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Sampled-Data Consensus of Multi-Agent System in the Presence of Packet Losses

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ABSTRACT The consensus problem is investigated in this paper for multi-agent systems subjected to sampled data and packet losses. The switching system, which simultaneously contains both stable and unstable subsystems, is applied to model the packet dropout in a deterministic way. The focus of this paper is to derive an improved condition stated elegantly in terms of linear matrix inequality (LMI) with less conservative and lower complexity such that the multi-agent system considered can reach consensus. In line with this, a novel nonlinear function is introduced in the construction of the piecewise differentiable Lyapunov functions, and the Wirtinger inequality is applied as well. Furthermore, several comparison results against other related ones are presented, which yield that our consensus protocols significantly improve upon the existing ones advocated thus far. We verify the validity, the effectiveness, and the practical applicability of our results in the simulation examples.

INDEX TERMS Consensus, sampling data, packet losses, switching system.

I. INTRODUCTION

Recently, as a representative large-scale system, multi-agent systems have provoked widespread attention because of their potential applications in a variety of areas including smart grids, networked systems and sensor networks, etc [1]–[9]. The emergence of the consensus in multi-agent systems, which aims to how to utilize the innumerable its neighbor's state data to reach a unified state, is an appealing research theme in various disciplines. Moreover, as a presentative collective behavior, the consensus is always a prerequisite to broad applications of multi-agent systems. Hence, fruitful results on how to guarantee multi-agent systems reaching consensus are springing up [10]–[16] (and their references therein). Up to now, numerous approaches have been come forth to address the consensus issue of multi-agent systems based on various control schemes, such as the sampled control [17], impulsive control [18]–[20], output feedback control [21], etc. It is well known that consensus protocols based on sampling patterns just utilize the state information of their neighbors at some prescribed discrete instants. Therefore, different from other methods, sampled control has evident merits in alleviating the risk of network congestion. Meanwhile, the interest in the consensus based on sampled data can be traced back to [22] but it continuous to fascinating the scientific community to date.

It should be pointed out that, in practical industrial applications, especially in network control systems, due to the non-ideal outer environment and inherent network-limited bandwidth, intermittent data packet dropout and signal transmission delay can not be overlooked [23]. On the other hand, there are many causes for the performance deterioration or even the impossibility of consensus for multi-agent systems. Some are related to the time delay which appears in signal transfer [24]. The packed losses in the information transfer can have a tremendous impact as well. Hence, there is a need to consider the packed loss simultaneously when the consensus of multi-agent systems is considered. Generally speaking, from the manner of its appearance, the packet dropouts can be roughly divided into two categories: random and deterministic packet losses. For the former case, a set of stochastic parameters fulfilling Bernoulli-distribution are introduced to describe such phenomenon of the packet dropouts. However, the deterministic packet loss is modeled by a switching system, which simultaneously contains stable and unstable subsystems, which is also the primary consideration of this paper.

Note that several consensus protocols based on sampling pattern measurements are derived by constructing a Lyapunov-Krasovskii functional(LKF), which will lead to certain degree of conservatism inevitably. Along with the rapid improvement of the requirement for sampled controller performance, it is a challenge to develop such novel consensus criteria which are of not only less conservative but also of lower complexity. In line with this, considerable efforts have been recently devoted(Please refer [25]–[28] and there references therein). Among this pioneering literature, it is not difficult to discover that these methods concerning the conservativeness reducing can be roughly divided into two categories: One is to construct an effective LKF which takes full advantage of the available information about the sample pattern [10], [13]. Another one is to make the estimation of the derivative of LKF as tight as possible by introducing some inequality, such as free matrix based integral inequality [29], Wirtinger inequality [30], reciprocally inequality [31], etc. Compared with the former, it is clear that the latter's ability to tackle this problem is limited. Therefore, it is meaningful to introduce some functions which capture the information about sampling pattern adequately and to construct corresponding LKFs to derive novel consensus criteria with less conservative character, which is the second motivation of this paper.

Inspired by the aforementioned literature, in this paper, we aim to investigate a sampled consensus of class of multi-agent systems with packet losses occurring in a deterministic manner. A switching system, which simultaneously contains both stable and unstable subsystems, is applied to model the packet dropout case. By introducing a novel nonlinear function in the construction of the piecewise differentiable Lyapunov functions and applying the Wirtinger inequality, an improved condition is stated elegantly in terms of Linear matrix inequality with less conservative and lower complexity such that the multi-agent system can reach consensus. Further, several comparison corollaries against other related results are presented, which yields that our consensus protocols significantly improve upon the existing ones advocated thus far.

The rest of this paper is organized as follows. Section II collects preliminaries and problem formulation of this paper. Section III establishes several sufficient conditions and the corresponding controller synthesis criterion. A simulation example is reported in Section IV, and this paper concludes in Section V.

Notations: Throughout this paper, we adapt the following notations if there is no special emphasis: $\mathbb{R}^n$ expresses the n-dimensional Euclidean space. $x \in \mathbb{R}^n$, denotes its transpose by x^{T}. $\mathbb{N}$ refers to be a set of natural numbers. Let $|\cdot|$ be the Euclidean norm of a vector or the induced norm of a matrix. Column vectors whose entries are all zero and all one are denoted by 0_n and 1_n, respectively. I_n stands for the identity matrix of order n; The notation $M > 0 (\geq 0)$means that M is positive definite (semipositive definite). $\mathbb{E}$ denotes the mathematical expectation. Let $\mathcal{G}(\mathcal{V}, \varepsilon, A)$ be a simple directed graph which is associated with interactions among agents consisting of a finite set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a finite set of directed edges $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ and a weighted adjacency matrix $A = [a_{ij}]_{N\times N}$. The adjacency matrix $A = [a_{ij}]$ associated with the interaction topology graph is defined as follows: $a_{ij} = 1$ if and only if there is edge connected from node j to node i, otherwise, $a_{ij} = 0$. Let the diagonal D be the degree matrix with $d_{ii} = \sum_{i=1}^{N} a_{ij}$. Based on this, the Laplacian matrix is $L = D - A$.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider the model of a multi-agent system composing of N agents and sharing the self-linear dynamics that can be represented by the following differential equation:

$$y_i(t) = Ay_i(t) + Bu_i(t) \tag{1}$$

where $y_i(t) = [y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t)]^{\mathrm{T}} \in \mathbb{R}^n$ denotes the state parameter of the ith node at time t, $i = 1, 2, \ldots, N$, and $u_i(t) \in \mathbb{R}^m$ stands for the control input to be determined. A and B designate constant matrices whose dimensions are supposed to be compatible. The initial condition is given by $y_i(0) = y_i^0$.

