes the protection measure dispensable.

[41] Performing a cost-benefit analysis, these results can be used to derive an optimal protection height, for which the total costs, comprising implementation costs and residual damages, are minimized.

6. Conclusions

[42] We have derived expressions for the expected damage from coastal floods and its standard deviation for a general case study region as a function of varying location and scale parameters. The findings are complemented with the corresponding expressions as a function of the protection height, i.e., the value below which any damage is suppressed.

[43] The relations are summarized in Table 1. In particular, we find that while the expectation value increases as a power law with the location parameter involving the damage function exponent γ, the standard deviation comprises an exponent $\gamma - 1$. Hence, the relative uncertainty, i.e., the ratio of both quantities decreases as μ^{-1} with the consequence that, from a relative perspective, the damages become more certain. For instance, the relative error of our estimation for Copenhagen, $\mathrm{STD}_D/\mathrm{E}_D$, which currently amounts to 34% would decrease to approximately 26%, supposing 20 cm of sea level rise. This also indicates, that the expected damages are increasingly determined by more common floods and that the contribution of "low-probability high-impact" events to the expected damages declines with rising sea levels.

[44] This does not hold for the quantities as a function of the scale parameter. Here the expectation value and the standard deviation increase with the same exponent and the relative uncertainty is constant. In fact, all relations are power laws with exponent γ, except the standard deviation as a function of μ which goes with a by 1 reduced exponent.

[45] Surprisingly, the expressions are *universal*, i.e., the expectation value and the standard deviation as a function of location and scale parameter are all independent of the shape parameter ξ of the extreme value distribution. This makes the results easy applicable to arbitrary regions, since no information about the extreme value behavior is necessary. Overall, the damage function exponent γ, that appears in all relations, is the most decisive factor for the estimation of future damages. If $\gamma > 1$, as in both case studies, the expected damages rise superlinear with the sea level.

[46] Investigating the influence of protection measures, we find different expressions for the different GEV types (see last column of Table 1). While the Gumbel case involves the scale parameter and the damage function exponent, the Fréchet case involves the shape parameter and the damage function exponent, and the Weibull case involves the shape parameter and surprisingly not the damage function exponent. Interestingly, in all cases, the expectation value and the standard deviation differ only by a factor 0.5 in the exponent. Accordingly, the relative uncertainty increases and the damages become more uncertain the higher the protection level is.

[47] Since, in general, sea level records follow an unknown distribution, we applied a block-maxima approach, which allows to describe the highest water level per year by one of three limiting distributions. These GEV distributions, in turn, can be fully examined. However, the drawback are inherent limitations. Since our block maxima approach considers only the largest flood event per year and neglects possible further floods. At the current conditions, this is only a minor shortcoming, since coastal floodings are per se rare in most areas, which is expected to change with rising mean sea levels. However, the integration of additional flood events (e.g., the second largest per year) in the framework would cause further inaccuracies, since several events in quick succession would not lead to independent damages.

[48] In summary, based on a set of assumptions, simple analytic expressions have been found for all dependencies. Although these results are derived for the asymptotic case, they showed good agreements with the numerical calculations from the two case studies. Therefore, the results provide a reasonable estimation for the development of future damages and could be employed in integrated assessment models in the context of climate impact research [e.g., *Tol*, 2002].

Appendix A: Asymptotic Relations

[49] The asymptotic relations in equations (5)–(10) are proven analytically in Theorems 1–5. Additionally, relations for the standard deviation of annual damages with respect to the protection height are provided. Some of the results (Theorems 3–5) are shown only for power damage functions with integer exponents, and their general validity could still be confirmed by numerical calculations following equations (3) and (4). For all considerations with regard to Fréchet-distributed water levels, the assumption $\gamma < 1/\xi$ has to be made [*Katz et al.*, 2002] in order to ensure the existence of the expected damage. An even stricter limitation of $\gamma < 0.5/\xi$ is necessary for the examination of the corresponding standard deviations. All following integrals without integration limits are meant to integrate over the largest possible interval, typically over the support of the corresponding probability distribution.

[50] We start with some general lemmas to arrange the subsequent theorems more clearly.

