Economic growth effects of alternative climate change impact channels in economic modeling

Appendix

A.1 Derivations

In this section we present the detailed derivation of the analytical results and the damage expressions in Section 2. Note again that all shocks assumed here are one-time shocks.

A.1.1 Growth rate after shock

We assume a Ramsey model with labor-augmenting technological progress $Y(t) = F[K(t), \chi(t)L(t)]$, which can be expressed in units of effective labor as $\bar{y} = f(\bar{k})$. The time paths of effective consumption $\bar{c}$ and effective capital $\bar{k}$ are determined by[1]

$$\dot{\bar{k}} = f(\bar{k}) - \bar{c} - (x + n + \delta)\,\bar{k} \qquad \text{(A 1)}$$

and

$$\frac{\dot{\bar{c}}}{\bar{c}} = \frac{\dot{c}}{c} - x = \frac{1}{\theta}[f'(\bar{k}) - \delta - \varepsilon - \theta x] \qquad \text{(A 2)}$$

Here, x is the growth rate of labor productivity, n is the population growth rate, δ is the capital depreciation rate, ε is the rate of time preference and Θ is the negative elasticity of marginal utility.

In order to provide a quantitative assessment of the speed of convergence, we use a log-linearized version of these equations, as provided by Barro & Sala-i-Martin (2004, p.111)

$$\ln[\bar{y}(t)] = e^{-\beta t}\ln[\bar{y}(0)] + \left(1 - e^{-\beta t}\right)\ln(\bar{y}^{*}) \qquad \text{(A 3)}$$

where β>0 is the convergence coefficient and the * marks the steady-state value.

[1] See Barro & Sala-i-Martin (2004) p. 97.

We assume that the shock occurs at t=0. The growth rate of effective output after the shock is then given by

$$\frac{d\ln\bar{y}(t)}{dt} = \beta e^{-\beta t}\ln\left[\frac{\bar{y}^*}{\bar{y}(0)}\right] \overset{t=0}{=} \beta\ln\left[\frac{\bar{y}^*}{\bar{\hat{y}}}\right] \tag{A 4}$$

Here, $\bar{\hat{y}}$ indicates the value of effective output after the shock. Per capita GDP is $y = \frac{Y}{L} = \frac{\chi Y}{\chi L} = \chi\bar{y}$ and labor productivity is assumed to grow exponentially with a constant growth rate x, described by $\chi(t) = \chi(0)e^{xt}$. It follows for the growth rate of per capita output after the shock

$$g = \frac{d\ \ln y}{dt} = \frac{d\ \ln\chi}{dt} + \frac{d\ln\bar{y}}{dt} = x + \beta\ln\left[\frac{\bar{y}^*}{\bar{\hat{y}}}\right] \tag{A 5}$$

while without a shock in the steady state g=x.

A.1.2 Shock on output

A direct, un-anticipated, one-time shock on output Ω changes GDP to an after-shock level of $\hat{Y} = (1-\Omega)Y^*$. In the long run, however, the economy will again converge to the (before shock) steady state intensive-form GDP level $\bar{y}^* = \frac{Y^*}{\chi L}$ and the differences in the GDP levels between the original and the disturbed economy diminishes, because total factor productivity is not lowered permanently. A lower GDP will also lower the capital stock after the shock. Therefore, the economy will experience additional growth in order to converge back to the steady-state level. Inserting the after-shock GDP level into eq. (A 5), we get for the after-shock growth rate in the case of an output shock

$$g_Y = x - \beta\ln(1-\Omega). \tag{A 6}$$

Note that $-\beta\ln(1-\Omega) > 0$ as $0 < \Omega < 1$. The GDP per capita growth after the shock will therefore be higher than without the shock. This increase in economic growth after a shock does not imply a higher social welfare as the GDP level remains below the counterfactual GDP of the economy without the shock.

A.1.2 Shock on capital stock

An immediate, un-anticipated marginal change in the capital stock $\frac{dK}{K}$ changes GDP by $\frac{d\ln Y}{d\ln K}\frac{dK}{K} = \frac{d\,Y}{d\,K}\frac{K}{Y}\frac{dK}{K} = \Gamma_K\frac{dK}{K}$ where Γ_K is the capital income share of the economy. Hence, for a marginal fall of the capital stock by Ω_K, the new immediate after-shock GDP $\hat{Y}$ will be

$$\hat{Y} = (1 - \Gamma_K\Omega_K)Y^*. \tag{A 7}$$

Like for a shock on output, in the long run, the shock effects will diminish over time and the GDP will return to the pre-shock level. Capital is now affected directly, leading also to additional convergence growth after the shock. Inserting eq. (A 7) into eq. (A 5) we get for the per capita GDP growth rate after a shock on capital

$$g_K = x - \beta\ln(1 - \Gamma_K\Omega_K). \tag{A 8}$$

The GDP per capita growth after the shock will be higher than without the shock as $0 < \Gamma_K\Omega_K < 1$.