Here, we suppose, at some prescribed instants t_k, $k \in \mathbb{N}$, each agent can detect the related state information from its neighbors. Hence, the consensus agreements based on sampling pattern is introduced:

$$u_i(t) = -K\sum_{i=1}^{N} a_{ij}(y_j(t) - y_i(t)), \quad t \in [t_k, t_{k+1}) \tag{2}$$

where K stands for the gain matrix to be designed later. t_k is called as sampling instants generated via Zero-order hold circuits, which satisfies $0 = t_0 < t_1 \cdots < t_k < \cdots$. In this paper, we consider a more general scenario of sampling pattern, namely, aperiodic sampling. Accordingly, set $t_{k+1} - t_k = h_k$. Let the admissible set of aperiodic sampling instants sequence be

$$\varrho(\iota, h) = \Big\{\Big\{t_k, k \in \mathbb{N}\Big\} : \iota \leq h_k < h\Big\}, \quad 0 < \iota < h$$

Here, h and ι refer to be the minimum and maximum sampling period, respectively.

Therefore, a compact form of the multi-agent system (1) with the agreement (2) is described as

$$y(t) = (I_N \otimes A)y(t) + (L \otimes BK)y(t_k), \quad t \in [t_k, t_{k+1}). \tag{3}$$

Taking into account the sampling measurements in the presence of deterministic packet losses, we introduce a piecewise constant function $\sigma(t) : [0, \infty) \rightarrow \mathcal{S} = \{1, 2\}$. If $\sigma(t) = 2$ implies that a control packet loss occurs during the period $[t_k, t_{k+1})$, otherwise, $\sigma(t) = 1$ for $t \in [t_k, t_{k+1})$. Based on this, the multi-agent system subjected to deterministic packet losses can be specified as follows.

$$y(t) = (I_N \otimes A)y(t) + (L \otimes BK_{\sigma(t)})y(t_k), \quad t \in [t_k, t_{k+1}) \tag{4}$$

where $K_{\sigma(t)}$ either equals to K or 0.

In what follows, we will address the consensus problems for multi-agent system without packet losses (3) and the system with packet losses (4), respectively. In order to explicitly describe the performance of sampled control and the disruptive behavior of packet losses, we suppose that A is not Hurwitz stable and $(I_N \otimes A, L \otimes BK)$ is controllable. Thus, system (4) can be viewed as a switch system which consists of both a stable subsystem $y(t) = (I_N \otimes A)y(t)+(L\otimes BK)y(t_k)$ and an unstable subsystem $y(t) = (I_N \otimes A)y(t)$.

To facilitate further analysis, it is reasonable to introduce the disagreement variable. Enlightened by the method proposed in [33], we are going to carry out the model transformation on systems (3) and (4). Firstly, the disagreement variables shall be formulated as follows:

$$z_i(t) = y_1(t) - y_{i+1}(t)$$

for $i = 1, 2, \ldots, N-1$ or more compact

$$z(t) = (U \otimes I_n)\, y(t)$$

and also

$$y(t) = 1_N \otimes z_1(t) + (W \otimes I_n)z(t)$$

where $z(t) = \mathrm{col}^i_{N-1}\{z_i(t)\}$, $U = [1_{N-1}, -I_{N-1}]$ and $W = [0^{\mathrm{T}}_{N-1}, -I_{N-1}]^{\mathrm{T}}$. With the above formulation, models (3) and (4), respectively, are adapted as the following closed loop systems:

$$\dot{z}(t) = (I_{N-1} \otimes A)z(t) + (ULW \otimes BK)z(t_k) \tag{5}$$

and

$$\dot{z}(t) = (I_{N-1} \otimes A)z(t) + (ULW \otimes BK_{\sigma(t)})z(t_k). \tag{6}$$

In this paper, our focus is to derive several improved condition stated elegantly in terms of Linear matrix inequality with less conservative and lower complexity such that multi-agent systems (3) and (4) can reach consensus and to highlight the packed losses effect on the multi-agent consensus issue simultaneously. It is a remarkable fact that the consensus problems for (3) and (4) are said to be solved if and only if the systems (5) and (6) are stable. To resolve these problems well, we are in a position to present the basic concept and some necessary Lemmas.

Definition 1: The consensus for the multi-agent systems (3) and (4) is said to be attained exponentially over $\varrho(\iota, h)$, if there exist positive scalars M_0, ϵ such that $\|y_i(t) - y_j(t)\| \leq M_0 \exp(-\epsilon(t - t_0))$, $i \neq j$, holds under any initial state $y_i(0)$. Moreover, the exponential convergence rate is denoted by ϵ.

Lemma 2 [34]: Define $a = \frac{1}{2}\sqrt{|4d - c^2|}$,

$$h = h(c, d) \triangleq \begin{cases} \frac{1}{a}\mathrm{arctanh}\frac{2a}{c}, & \text{if } c > 2\sqrt{d} \\ \frac{2}{c}, & \text{if } c = 2\sqrt{d} \\ \frac{1}{a}\mathrm{arctanh}\frac{2a}{c} & \text{if } 0 < c < 2\sqrt{d} \\ \frac{\pi}{2a}, & \text{if } 2\sqrt{d} < c < 0, \end{cases} \tag{7}$$

and set $\varphi(t, \varphi_0)$ be the unique solution of the initial value problem described as follows.

$$\begin{cases} \dot{\varphi}(t) = -\frac{\nu}{\tau + \epsilon}\left(\varphi^2 + c\varphi + d\right), & t \geq \alpha \\ \varphi(t_k) = \varphi_{0k} \end{cases}$$

where α, δ and ϵ be known positive constants, $\tau \in (0, \delta]$. If there exist constants c and d satisfying $d > 0$, $c > -2\sqrt{d}$, then, there exists a positive constant $\rho(\epsilon, c, d, \delta)$ such that the solution $\varphi(t) = \varphi(t, \varphi^*_0)$ satisfies the following condition for some initial value $\varphi^*_0 \in (0, \rho)$:

$$\varphi(\alpha + \tau) = 0, \quad \dot{\varphi}(t) < 0 \quad \text{for } t \in [\alpha, \alpha + \tau].$$

Lemma 3 [33]: Consider any symmetric positive matrix V, and constants t, h satisfying $t > t - h \geq 0$. If the following integrations are well defined, we have

$$\int_{t-h}^{t} \dot{x}(\alpha)^{\mathrm{T}} V \dot{x}(\alpha) d\alpha \geq \frac{1}{h}\int_{t-h}^{t} \dot{x}^{\mathrm{T}}(\alpha) d\alpha V \int_{t-h}^{t} \dot{x}^{\mathrm{T}}(\alpha) d\alpha + \frac{3}{h}\Sigma^{\mathrm{T}} V \Sigma$$

with

$$\Sigma = x(t-h) + x(t) - \frac{2}{h}\int_{t-h}^{t} x(\alpha) d\alpha.$$

Lemma 4 (Average Dwell Time) [32]: Set $\mathcal{N}_\sigma(t_1, t_2)$ be the number of switchings of $\sigma(t)$ on $[t_1, t_2)$. The scalar τ_a and $\mathcal{N}_0$, respectively, are said to be the average dwell time and the chatter bound if the following condition is hold.

$$\mathcal{N}_\sigma(t_1, t_2) \leq \mathcal{N}_0 + \frac{t_2 - t_1}{\tau_a}$$

III. MAIN RESULTS

In this section, we aim to tackle the consensus problem formulated in the previous section. By constructing a new piecewise differentiable Lyapunov function, which can fully capture the information on sampling pattern, some less conservative and lower complexity criteria formulated by LMIs are put forward for assuring that the multi-agent system (4) reaches consensus.