[51] **Lemma 1.** *Given a probability density p with existing rth moment $m_r = \int x^r p(x) \mathrm{d}x$ and $r, a, b \in \mathbb{R}^+$, for the integral*

$$I_{a,b} := \int (a + bx)^r p(x) \mathrm{d}x \tag{A1}$$

holds

$$\lim_{a\to\infty} \frac{I_{a,b}}{a^r} = 1 \quad \text{and} \quad \lim_{b\to\infty} \frac{I_{a,b}}{b^r} = m_r. \tag{A2}$$

[52] **Proof.** The integral can be written as

$$I_{a,b} = a^r \int \left(1 + \frac{b}{a}x\right)^r p(x) \mathrm{d}x = b^r \int \left(\frac{a}{b} + x\right)^r p(x) \mathrm{d}x,$$

and the uniform convergence of the integrands for $a \to \infty$ and $b \to \infty$, respectively, allows the swapping of the integral and the limit. It follows

$$\lim_{a\to\infty} \frac{I_{a,b}}{a^r} = \int \lim_{a\to\infty} \left(1 + \frac{b}{a}x\right)^r p(x)\mathrm{d}x = \int p(x)\mathrm{d}x = 1.$$

and

$$\lim_{b\to\infty} \frac{I_{a,b}}{b^r} = \int \lim_{b\to\infty} \left(\frac{a}{b} + x\right)^r p(x)\mathrm{d}x = \int x^r p(x)\mathrm{d}x = m_r.$$

□

[53] **Lemma 2.** *Given a probability density p with existing 2rth moment* $m_{2r} = \int x^{2r} p(x)\mathrm{d}x$ *and* $r, b \in \mathbb{R}^+$*, for the integral*

$$I_a := \int (a + bx)^{2r} p(x)\mathrm{d}x - \left(\int (a + bx)^r p(x)\mathrm{d}x\right)^2 \quad \text{(A3)}$$

holds

$$\lim_{a\to\infty} \frac{I_a}{a^{2r-2}} = \mathrm{const}. \neq 0. \quad \text{(A4)}$$

[54] **Proof.** We start with the Taylor expansion of $(a + bx)^r$ around $x = 0$:

$$(a + bx)^r = \sum_{i=0}^{\infty} \binom{r}{i} a^{r-i} b^i x^i = a^r + r a^{r-1} bx + \frac{r(r-1)}{2} a^{r-2} b^2 x^2 + \mathcal{O}(a^{r-3}),$$

where $\binom{r}{i}$ denotes the generalized binomial coefficient $\binom{r}{n} = r(r-1)\cdots(r-n+1)/n!$. Applying this also to the term $(a + bx)^{2r}$, it follows

$$\begin{aligned} I_a &= \int \left[a^{2r} + 2r a^{2r-1} bx + \frac{2r(2r-1)}{2} a^{2r-2} b^2 x^2 + \mathcal{O}(a^{2r-3})\right] p(x)\mathrm{d}x - \left(\int \left[a^r + r a^{r-1} bx + \frac{r(r-1)}{2} a^{r-2} b^2 x^2 + \mathcal{O}(a^{r-3})\right] p(x)\mathrm{d}x\right)^2 \\ &= \frac{2r(2r-1)}{2} a^{2r-2} b^2 m_2 - r^2 a^{2r-2} b^2 m_1^2 - r(r-1) a^{2r-2} b^2 m_2 + \int \mathcal{O}(a^{2r-3}) p(x)\mathrm{d}x \\ &= (m_2 - m_1^2) r^2 b^2 a^{2r-2} + \int \mathcal{O}(a^{2r-3}) p(x)\mathrm{d}x. \end{aligned}$$

[55] Since the integrand is uniform convergent again, swapping the integral and the limit leads to

$$\lim_{a\to\infty} \frac{I_a}{a^{2r-2}} = \lim_{a\to\infty} (m_2 - m_1^2) r^2 b^2 + \lim_{a\to\infty} \int \underbrace{\mathcal{O}(a^{-1})}_{\to 0} p(x)\mathrm{d}x = (m_2 - m_1^2) r^2 b^2,$$

which is constant and in general nonzero. In the special case of $r \in \mathbb{N}$ the Taylor expansions are finite and the result can be obtained more easily. □

[56] **Lemma 3.** *Given a probability density p with existing 2rth moment* $m_{2r} = \int x^{2r} p(x)\mathrm{d}x$ *and* $r, a, b \in \mathbb{R}^+$*, for the integral*

$$I_b := \int (a + bx)^{2r} p(x)\mathrm{d}x - \left(\int (a + bx)^r p(x)\mathrm{d}x\right)^2 \quad \text{(A5)}$$

holds

$$\lim_{b\to\infty} \frac{I_b}{b^{2r}} = \mathrm{const}. \neq 0. \quad \text{(A6)}$$

[57] **Proof.** Using the uniform convergence of the integrands, it holds

$$\lim_{b\to\infty} \frac{I_b}{b^r} = \int \lim_{b\to\infty} \left(\frac{a}{b} + x\right)^{2r} p(x)\mathrm{d}x - \left(\int \lim_{b\to\infty} \left(\frac{a}{b} + x\right)^r p(x)\mathrm{d}x\right)^2 = \int x^{2r} p(x)\mathrm{d}x - \left(\int x^r p(x)\mathrm{d}x\right)^2,$$

which is $m_{2r} - m_r^2$ and in general a nonzero constant. □

[58] Now we can proof the statements from the main text more easily, starting with the expected damage as functions of the location μ and the scale σ.