A.1.3 Shock on labor productivity

Again, consider the case of a steady state of the economy, i.e. effective capital $\bar{k}^* = \frac{K^*}{\chi L}$ is constant while labor productivity χ grows at constant rate x. An un-anticipated marginal relative reduction in labor productivity by Ω_χ reduces labor productivity to $\hat{\chi} = (1 - \Omega_\chi)\chi$.

A marginal relative change of effective labor $d\ln\chi L$ leads to a marginal relative change in GDP by $\frac{d\ln Y}{d\ln\chi L} = \frac{d\,Y}{d\,\chi L}\frac{\chi L}{Y} = Y_{\chi L}\frac{\chi L}{Y} = \Gamma_\chi$ (labor income share). Hence, a marginal relative reduction of effective labor by Ω_χ implies an approximate immediate GDP response of $\Omega_\chi\Gamma_\chi$ and the after-shock GDP will be

$$\hat{Y} = \left(1 - \Gamma_\chi\Omega_\chi\right)Y^* \tag{A 9}$$

In the steady state, consumption per effective worker is constant. Using the equation for the time path of effective consumption in a Ramsey model given by eq. (A 2), constant consumption implies $f'\left(\bar{k}^*\right) = \delta + \rho + \theta x$. Thus, the capital per effective worker in the steady state cannot change when the labor productivity is lower which implies that the new (long-run) steady state capital stock after the shock equals the original one, $\hat{\bar{k}}^* = \bar{k}^*$, or, equivalently, total capital stock $\widehat{K}^* = (1 - \Omega_\chi)\, K^*$. GDP in the new steady state, $\hat{Y}^*$, will at every point of time be permanently lower than the original GDP Y^*:

$$\hat{Y}^* = F\big((1-\Omega_\chi)\, K^*, (1-\Omega_\chi)\, \chi L^*\big) = (1-\Omega_\chi)\, F(K^*, \chi L^*) = (1-\Omega_\chi)\, Y^*. \qquad \text{(A 10)}$$

Directly after the shock, the effective capital stock exceeds the steady state capital stock, $\bar{k}' = \frac{\bar{k}^*}{(1-\Omega_\chi)} > \bar{k}^*$. Therefore the economy will experience negative convergence growth to the new steady state capital stock. Using equations (A 9) and (A 10), we get for the effective long-term GDP $\hat{\bar{y}}^* = \frac{\hat{Y}^*}{(1-\Omega_\chi)\,\chi L} = \frac{Y^*}{\chi L}$ and for the effective GDP right after the shock $\hat{\bar{y}} = \frac{\hat{Y}}{(1-\Omega_\chi)\chi L} = \frac{[1-\Gamma_\chi \Omega_\chi]}{(1-\Omega_\chi)} \frac{Y^*}{\chi L}$. The GDP per capita growth rate after shock follows from inserting these expressions into eq. (A 5), resulting in

$$g_A = x + \beta \ln\left(\frac{1-\Omega_\chi}{1-\Gamma_\chi \Omega_\chi}\right) \qquad \text{(A 11)}$$

Note that $\beta \ln\left(\frac{1-\Omega_\chi}{1-\Gamma_\chi \Omega_\chi}\right) < 0$ as $0 < \Gamma_\chi < 1$ and $0 < \Omega_\chi < 1$. Contrary to the destruction of capital, GDP per capita growth after the shock will be lower than without the shock.

A.2 Derivation of damage factors in the comparability approach

The comparability approach yields the channel-specific damage factors according to $\Omega_t^Y = \frac{Y_t^{N,Y}}{Y_t^G} = \frac{Y_t^{N,\chi\_L}}{Y_t^G} = \frac{Y_t^{N,K}}{Y_t^G} = \frac{Y_t^{N,L}}{Y_t^G}$.