For ease of derivation, one piecewise linear function and some brief notations are introduced.

$$\beta(t) = t_{k+1} - t, \quad \eta(t) = z(t) - z(t_k), \quad t \in [t_k, t_{k+1})$$

$$\Sigma_1(\alpha)=\begin{bmatrix}\Gamma_{11}+h\Lambda_{11} & \Pi_{12}+h\Sigma_{12} & hF^{\mathrm{T}}(V+W+Q) & \frac{h^2e^{\alpha h}}{2}M\\ * & -\frac{\nu}{h}(R_1+hR_2) & 0 & 0\\ * & * & -h(V+W+Q) & 0\\ * & * & * & -\frac{h^2e^{\alpha h}}{2}W\end{bmatrix}$$

$$\Sigma_2(\alpha)=\begin{bmatrix}\Gamma_{11} & \Pi_{12} & he^{\alpha h}G & hF^{\mathrm{T}}(V+W) & \frac{h^2e^{\alpha h}}{2}M\\ * & -\frac{\nu}{h}R_1 & 0 & 0 & 0\\ * & * & -Qhe^{\alpha h} & 0 & 0\\ * & * & * & -h(V+W) & 0\\ * & * & * & * & -\frac{h^2e^{\alpha h}}{2}W\end{bmatrix}$$

$$\begin{aligned}\xi^{\mathrm{T}}(t)&=\left[z^{\mathrm{T}}(t)\; z^{\mathrm{T}}(t_k)\; z^{\mathrm{T}}(t-h)\; \int_{t-h}^{t} z^{\mathrm{T}}(s)ds\right],\\ \zeta^{\mathrm{T}}(t)&=\left[\xi^{\mathrm{T}}(t)\;\; \varphi(t)\eta(t)\right],\\ z_t&=z(t+\theta),\quad \text{for } \theta\in[-\tau,0]\end{aligned}$$

So as to get rid of the redundant matrix representation, several block matrices shall be introduced:

$$\begin{aligned}\mathcal{I}_1&=\begin{bmatrix}I_{(N-1)n} & 0 & 0 & 0\end{bmatrix},\\ \mathcal{I}_2&=\begin{bmatrix}0 & I_{(N-1)n} & 0 & 0\end{bmatrix},\\ \mathcal{I}_{12}&=\begin{bmatrix}I_{(N-1)n} & -I_{(N-1)n} & 0 & 0\end{bmatrix},\\ \varepsilon&=\begin{bmatrix}I_{(N-1)n} & 0 & I_{(N-1)n} & -\frac{2}{h}I_{(N-1)n}\end{bmatrix}.\end{aligned}$$

A. CONSENSUS OF MULTI-AGENT SYSTEMS WITHOUT PACKET LOSSES

Theorem 1: Assume that the communication topology $\mathcal{G}$ contains a directed spanning tree. System (3) reaches consensus over $\varrho(\iota, h)$, if there exit constants c, d satisfying Lemma 2, $(N-1)n\times(N-1)n$ matrices $P_1>0, Q>0, V>0, W>0, R_1>0, R_2\geq 0$, $4(N-1)n\times(N-1)n$ matrices G, M and a positive constant α such that the following LMIs hold:

$$\Sigma_i(0)<0,\quad i=1,2 \tag{8}$$

where, $\Sigma_1(\alpha)$ and $\Sigma_2(\alpha)$, as shown at the top of this page, with

$$\begin{aligned}\Gamma_{11}&=\alpha\mathcal{I}_1^{\mathrm{T}}P_1\mathcal{I}_1+\mathcal{I}_1^{\mathrm{T}}P_1F+F^{\mathrm{T}}P_1\mathcal{I}_1-\frac{\nu d}{h}\mathcal{I}_{12}^{\mathrm{T}}R_1\mathcal{I}_{12}\\ &\quad-\frac{3e^{-\alpha h}}{h}\varepsilon^{\mathrm{T}}V\varepsilon-\frac{e^{-\alpha h}}{h}\mathcal{I}_{13}^{\mathrm{T}}V\mathcal{I}_{13}+G\mathcal{I}_1+\mathcal{I}_1^{\mathrm{T}}G^{\mathrm{T}}\\ &\quad-G\mathcal{I}_3-\mathcal{I}_3^{\mathrm{T}}G^{\mathrm{T}}+he^{-\alpha h}M\mathcal{I}_1+he^{-\alpha h}\mathcal{I}_1^{\mathrm{T}}M^{\mathrm{T}}\\ &\quad-e^{-\alpha h}M\mathcal{I}_4-e^{-\alpha h}\mathcal{I}_4^{\mathrm{T}}M^{\mathrm{T}},\\ \Pi_{11}&=\Gamma_{11}+\frac{h^2e^{-\alpha h}}{2}MW^{-1}M^{\mathrm{T}}+hF^{\mathrm{T}}(V+W)F,\\ \Pi_{12}&=\frac{\alpha}{2}R_1\mathcal{I}_{12}-\frac{\nu c}{2h}R_1\mathcal{I}_{12}+R_1F-\frac{1}{2}R_2\mathcal{I}_{12},\\ \Lambda_{11}&=-\frac{\nu d}{h}\mathcal{I}_{12}^{\mathrm{T}}R_2\mathcal{I}_{12},\\ \Sigma_{11}&=\Lambda_{11}+hF^{\mathrm{T}}QF,\\ \Sigma_{12}&=\frac{\alpha}{2}R_2\mathcal{I}_{12}-\frac{\nu c}{2h}R_2\mathcal{I}_{12}+R_2F,\\ F&=(I_{N-1}\otimes A)\mathcal{I}_1+(ULW\otimes BK)\mathcal{I}_2.\end{aligned}$$