[59] **Theorem 1.** *Assuming GEV-distributed maximum water levels with density function* $p_{\xi,\sigma,\mu}$ *and a power damage function* $F(x) = x^\gamma$ $(0 < \gamma < 1/\xi)$*, for the expected annual damage* E_D *holds asymptotically (for large* μ *and* σ*, respectively)*

$$\mathrm{E}_D(\mu) \sim \mu^\gamma \quad \text{and} \quad \mathrm{E}_D(\sigma) \sim \sigma^\gamma. \quad \text{(A7)}$$

[60] **Proof.** It holds

$$\mathrm{E}_D = \int F(x) p_{\xi,\sigma,\mu}(x)\mathrm{d}x = \int (\mu + \sigma x)^\gamma p_{\xi,1,0}\mathrm{d}x.$$

Lemma 1 now provides $\mathrm{E}_D(\mu)/\mu^\gamma \overset{\mu\to\infty}{\to} 1$ and $\mathrm{E}_D(\sigma)/\sigma^\gamma \overset{\sigma\to\infty}{\to} m_\gamma$, where m_γ denotes the γth moment of the GEV-distributed water levels. Since $\gamma < 1/\xi$ is presumed, $m_\gamma \neq 0$ exists, which proves the lemma. □

[61] **Theorem 2.** *Assuming GEV-distributed maximum water levels with density function $p_{\xi,\sigma,\mu}$ and a power damage function $F(x) = x^\gamma$ ($0 < \gamma < 0.5/\xi$), for the standard deviation STD_D of the annual damage holds asymptotically (for large μ and σ, respectively)*

$$\mathrm{STD}_D(\mu) \sim \mu^{\gamma-1} \quad \text{and} \quad \mathrm{STD}_D(\sigma) \sim \sigma^\gamma. \tag{A8}$$

[62] **Proof.** For the variance holds

$$\mathrm{Var}_D = \int (\mu + \sigma x)^{2\gamma} p_{\xi,1,0} \mathrm{d}x - \left(\int (\mu + \sigma x)^\gamma p_{\xi,1,0} \mathrm{d}x \right)^2.$$

Lemmas 2 and 3 provide $\mathrm{Var}_D(\mu)/\mu^{2\gamma-2} \overset{\mu\to\infty}{\longrightarrow} \mathrm{const.} \neq 0$ and $\mathrm{Var}_D(\sigma)/\sigma^{2\gamma} \overset{\sigma\to\infty}{\longrightarrow} \mathrm{const.} \neq 0$, respectively. Consequently, the same holds for $\mathrm{STD}_D = \mathrm{Var}_D^{1/2}$. □

[63] Now, we provide results to proof the relations for the dependencies on the protection height ω.

[64] **Lemma 4**. *Considering the probability density function $p_{\xi,\sigma,\mu}$ of a Fréchet distribution (i.e., $\xi > 0$) and $n \in \mathbb{N}$ ($n < 1/\xi$), it holds asymptotically (for large ω)*

$$I_n := \int_\omega^\infty x^n p_{\xi,\sigma,\mu}(x) \mathrm{d}x \sim \omega^{n-1/\xi}. \tag{A9}$$

[65] **Proof.** We start with the integral

$$I_n = \frac{1}{\sigma} \int_\omega^\infty x^n \left(1 + \xi \frac{x-\mu}{\sigma}\right)^{-1/\xi-1} \cdot \exp\left(-\left(1 + \xi \frac{x-\mu}{\sigma}\right)^{-1/\xi}\right) \mathrm{d}x$$

and substitute $z := \left(1 + \xi \frac{x-\mu}{\sigma}\right)^{-1/\xi}$:

$$\begin{aligned} I_n &= \int_0^{\left(1+\xi\frac{\omega-\mu}{\sigma}\right)^{-1/\xi}} \left(\frac{\sigma}{\xi} z^{-\xi} - \frac{\sigma}{\xi} + \mu\right)^n \mathrm{e}^{-z} \mathrm{d}z \\ &= \sum_{i=0}^n \underbrace{\binom{n}{i} \left(\mu - \frac{\sigma}{\xi}\right)^{n-i} \left(\frac{\sigma}{\xi}\right)^i}_{=:K_{n,i}} \cdot \underbrace{\int_0^{\left(1+\xi\frac{\omega-\mu}{\sigma}\right)^{-1/\xi}} z^{-i\xi} \mathrm{e}^{-z}}_{=\text{lower incomplete Gamma function } \gamma} \mathrm{d}z \\ &= \sum_{i=0}^n K_{n,i} \gamma\left(1 - i\xi, \left(1 + \xi \frac{\omega-\mu}{\sigma}\right)^{-1/\xi}\right) \\ &= \sum_{i=0}^n \underbrace{K_{n,i}}_{\text{const.}} \underbrace{\left(1 + \xi \frac{\omega-\mu}{\sigma}\right)^{i-1/\xi}}_{\to \omega^{i-1/\xi}} \cdot \underbrace{\exp\left(-\left(1 + \xi \frac{\omega-\mu}{\sigma}\right)^{-1/\xi}\right)}_{\to 1} \\ &\quad \cdot \underbrace{\sum_{k=0}^\infty \frac{\left(1 + \xi \frac{\omega-\mu}{\sigma}\right)^{-k/\xi}}{(1-i\xi)(2-i\xi) \cdot \ldots \cdot (1+k-i\xi)}}_{\to \text{const.}} \sim \omega^{n-1/\xi}, \end{aligned} \tag{A10}$$

for large ω. The properties of the incomplete Gamma function can be found in *Arfken and Weber* [2005]. □

[66] **Theorem 3 (ω-relation, Fréchet).** *Given Fréchet-distributed maximum water levels with density function $p_{\xi,\sigma,\mu}$ ($\xi > 0$), a power damage function $F(x) = x^\gamma$ ($\gamma \in \mathbb{N}, \gamma < 0.5/\xi$) and an implemented protection measure of height ω, for the annual damage holds asymptotically (for large ω)*

$$\mathrm{E}_D(\omega) \sim \omega^{\gamma-1/\xi} \quad \text{and} \quad \mathrm{STD}_D(\omega) \sim \omega^{\gamma-0.5/\xi}. \tag{A11}$$

[67] **Proof.** Lemma 4 gives the relation for the expectation value E_D (setting $n = \gamma$) and $\mathrm{Var}_D - \mathrm{E}_D^2 \sim \omega^{2\gamma-1/\xi}$ (setting $n = 2\gamma$). The expression for the standard deviation follows immediately. □

[68] Please note, that the case $\gamma \notin \mathbb{N}$ is not included in the theorem. Nevertheless, its validity could be confirmed by numerical calculations.

[69] **Lemma 5.** *Considering the probability density function $p_{\xi,\sigma,\mu}$ of a Weibull distribution (i.e., $\xi < 0$) and $n \in \mathbb{N}$, for ω close to the maximum possible sea level $x_{\max} := \mu - \sigma/\xi$ holds*

$$I_n := \int_\omega^R x^n p_{\xi,\sigma,\mu}(x) \mathrm{d}x \sim (x_{\max} - \omega)^{-1/\xi}. \tag{A12}$$

[70] **Proof.** We obtain equation (A10) similarly as in the proof above. For ω close to $x_{\max}$ follows

$$\begin{aligned} I_n &= \text{(A10)} \\ &= \sum_{i=0}^n \underbrace{K_{n,i}}_{\text{const.}} \underbrace{\left(1 + \xi \frac{\omega-\mu}{\sigma}\right)^{i-1/\xi}}_{\to (x_{\max}-\omega)^{i-1/\xi}} \cdot \underbrace{\exp\left(-\left(1 + \xi \frac{\omega-\mu}{\sigma}\right)^{-1/\xi}\right)}_{\to 1} \\ &\quad \cdot \underbrace{\sum_{k=0}^\infty \frac{1}{(1-i\xi)(2-i\xi) \cdot \ldots \cdot (1+k-i\xi)}}_{\text{const.}} \sim (x_{\max} - \omega)^{-1/\xi}. \end{aligned} \tag{A13}$$