A.2.1 Capital channel

$$\Omega_t^Y = \frac{Y_t^{N,K}}{Y_t^G} = \frac{a_0[\alpha K_t^{*\sigma} + \beta(\chi_t^L L_t)^\sigma]^{1/\sigma}}{a_0[\alpha K_t^{\sigma} + \beta(\chi_t^L L_t)^\sigma]^{1/\sigma}} \qquad \text{(A 12)}$$

Using $K_t^* = \Omega_t^K K_t$ and $\Omega_t^Y = 1 - \Omega_t$, we solve for Ω_t^K as

$$\Omega_t^K = \left[\left\{\left(\frac{\Omega_t^Y Y_t^G}{a_0}\right)^\sigma - \beta(\chi_t^L L_t)^\sigma\right\}\frac{1}{\alpha}\right]^{1/\sigma} \frac{1}{K_t} \qquad \text{(A 13)}$$

$$= \left[\left(\frac{\Omega_t^Y a_0 (\alpha K_t^\sigma + \beta(\chi_t^L L_t)^\sigma)^{1/\sigma}}{a_0 \alpha^{1/\sigma} K_t} \right)^\sigma - \frac{\beta(\chi_t^L L_t)^\sigma}{\alpha K_t^\sigma} \right]^{1/\sigma}$$

$$= \left[\Omega_t^{Y\sigma} \left(1 + \frac{\beta(\chi_t^L L_t)^\sigma}{\alpha K_t^\sigma} \right) - \frac{\beta(\chi_t^L L_t)^\sigma}{\alpha K_t^\sigma} \right]^{1/\sigma}$$

$$= \left[\Omega_t^{Y\sigma} + (\Omega_t^{Y\sigma} - 1) \frac{\beta(\chi_t^L L_t)^\sigma}{\alpha K_t^\sigma} \right]^{1/\sigma}$$

$$= \left[(1 - \Omega_t)^\sigma - (1 - (1 - \Omega_t)^\sigma) \frac{\beta(\chi_t^L L_t)^\sigma}{\alpha K_t^\sigma} \right]^{1/\sigma}.$$

A.2.2 Labor and labor productivity channel

The other channels are derived similarly. Due to the labor-augmenting technical progress it does not matter if the damage affects labor or labor productivity in the derivation of the damage factor, however the effects are different due to the different growth of the two factors. We show the derivation here for labor.

$$\Omega_t^Y = \frac{Y_t^{N,L}}{Y_t^G} = \frac{a_0[\alpha K_t{}^\sigma + \beta(\chi_t^L L_t^*)^\sigma]^{1/\sigma}}{a_0[\alpha K_t{}^\sigma + \beta(\chi_t^L L_t)^\sigma]^{1/\sigma}} \quad \text{(A 14)}$$

Using $L_t^* = \Omega_t^L L_t$ and $\Omega_t^Y = 1 - \Omega_t$, we solve for Ω_t^L as

$$\Omega_t^L = \left[\left\{ \left(\frac{\Omega_t^Y Y_t^G}{a_0} \right)^\sigma - \alpha K_t^\sigma \right\} \frac{1}{\beta} \right]^{1/\sigma} \frac{1}{\chi_t^L L_t} \quad \text{(A 15)}$$

$$= \left[\left(\frac{\Omega_t^Y a_0 (\alpha K_t^\sigma + \beta(\chi_t^L L_t)^\sigma)^{1/\sigma}}{a_0 \beta^{1/\sigma} \chi_t^L L_t} \right)^\sigma - \frac{\alpha K_t^\sigma}{\beta(\chi_t^L L_t)^\sigma} \right]^{1/\sigma}$$

$$= \left[\Omega_t^{Y\sigma} \left(1 + \frac{\alpha K_t^\sigma}{\beta(\chi_t^L L_t)^\sigma} \right) - \frac{\alpha K_t^\sigma}{\beta(\chi_t^L L_t)^\sigma} \right]^{1/\sigma}$$

$$= \left[\Omega_t^{Y\sigma} + (\Omega_t^{Y\sigma} - 1) \frac{\alpha K_t^\sigma}{\beta(\chi_t^L L_t)^\sigma} \right]^{1/\sigma}$$

$$= \left[(1-\Omega_t)^\sigma - (1-(1-\Omega_t)^\sigma)\frac{\alpha K_t^\sigma}{\beta(\chi_t^L L_t)^\sigma}\right]^{1/\sigma}.$$