Proof: We adopt the following Lyapunov functional:

$$V(t,z_t)=\sum_{k=1}^{4}V(t,z_t) \tag{9}$$

with

$$\begin{aligned}V_1(t,z_t)&=z(t)P_1z(t)+\beta(t)\int_{t_k}^{t}e^{-\alpha(t-s)}\dot{z}^{\mathrm{T}}(s)Q\dot{z}(s)\\ V_2(t,z_t)&=\varphi(t)\left[z(t)-z(t_k)\right]^{\mathrm{T}}\left(R_1+\beta(t)R_2\right)\left[z(t)-z(t_k)\right]\\ V_3(t,z_t)&=\int_{-h}^{0}\int_{t+\zeta}^{t}e^{-\alpha(t-s)}\dot{z}^{\mathrm{T}}(s)V\dot{z}(s)dsd\zeta\\ V_4(t,x_t)&=\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\zeta}^{t}e^{-\alpha(t-s)}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)dsd\zeta d\theta\end{aligned}$$

where $\varphi(t)=\varphi_k(t)$ for $t\in[t_k,t_{k+1})$ with $\varphi_k(t)$ is defined by the unique solution of the initial value problem described as follows:

$$\begin{cases}\dot{\varphi}(t)=-\dfrac{\nu}{h_k+\epsilon}\left(\varphi^2+c\varphi+d\right), & t\in[t_k,t_{k+1})\\ \varphi(t_k)=\varphi_{0k}\end{cases}$$

By simple calculation, we get

$$\begin{aligned}&\dot{V}_1(t,z_t)\\ &=2z^{\mathrm{T}}(t)P_1\dot{z}(t)-\int_{t_k}^{t}e^{-\alpha(t-s)}\dot{z}^{\mathrm{T}}(s)Q\dot{z}(s)\\ &\quad+\beta(t)\dot{z}^{\mathrm{T}}(t)Q\dot{z}(t)\\ &\leq-\alpha V_1(t)+\alpha z^{\mathrm{T}}(t)P_1z(t)+2z^{\mathrm{T}}(t)P_1\dot{z}(t)\\ &\quad-\int_{t_k}^{t}e^{-\alpha h}\dot{z}^{\mathrm{T}}(s)Q\dot{z}(s)+\beta(t)\dot{z}^{\mathrm{T}}(t)Q\dot{z}(t)\end{aligned} \tag{10}$$

$$\begin{aligned}&\dot{V}_2(t,z_t)\\ &=\dot{\varphi}(t)\left[z(t)-z(t_k)\right]^{\mathrm{T}}\left(R_1+\beta(t)R_2\right)\left[z(t)-z(t_k)\right]\\ &\quad+2\varphi(t)\left[z(t)-z(t_k)^{\mathrm{T}}\right]\left(R_1+\beta(t)R_2\right)\dot{z}(t)\\ &\quad-\varphi(t)\left[z(t)-z(t_k)\right]^{\mathrm{T}}R_2\left[z(t)-z(t_k)\right]\end{aligned}$$

$$
\begin{aligned}
&\leq -\alpha V_2(t,z_t) + \alpha\varphi(t)\,[z(t)-z(t_k)]^{\mathrm{T}} R_1\,[z(t)-z(t_k)] \\
&\quad + \alpha\beta(t)\varphi(t)\,[z(t)-z(t_k)]^{\mathrm{T}} R_2\,[z(t)-z(t_k)] \\
&\quad - \frac{\nu}{h}\left(\varphi^2 + c\varphi + d\right)[z(t)-z(t_k)]^{\mathrm{T}} R_1\,[z(t)-z(t_k)] \\
&\quad - \beta(t)\frac{\nu}{h}\left(\varphi^2 + c\varphi + d\right)[z(t)-z(t_k)]^{\mathrm{T}} R_2\,[z(t)-z(t_k)] \\
&\quad + 2\varphi(t)\left[z(t)-z(t_k)^{\mathrm{T}}\right](R_1+\beta(t)R_2)\,\dot{z}(t) \\
&\quad - \varphi(t)\,[z(t)-z(t_k)]^{\mathrm{T}} R_2\,[z(t)-z(t_k)] \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
&\dot{V}_3(t,z_t) \\
&= h\dot{z}^{\mathrm{T}}(t)V\dot{z}(t) - \int_{-h}^{0} e^{\alpha s}\dot{z}^{\mathrm{T}}(t+s)V\dot{z}(t+s)ds \\
&\leq -\alpha V_3(t,z_t) + h\dot{z}^{\mathrm{T}}(t)V\dot{z}(t) \\
&\quad - e^{-\alpha h}\int_{t-h}^{t}\dot{z}^{\mathrm{T}}(s)V\dot{z}(s)ds \qquad (12)
\end{aligned}
$$

By applying Lemma 2, then

$$
\begin{aligned}
\dot{V}_3(t,z_t) &\leq -\alpha V_3(t,z_t) + h\dot{z}^{\mathrm{T}}(t)V\dot{z}(t) - \frac{3e^{-\alpha h}}{h}\Sigma^{\mathrm{T}}V\Sigma \\
&\quad \times \frac{-e^{-\alpha h}}{h}\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(\alpha)d\alpha V\int_{t-h}^{t}\dot{z}^{\mathrm{T}}(\alpha)d\alpha \\
&\leq -\alpha V_3(t,z_t) + h\dot{z}^{\mathrm{T}}(t)V\dot{z}(t) - \frac{3e^{-\alpha h}}{h}\Sigma^{\mathrm{T}}V\Sigma \\
&\quad - \frac{e^{-\alpha h}}{h}[z(t)-z(t-h)]^{\mathrm{T}}V[z(t)-z(t-h)]
\end{aligned} \qquad (13)
$$

$$
\begin{aligned}
\dot{V}_4(t,z_t) &= e^{-\alpha h}\frac{h^2}{2}\dot{z}^{\mathrm{T}}(t)W\dot{z}(t) \\
&\quad - e^{-\alpha h}\int_{-h}^{0}\int_{t+\zeta}^{t}\dot{z}^{\mathrm{T}}(s)W\dot{z}(s)dsd\zeta \qquad (14)
\end{aligned}
$$

Given any $4(N-1)n\times(N-1)n$ matrices G and M, one gets

$$
\begin{aligned}
0 &= 2\xi^{\mathrm{T}}M\left[z(t)-z(t_k)-\int_{t_k}^{t}\dot{z}(s)ds\right] \\
&\leq \xi^{\mathrm{T}}(t)\left[G\mathcal{I}_1 + \mathcal{I}_1^{\mathrm{T}}G^{\mathrm{T}} - G\mathcal{I}_3 - \mathcal{I}_3^{\mathrm{T}}G^{\mathrm{T}}\right]\xi(t) \\
&\quad + e^{\alpha h}(t-t_k)\xi^{\mathrm{T}}(t)GQ^{-1}G^{\mathrm{T}}\xi(t) \\
&\quad + e^{-\alpha h}\int_{t_k}^{t}\dot{z}^{\mathrm{T}}(s)Q\dot{z}(s) \qquad (15)
\end{aligned}
$$