□

[71] **Theorem 4 (ω-relation, Weibull).** *Given Weibull-distributed maximum water levels with density function $p_{\xi,\sigma,\mu}$ ($\xi < 0$), a power damage function $F(x) = x^\gamma$ ($\gamma \in \mathbb{N}, \gamma < 0.5/\xi$) and an implemented protection measure of height ω close to the maximum possible sea level $x_{\max} := \mu - \sigma/\xi$, for the annual damage holds*

$$\mathrm{E}_D(\omega) \sim (x_{\max} - \omega)^{-1/\xi} \tag{A14}$$

and

$$\mathrm{STD}_D(\omega) \sim (x_{\max} - \omega)^{-0.5/\xi}. \tag{A15}$$

[72] **Proof.** Lemma 5 gives the relation for the expectation value E_D (setting $n = \gamma$) and $\mathrm{Var}_D - \mathrm{E}_D^2 \sim \omega^{2\gamma-1/\xi}$ (setting $n = 2\gamma$). The expression for the standard deviation follows immediately. □

[73] As before, please note that the case $\gamma \notin \mathbb{N}$ is not included in the theorem. Nevertheless, its validity could be confirmed by numerical calculations.

[74] **Lemma 6.** *Considering the probability density function $p_{0,\sigma,\mu}$ of a Gumbel distribution and $n \in \mathbb{N}$, it holds asymptotically (for large ω)*

$$I_n := \int_{\omega}^{\infty} x^n p_{\xi,\sigma,\mu}(x)\mathrm{d}x \sim \omega^n \mathrm{e}^{-\omega/\sigma}. \tag{A16}$$

[75] **Proof.** Starting with the substitution $z := \frac{x-\mu}{\sigma}$ we obtain

$$\begin{aligned}\frac{I_n}{\omega^n \mathrm{e}^{-\omega/\sigma}} = \frac{\sigma^n}{\omega^n \mathrm{e}^{-\omega/\sigma}} \int_{\frac{\omega-\mu}{\sigma}}^{\infty} \left(\sum_{i=0}^{n} \binom{n}{i} \left(\frac{\mu}{\sigma}\right)^{n-i} z^i \right) \\ \cdot \exp(-z) \underbrace{\exp(-\exp(-z))}_{\leq 1} \mathrm{d}z\end{aligned} \tag{A17}$$

$$\begin{aligned}&\leq \frac{\sigma^n}{\omega^n \mathrm{e}^{-\omega/\sigma}} \int_{\frac{\omega-\mu}{\sigma}}^{\infty} \left(\sum_{i=0}^{n} \underbrace{\binom{n}{i} \left(\frac{\mu}{\sigma}\right)^{n-i}}_{\leq \mathrm{const.}} z^i \right) \cdot \exp(-z)\mathrm{d}z \\ &\leq \mathrm{const.} \frac{1}{\omega^n \mathrm{e}^{-\omega/\sigma}} \sum_{i=0}^{n} \int_{\frac{\omega-\mu}{\sigma}}^{\infty} z^i \exp(-z)\mathrm{d}z \\ &= \mathrm{const.} \frac{1}{\omega^n \mathrm{e}^{-\omega/\sigma}} \sum_{i=0}^{n} \Gamma\left(n+1, \frac{\omega-\mu}{\sigma}\right),\end{aligned} \tag{A18}$$

which is constant for $\omega \to \infty$ with the upper incomplete Gamma function Γ. equation (A17) can also be bounded from below:

$$\begin{aligned}(\mathrm{A17}) &\geq \mathrm{const.} \frac{1}{\omega^n \mathrm{e}^{-\omega/\sigma}} \sum_{i=0}^{n} \int_{\frac{\omega-\mu}{\sigma}}^{\infty} z^n \exp(-z)\mathrm{d}z \\ &= \mathrm{const.} \frac{1}{\omega^n \mathrm{e}^{-\omega/\sigma}} \sum_{i=0}^{n} \Gamma\left(n+1, \frac{\omega-\mu}{\sigma}\right),\end{aligned}$$

which is again constant for $\omega \to \infty$ and finishes the proof.□

[76] **Theorem 5 (ω-relation, Gumbel).** *Given Gumbel-distributed maximum water levels with density function $p_{0,\sigma,\mu}$, a power damage function $F(x) = x^\gamma$ ($\gamma > 0$) and an implemented protection measure of height ω, for the annual damage E_D holds asymptotically (for large ω)*

$$\mathrm{E}_D(\omega) \sim \omega^\gamma \mathrm{e}^{-\omega/\sigma} \quad \text{and} \quad \mathrm{STD}_D(\omega) \sim \omega^\gamma \mathrm{e}^{-\omega/(2\sigma)}. \tag{A19}$$

[77] **Proof.** The theorem follows immediately with Lemma 6. □

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