A.3 Additional illustrations and discussions

A.3.1 Labor and productivity growth rates after a factor-specific shock

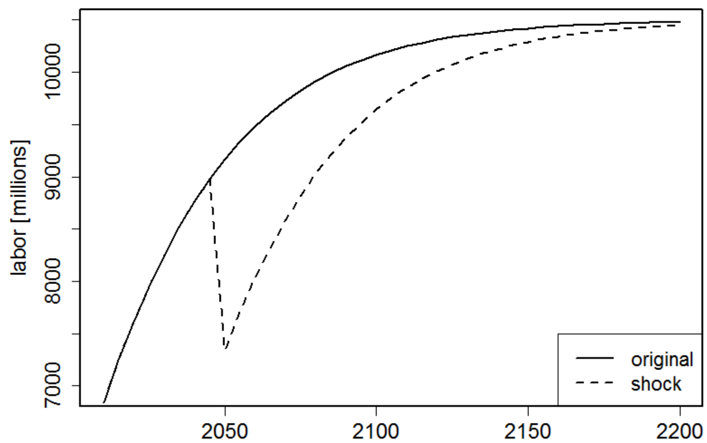

Figure S 1: Evolution of the labor force in the baseline (solid line) and in case of a shock in 2050 (dashed line). Due to the formulation of labor growth, the asymptotical growth towards the final number of 10.5 billion people increases after the shock. In the baseline population is equal to labor force. In case of the shock, the labor force affected by the shock enters the production function, but the baseline population remains unchanged, entering the utility function.

A one-time, temporary shock Ω^{χ} at t_s affects labor productivity as $\chi^*_{t_s} = \Omega^{\chi}\chi_{t_s}$. In the following we derive the growth rates at the time of the shock, expressed as $g^*_{t_s} = \left(\frac{\chi^*_{t_s+1}}{\chi^*_{t_s}}\right)^{1/\Delta t} - 1$, using both the permanent and the dissipative productivity formulation given by equations (2)and (4), respectively.

For the permanent case, the productivity after the shock is given by $\chi^*_{t_s+1} = \frac{\chi^*_{t_s}}{1-g^L_{t_s}} = \frac{\Omega^{\chi}\chi_{t_s}}{1-g^L_{t_s}} = \Omega^{\chi}\chi_{t_s+1}$. For the growth rate it follows that $g^*_{t_s} = \left(\frac{\chi^*_{t_s+1}}{\chi^*_{t_s}}\right)^{1/\Delta t} - 1 = \left(\frac{\Omega^{\chi}\chi_{t_s+1}}{\Omega^{\chi}\chi_{t_s}}\right)^{\frac{1}{\Delta t}} - 1 = \left(\frac{\chi_{t_s+1}}{\chi_{t_s}}\right)^{\frac{1}{\Delta t}} - 1 = g_{t_s}$, i.e. the growth rate stays the same and the shock remains permanent.

The productivity after the shock in the dissipative case is given by $\chi^*_{t_s+1} = (1-\delta^{\chi})^{\Delta t}\chi^*_{t_s} + P^{\chi}_{t_s} = (1-\delta^{\chi})^{\Delta t}\Omega^{\chi}\chi_{t_s} + P^{\chi}_{t_s}$. Then $g^*_{t_s} = \left(\frac{\chi^*_{t_s+1}}{\chi^*_{t_s}}\right)^{1/\Delta t} - 1 = \left(\frac{(1-\delta^{\chi})^{\Delta t}\Omega^{\chi}\chi_{t_s} + P^{\chi}_{t_s}}{\Omega^{\chi}\chi_{t_s}}\right)^{\Delta t} - 1 = \left[(1-\delta^{\chi})^{\Delta t} + \frac{P^{\chi}_{t_s}}{\Omega^{\chi}\chi_{t_s}}\right]^{\Delta t} - 1$. Without shock, $g_{t_s} = \left(\frac{\chi_{t_s+1}}{\chi_{t_s}}\right)^{1/\Delta t} - 1 = \left[(1-\delta^{\chi})^{\Delta t} + \frac{P^{\chi}_{t_s}}{\chi_{t_s}}\right]^{\Delta t} - 1$. In this case, the growth rate is influenced by the shock and as $\Omega^{\chi} < 1$, it follows that $g^*_{t_s} > g_{t_s}$. Therefore the shock will dissipate over time. Both cases are shown, together with the corresponding productivity pathways, are shown in Figure S2.