and

$$
\begin{aligned}
0 &= 2\xi^{\mathrm{T}}(t)M \\
&\quad \times\left[hz(t) - \int_{t-h}^{t}z(s)ds - \int_{-h}^{0}\int_{t+\theta}^{t}\dot{z}(s)dsd\theta\right] \\
&\leq 2h\xi^{\mathrm{T}}(t)Mz(t) - 2\xi^{\mathrm{T}}M\int_{t-h}^{t}z(s)ds \\
&\quad + \frac{h^2}{2}\xi^{\mathrm{T}}(t)MW^{-1}M^{\mathrm{T}}\xi(t) \\
&\quad + \int_{-h}^{0}\int_{t+\theta}^{t}\dot{z}^{\mathrm{T}}(s)W\dot{z}(s)dsd\theta \qquad (16)
\end{aligned}
$$

which yields

$$
\begin{aligned}
\dot{V}_4(t,z_t) &\leq -\alpha V_4(t,z_t) + e^{-\alpha h}\frac{h^2}{2}\dot{z}^{\mathrm{T}}(t)W\dot{z}(t) \\
&\quad + e^{-\alpha h}\Big\{2h\xi^{\mathrm{T}}(t)Mz(t) - 2\xi^{\mathrm{T}}M\int_{t-h}^{t}z(s)ds \\
&\quad + \frac{h^2}{2}\xi^{\mathrm{T}}(t)MW^{-1}M^{\mathrm{T}}\xi(t)\Big\}. \qquad (17)
\end{aligned}
$$

Combing (11)-(18) implies

$$
\dot{V}(t,z_t) \leq -\alpha V(t,z_t) + \zeta^{\mathrm{T}}(t)\Omega\zeta(t), \quad t\in[t_k,t_{k+1}) \qquad (18)
$$

where $\Omega = \Pi + \beta(t)\Upsilon + (h-\beta(t))\Phi$ with $\Pi = \begin{bmatrix}\Pi_{11} & \Pi_{12}^{\mathrm{T}} \\ * & -\frac{\nu}{h}R_1\end{bmatrix}$, $\Upsilon = \begin{bmatrix}\Upsilon_{11} & \Upsilon_{12}^{\mathrm{T}} \\ * & -\frac{\nu}{h}R_2\end{bmatrix}$, $\Phi = \mathrm{diag}\{e^{\alpha h}GQ^{-1}G^{\mathrm{T}}, 0\}$. Clearly, $\dot{V}(t,z_t) \leq -\alpha V(t,z_t)$, if $\Omega < 0$, which also implies that system (5) is globally exponentially stable. Due to the fact that Ω is a convex combination of Υ and Φ on $\beta(t)\in[0,h]$, it is straightforward to notice that $\Omega < 0$ if the following condition is satisfied.

$$
\Upsilon + h\Pi < 0, \quad \Phi + h\Pi < 0 \qquad (19)
$$

By applying Schur Complement to the LMI in theorem 1, one gets (19). Thus, the proof is completed.

Remark 1: In order to fully capture the useful information of the sampled data pattern, the nonlinear function $\varphi(t)$ instead of an augmented matrix is introduced in the construction the establishment of the LKF here. It is worthy noting that the introduction of augmented matrices implies greater computational complexity of LMIs. Moreover, quite different with the LKF constructed in previous papers, a triple integral term is introduced, which leads to less conservatism than the quadratic Lyapunov function used in [10]. Hence, quite different to some existing results, our methods reduce the conservatism of the LMIs to certain degree.

Remark 2: Based on a literature survey, it is important to note that few researchers have worked on deriving sufficient conditions by considering the terms $\int_{t_k}^{t}\dot{z}(s)ds$. Fortunately, this term which can reduce the conservativeness of the condition is fully utilized in this paper. In addition, here we consider time-dependent Lyapunov functions. Moreover, the LKF considered here depends on both t_k and t_{k+1} that allows us to make use of full information of lower and upper bounds sampling instants. Thus, our method could move up the reduction of the conservatism to a new level.

B. CONSENSUS OF MULTI-AGENT SYSTEMS WITH PACKET LOSSES

So as to clarify effects of packet losses, we introduce $\lambda_1(t_1,t_2)$ and $\lambda_2(t_1,t_2)$, which are utilized to indicate the total activation time of the stable system and the unstable system during interval $[t_1,t_2)$, respectively. Define $\mu_1(t_1,t_2) = \frac{\lambda_1(t_1,t_2)}{t_2-t_1}$ and $\mu_2(t_1,t_2) = \frac{\lambda_2(t_1,t_2)}{t_2-t_1}$. Then, $\mu_1(t_1,t_2)$ and $\mu_2(t_1,t_2)$ are called the attention rate and the packet loss rate, respectively. Apparently, $\mu_1(t_1,t_2)+\mu_2(t_1,t_2)=1$.

The primary object of this subsection is to assess an upper bound $\mu_{\max}$ of $\mu_2(t_0, t)$ such that the multi-agent system with packet losses (4) reaches consensus based on the assumption that the consensus of multi-agent system without packet losses (3) has been well solved over $\varrho(\iota, h)$.

In the follow sequel, another novel condition is developed to highlight the consensus issue for multi-agent system with packet losses in a deterministic manner.

Theorem 2: Consider the multi-agent system (4) with sampling instants $t_k \in \varrho(\iota, h)$. Suppose that there exit constants c, d satisfying Lemma 2, $(N-1)n \times (N-1)n$ matrces $P_1 > 0$, $P_2 > 0$, $Q > 0$, $V > 0$, $W > 0$, $R_1 > 0$, $R_2 \geq 0$, $4(N-1)n \times (N-1)n$ matrices G and M, and scalars $u_i, i = 1, 2$ such that the conditions in Theorem 1 and the following LMIs hold:

$$P_2(A \otimes I_{N-1}) + (A \otimes I_{N-1})^{\mathrm{T}} P_2 - \delta P_2 < 0 \tag{20}$$

$$P_1 \leq u_1 P_2, \quad P_2 \leq u_2 P_1 \tag{21}$$

and if $\tau_a > \frac{\ln\sqrt{u_1 u_2}}{\alpha}$ and $\mu_{\max} < \frac{\alpha - \ln\sqrt{u_1 u_2}/\tau_a}{\alpha+\delta}$ for any positive constants α and δ , then, the multi-agent system with packet losses (4) reaches consensus.

Proof: Without loss of generality, suppose that there are m numbers of switchings taken place on the interval $[t_0, t)$, and $t \in [t_{k_0}, t_{k_0+1})$, where k_0 denotes the $k_0 th$ sampling. Set $l_1, l_2, \cdots, l_m$ be the corresponding switching instants satisfying $l_1 < l_2 < \cdots < l_m$. Then for each l_i, $i \in \{1, 2, \ldots, m\}$, there exists $k_0^* \in \{1, 2, \ldots, k_0\}$ such that $l_i = t_{k_0^*}$.

We consider the candidate multiple Lyapunov functions

$$V(s, x_s) = V_{\sigma(s)}(s, x_s) \tag{22}$$

where $V_1(s, x_s)$ is defined in (9) and $V_2(s, x_s) = x^{\mathrm{T}}(s) P_2 x(s)$. By applying Theorem (1) and conditions (20)(21), one deduces either

$$\begin{cases} V_1(s) \leq \exp(-(\alpha+\iota)(s-l_k))V_1(l_k), & s \in [l_k, l_{k+1}) \\ V_1(l_k) \leq u_1 V_2(l_k^-), & \text{if } \sigma(l_k) = 1 \end{cases} \tag{23}$$

or

$$\begin{cases} V_2(s) \leq \exp((\delta-\iota)(s-l_k))V_2(l_k), & s \in [l_k, l_{k+1}) \\ V_2(l_k) \leq u_2 V_1(l_k^-), & \text{if } \sigma(l_k) = 2 \end{cases} \tag{24}$$

where $V_i(l_k^-) = \lim_{\nu \to 0}(V_i(l_k - \nu)), i = 1, 2$.

Define $\varpi(\theta) = \begin{cases} -(\alpha+\iota) & \text{if } \sigma(\theta) = 1 \\ \delta - \iota & \text{if } \sigma(\theta) = 2 \end{cases}$. Without loss of generality, let $l_0 = t_0$. Denote $\rho_{us}(t_0, s)$ and $\rho_{su}(t_0, s)$, respectively, the number of switchings from the unstable subsystems to the stable ones and that from the stable subsystems to unstable ones over the interval $[t_0, s)$. Accordingly, one extracts that either $\rho_{us}(t_0, s) = \rho_{su}(t_0, s)$ or $\rho_{us}(t_0, s) = \rho_{su}(t_0, s) - 1$. Therefore, it yields that

$$u_1^{\rho_{su}(t_0,s)} u_2^{\rho_{us}(t_0,s)} \leq u^{\frac{1}{2}N_\sigma(t_0,s)} \tag{25}$$

where $u = \max\left\{\sqrt{\frac{u_1}{u_2}}, 1\right\}$.

Hence, combing (23)-(25), one gets, for any $s \in [t_{k_0}, t)$

$$\begin{aligned} V(s) &\leq e^{\varpi(l_j)(s-l_j)} V_{\sigma(l_j)}(l_j) \\ &\leq u_1^{\rho_{su}(t_0,s)} u_2^{\rho_{us}(t_0,s)} V_{\sigma(l_0)}(l_0) \\ &\quad \times \exp\left(\sum_{i=0}^{j-1} \varpi(l_i)(l_{i+1}-l_i) + \varpi(l_j)(s-l_j)\right) \\ &\leq u^{\frac{1}{2}} N_\sigma(t_0, s) V_{\sigma(l_0)}(l_0) \\ &\quad \times \exp\left(\sum_{i=0}^{j-1} -(\alpha+\iota)(l_{i+1}-l_i) + (\delta-\iota)(s-l_j)\right) \\ &\leq u V_1(t_0) e^{-\iota(s-t_0)} \\ &\quad \times \left(e^{\frac{\ln\sqrt{u_1u_2}}{\tau_a} - \alpha + (\alpha+\delta)\mu_2(t_0,s)}\right)^{s-t_0}, \end{aligned} \tag{26}$$

which completes the proof.

As a final theorem in this paper, we want to focus on the existence condition for the consensus controller for the multi-agent system with packet losses (4).

Theorem 3: Given constants, $h > 0$, $\alpha > 0$, $\kappa_j > 0$, $j = 1, 2, 3$, and constants c, d satisfying Lemma 2, system (4) reaches consensus exponentially over $\varrho(h)$, if the graph G contains a directed spanning tree and there exit $(N-1)n \times (N-1)n$ real matrices $\widetilde{P}_i > 0$, R_i, $i = 1, 2$, $\widetilde{Q} > 0$, $\widetilde{V} > 0$, $\widetilde{W} > 0$, $4(N-1)n \times (N-1)n$ matrices $\widetilde{G}$, $\widetilde{M}$ such that the following LMIs hold, (27)–(31), as shown at the bottom of the next page, where

$$\begin{aligned} \Gamma_{11} &= \alpha \mathcal{I}_1^{\mathrm{T}} \bar{P}_1 \mathcal{I}_1 + \mathcal{I}_1^{\mathrm{T}} \widetilde{F} + \widetilde{F}^{\mathrm{T}} \mathcal{I}_1 + \widetilde{G} \mathcal{I}_1 \\ &\quad - \frac{\kappa_1 \nu d}{h} \mathcal{I}_{12}^{\mathrm{T}} \bar{P}_1 \mathcal{I}_{12} - \frac{3e^{-\alpha h}}{h} \varepsilon^{\mathrm{T}} \widetilde{V} \varepsilon + \mathcal{I}_1^{\mathrm{T}} \widetilde{G}^{\mathrm{T}} \\ &\quad - \frac{e^{-\alpha h}}{h} \mathcal{I}_{13}^{\mathrm{T}} \widetilde{V} \mathcal{I}_{13} - \widetilde{G} \mathcal{I}_3 - \mathcal{I}_3^{\mathrm{T}} \widetilde{G}^{\mathrm{T}} + h e^{-\alpha h} \widetilde{M} \mathcal{I}_1 \\ &\quad + h e^{-\alpha h} \mathcal{I}_1^{\mathrm{T}} \widetilde{M}^{\mathrm{T}} - e^{-\alpha h} \widetilde{M} \mathcal{I}_4 - e^{-\alpha h} \mathcal{I}_4^{\mathrm{T}} \widetilde{M}^{\mathrm{T}}, \\ \Pi_{12} &= \frac{\alpha \kappa_1}{2} \bar{P}_1 \mathcal{I}_{12} - \frac{\nu c \kappa_1}{2h} \bar{P}_1 \mathcal{I}_{12} \\ &\quad + \kappa_1 \widetilde{F} - \frac{\kappa_2}{2} \bar{P}_1 \mathcal{I}_{12}, \\ \Lambda_{11} &= -\frac{\nu d \kappa_2}{h} \mathcal{I}_{12}^{\mathrm{T}} \bar{P}_1 \mathcal{I}_{12}, \\ \Sigma_{12} &= \frac{\alpha \kappa_2}{2} \bar{P}_1 \mathcal{I}_{12} - \frac{\nu c \kappa_2}{2h} \bar{P}_1 \mathcal{I}_{12} + \kappa_2 \widetilde{F}, \\ F &= \left(I_{N-1} \otimes A \widetilde{P}_1\right) \mathcal{I}_1 + \left(ULW \otimes B \widetilde{K}\right) \mathcal{I}_2 \\ \bar{P}_1 &= I_{N-1} \otimes \widetilde{P}_1, \\ \tilde{\Upsilon} &= \kappa_3^2 (\widetilde{V} + \widetilde{W}) - 2\kappa_3 \bar{P}. \end{aligned}$$

Moreover, the consensus controller gain is determined by $K = \widetilde{K} \widetilde{P}_1^{-1}$.