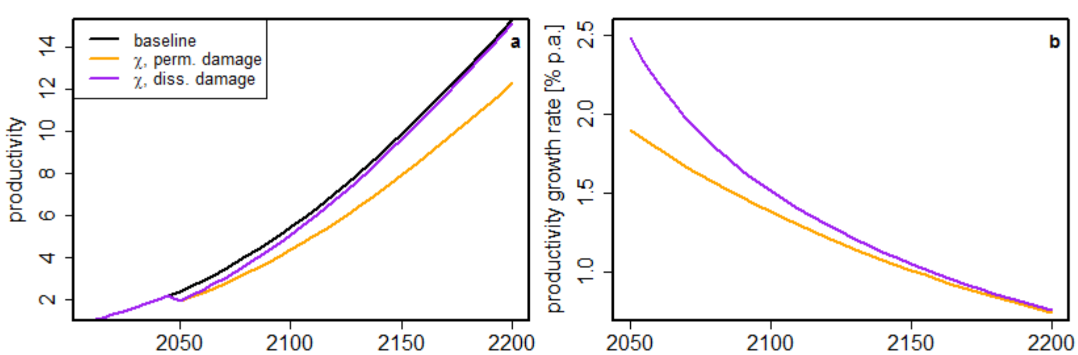

Figure S 2: Growth dynamics for the two different labor productivity formulations. Panel a shows productivity χ_t, panel b the corresponding growth rate. In black is the baseline case, in orange and purple the two with a 15% GDP shock in 2050. Orange

is the case with a permanent effect, using the original DICE productivity equation ($\chi_{t+1}^{L,p} = \chi_t^{L,p}/(1-g_t^L)$), in purple the formulation with a dissipating effect based on the endogenous growth formulation but using an exogenous investment path as driver ($\chi_{t+1}^{L,d} = (1-\delta^\chi)^{\Delta t}\chi_t^{L,d} + P_t^\chi$). In the dissipating case, the productivity growth rate after the shock is increased, while in the permanent case the growth rate is unchanged from the baseline growth rate (i.e. the orange line lies on top of the black line in the right-hand panel).

A.3.2 Further sensitivity cases: anticipated shocks, shock magnitude, capital adjustment costs

In our standard setting, the shock is not anticipated. In fact, the savings rate is fixed until the time of the shock itself (2050), which means that any adaptive reaction to the shock can only happen in the time step following the shock. The technical reason for this is to ensure the comparability of the shock when it hits. It can be interpreted as the dynamics under uncertainty in the presence of low probability high impact shocks. When the system can react immediately at the time of the shock (i.e. the savings rate is endogenous in 2050), the reactions in terms of savings rate adjustments are stronger than with the delay in both directions (increased savings for the capital channel, decreased savings for the others, Figure S 3 panel b). If the savings rate is completely endogenous, i.e. the shock is anticipated from the start, for all channels there is an increase in savings before the shock (Figure S 3 panel d). Table S1 shows results in terms of the change in BGE. With exogenous growth those changes are all welfare increasing, i.e. the change in BGE decreases. With endogenous growth, the BGE changes are larger and the welfare effect depends on the savings reaction at the time of the shock. If that is positive, as in the capital channel, again immediate adaptation and even more so anticipation increase welfare. If savings are reduced at the time of the shock due to an overcapitalization (all other channels), this affects also productivity through the investment spillover, leading to a reduction in welfare compared to the standard case and therefore an increase in ΔBGE (bottom part of Table S1). This can be remedied to some degree by full anticipation and related adaptation, but only for the output channel is that adaptation strong enough to result in a smaller change in BGE than for the standard case. With both exogenous and endogenous growth the capital channel benefits most from the adaptation option through anticipation or immediate reaction at the time of shock.

Table S1: Change in balanced growth equivalent (ΔBGE) in % in the different channels and with the different savings rate settings for exogenous growth (top 3 rows) and endogenous growth (bottom rows).