Proof: Define $P_1 = I_{N-1} \otimes U$, $R_i = \kappa_i P_1$, $\tilde{P}_1 = U_1^{-1}$, $\tilde{Q} = P_1^{-1} Q P_1^{-1}$, $\tilde{V} = P_1^{-1} V P_1^{-1}$, $\tilde{W} = P_1^{-1} W P_1^{-1}$, $J = \mathrm{diag}\{P_1^{-1}, P_1^{-1}, P_1^{-1}, P_1^{-1}\}$ $\tilde{G} = JGJ$, $\bar{P}_1 = I_{N-1} \otimes \tilde{P}_1$, $\tilde{K} = KU^{-1}$, $\tilde{M} = JMJ$, $P_2 = \tilde{P}_2^{-1}$. Pre-and post multiplying both sides of (8) by $\mathrm{diag}\{J, P_1^{-1}, (V+W+Q)^{-1}, P^{-1}\}$, both sides of (9) by $\mathrm{diag}\{J, P_1^{-1}, (V+W)^{-1}, P^{-1}\}$, both sides of (21) by P_2^{-1}, both sides of the first inequality by $\text{diag}\{P_2^{-1}, I_{(N-1)n\times(N-1)n}\}$ and both sides of the second inequality by $\text{diag}\{P_1^{-1}, I_{(N-1)n\times(N-1)n}\}$, respectively. Applying the matrix inequality $-P_1^{-1}(V+W)^{-1}P_1 \leq \kappa_3^2(V+W)-2\kappa_3 P$, one obtains the corresponding LMIs in theorem 3.

IV. NUMERICAL EXAMPLES

In this section, a numerical example is presented to verify the effectiveness of our results.

Consider a system consisting of three networked quadrotors [33] and each quadrotor share the same second order dynamical model described as

$$\ddot{\phi}_i + a\dot{\phi}_i + b\phi_i = u_i(\theta(t_k)) \tag{32}$$

where ϕ_i stands for the position parameter of the quadrotor. a and b denote the damping and spring constants, respectively. The equation (32) can be rewritten as the following format

$$\begin{bmatrix} \dot{\phi}_i \\ \ddot{\phi}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} \phi_i \\ \dot{\phi}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i(\theta(t_k))$$

In order to investigate the related formation behavior, the external control input is chosen as

$$u_i(t_k) = c\alpha_i - \sum_{j\neq i, j=1}^{N} a_{ij}(\theta(t_k))K(\phi_i - \alpha_i - \phi_j + \alpha_j)$$

Let $x_i = \left[\phi_i - \alpha_i \ \dot{\phi}_i\right]^{\mathrm{T}}$, and choose $a = 0.1$, $b = 0$. Without loss of generality, the communication with a directed network topology is depicted in Fig.1 and the corresponding Laplacian matrix in this scenario is presented as follows.

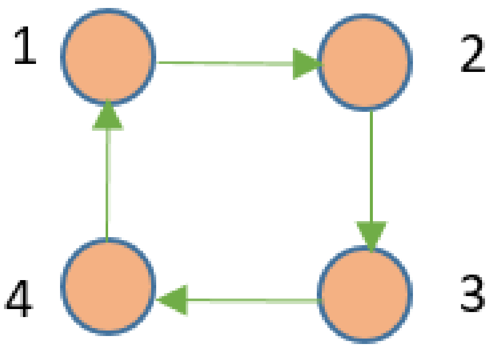

FIGURE 1. The communication topology graph.

$$L_1 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \tag{33}$$

Note that the topology contains a directed spanning tree. Firstly, we shall determine the maximal sampling interval h guaranteeing the feasibility of LMIs in Theorem 1. By solving the LMIs in Theorem 2, one obtains $h < 1.7210$. Similar as [33], the related distances among agents are defined as $\alpha_1 = 1$, $\alpha_2 = 2$, $\alpha_3 = 3$ and $\alpha_4 = 4$, The initial position and velocity values are $\phi_1 = 1$, $\phi_2 = 3$, $\phi_3 = 6$, $\phi_4 = 10$ and $\dot{\phi}_1 = 4$, $\dot{\phi}_2 = 3$, $\dot{\phi}_3 = -2$, $\dot{\phi}_4 = -3$ respectively. The corresponding state trajectories for $q_i(t)$ and $\dot{q}_i(t)$ for four agents are depicted in Fig. 2. Moreover, set the maximum sampling interval $h = 0.8$, the average dwell time $\tau_a = 1.0134$, and the packet losses rate $\tau = 0.25$, set $c = 0.2$, $d = 0.1$, $\alpha = 0.3$. Then, by solving the LMIs in Theorem 3,

$$\begin{bmatrix} \widetilde{\Gamma}_{11} + h\widetilde{\Lambda}_{11} & \widetilde{\Pi}_{12} + h\widetilde{\Sigma}_{12} & h\widetilde{F}^{\mathrm{T}} & \frac{h^2 e^{\alpha h}}{2}\widetilde{M} \\ * & -\frac{\nu}{h}(\kappa_1 + h\kappa_2) & 0 & 0 \\ * & * & h(\widetilde{\Upsilon} + \kappa_3^2\widetilde{Q}) & 0 \\ * & * & * & -\frac{h^2 e^{\alpha h}}{2}\widetilde{W} \end{bmatrix} < 0 \tag{27}$$

$$\begin{bmatrix} \widetilde{\Gamma}_{11} & \widetilde{\Pi}_{12} & he^{\alpha h}G & h\widetilde{F}^{\mathrm{T}} & \frac{h^2 e^{\alpha h}}{2}\widetilde{M} \\ * & -\frac{\nu}{h}R_1 & 0 & 0 & 0 \\ * & * & -\widetilde{Q}he^{\alpha h} & 0 & 0 \\ * & * & * & h\widetilde{\Upsilon} & 0 \\ * & * & * & * & -\frac{h^2 e^{\alpha h}}{2}\widetilde{W} \end{bmatrix} < 0 \tag{28}$$

$$\widetilde{P}_2(A \otimes I_{N-1}) + A \otimes I_{N-1})^{\mathrm{T}}\widetilde{P}_2 - \delta\widetilde{P}_2 < 0 \tag{29}$$

$$\begin{bmatrix} -u_1\widetilde{P}_2 & \widetilde{P}_2 \\ * & -\widetilde{P}_1 \end{bmatrix} < 0 \tag{30}$$

$$\begin{bmatrix} -u_2\widetilde{P}_1 & \widetilde{P}_1 \\ * & -\widetilde{P}_2 \end{bmatrix} < 0 \tag{31}$$

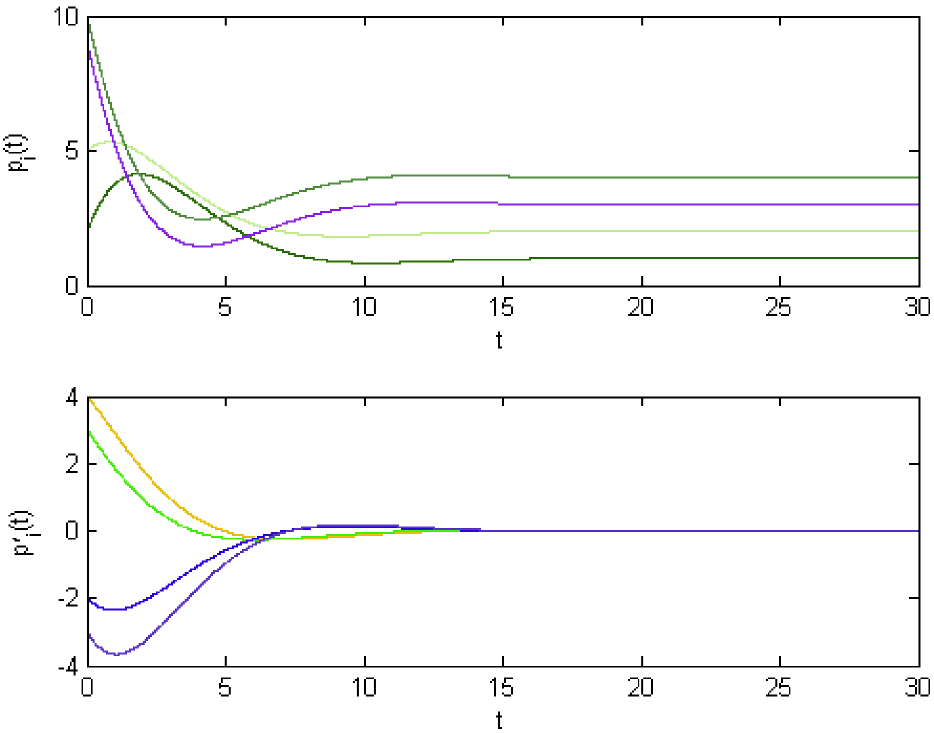

FIGURE 2. State trajectories of position $p_i(t)$ and $\dot{p}_i(t)$ for four agents.

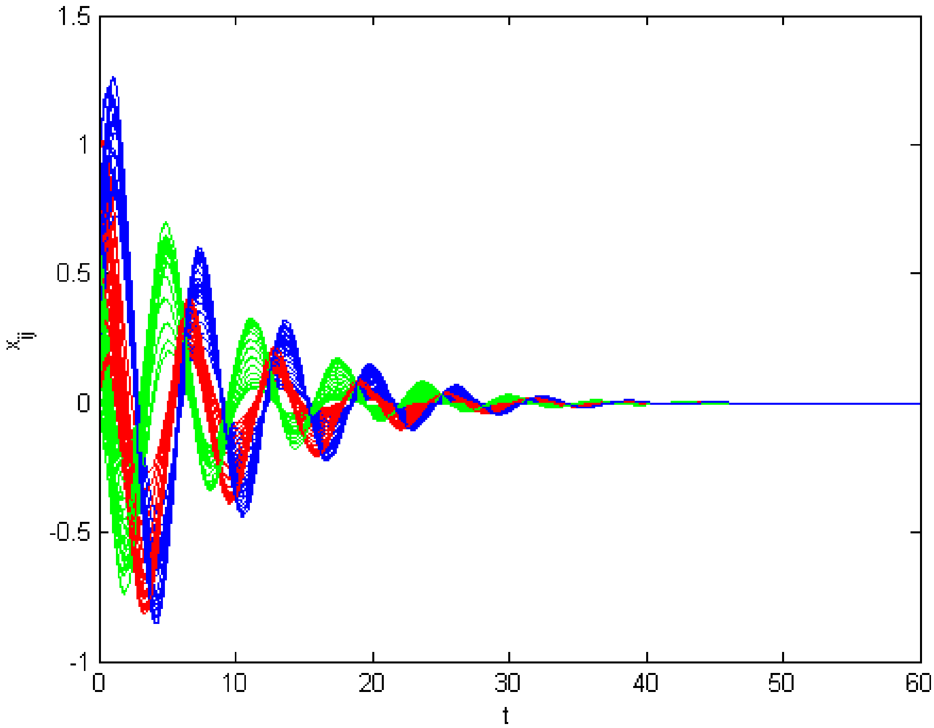

FIGURE 3. State trajectories of agents in (4) with packet losses occurred in deterministic manner.

the gain matrix K is determined as

$$K = \begin{bmatrix} -2.9217 & -1.4398 \\ -1.3932 & -1.6429 \end{bmatrix}. \tag{34}$$

Example 2: As a matter of fact, multi-agent systems is a large-scale one, in which contains innumerable agents. Hence, here, we will consider the multi-agent systems with 20 agents satisfying the nearest neighbor connecting rule,

TABLE 1. Maximum allowable sampling interval of related literatures.

Methods	[10]	[29]	[33]	Corollary1
Maximal sampling period	0.4662	0.6093	0.7479	0.8394

whose Laplacian matrix is described as follows:

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & \cdots & -1 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & 0 \\ -1 & 0 & 0 & \cdots & \cdots & 2 \end{bmatrix}_{20\times 20} \tag{35}$$

In this scenario, the other parameter given by

$$A = \begin{bmatrix} 0.4 & 0.6 & -0.7 \\ -0.5 & -0.5 & -0.5 \\ 1 & 1 & 0 \end{bmatrix}; \tag{36}$$

and $B = I_3$. Accordingly, we can obtain the maximal sampling period $h = 0.79$ by solving the LMIs in in Theorem 1. The state trajectories of this example is shown in Fig. 3, which confirms the validation of our results effectively .

V. CONCLUSIONS

In this paper, we have been focused on consensus problem for a class of multi-agent systems in presence of packet losses in a in a deterministic manner. By exploiting the joint effect of packet losses and sampled data measurements on the consensusability, new results on maximal sampled period have been derived via the establishment of the Lyapunov Krasovskii functional. By introduced some novel terms and a nonlinear function which lead to less conservatism into the construction of the LKFs, a larger permissable sampling period was obtained. Meanwhile, so as to well illustrate the superiority of our method, some comparison results focusing on the conservatism issue for the obtained conditions was put forward. The results of this paper extend previous results. For future research, it is interesting to consider consensus of multi-agent systems with non-identical nonlinear dynamics and asynchronous sampled-data with packet losses.

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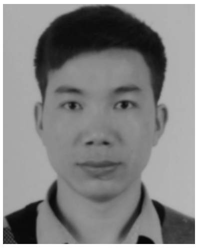

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