	Y damage	K damage	L damage	χ, perm. damage	χ, diss. damage
Standard (free S after 2050)	0.95	1.67	4.17	9.22	4.52
Free S from 2050 onward	0.93	1.64	4.16	9.22	4.51
Free S (anticipation case)	0.93	1.6	4.16	9.21	4.51
EG (free S after 2050)	1.23	1.91	5.97	-	6.71
EG (free S from 2050 onward)	1.45	1.59	6.15	-	6.92
EG (free S, anticipation case)	1.09	0.77	6.04	-	6.82

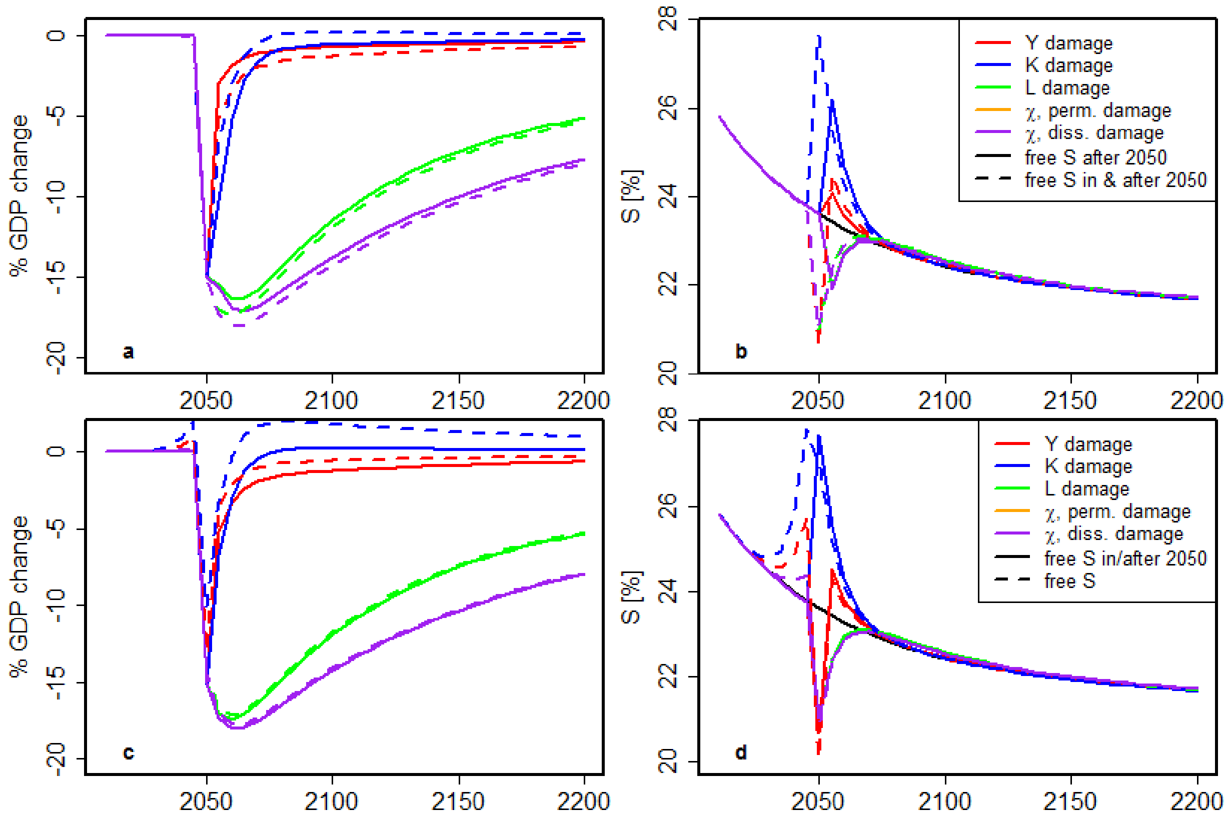

Figure S 3: GDP change and savings rate for the different impact channels (colors). Panels a and b compare the standard case with an endogenous savings rate after the shock to a case where the savings rate only fixed until the shock (solid vs. dashed lines). Panels A and B compare the case with full anticipation (completely endogenous savings rate, dashed lines) to the case where the savings rate is endogenous from the shock onward (solid lines – equal to the dashed line case in the top panels).

Figure S 4 and Figure S 5 show the effects of different shock magnitudes on the results. The principle long-term shock dynamics are not affected by the size of the shock, but the half-life times change in different directions depending on the impact channel. For the output, capital, labor and permanent productivity damage channels, half-life times increase somewhat with the shock magnitude, driven by the indirect capital effect (Figure S 5, panel a). However, for the dissipative productivity damage half-life times decrease. This is a savings rate effect. While the labor formulation is designed to asymptotically reach a long-term level of 10.5 billion people despite the shocks, the productivity, despite the in-build increase in the growth rate in response to the shock, will stay increasingly below the original level with higher shocks, having larger and larger long-term effects. This triggers a change in the savings rate response with higher damages – the decrease after the shock is lower and the following increase higher, leading to a quicker dissipation (Figure S 5, panel b).

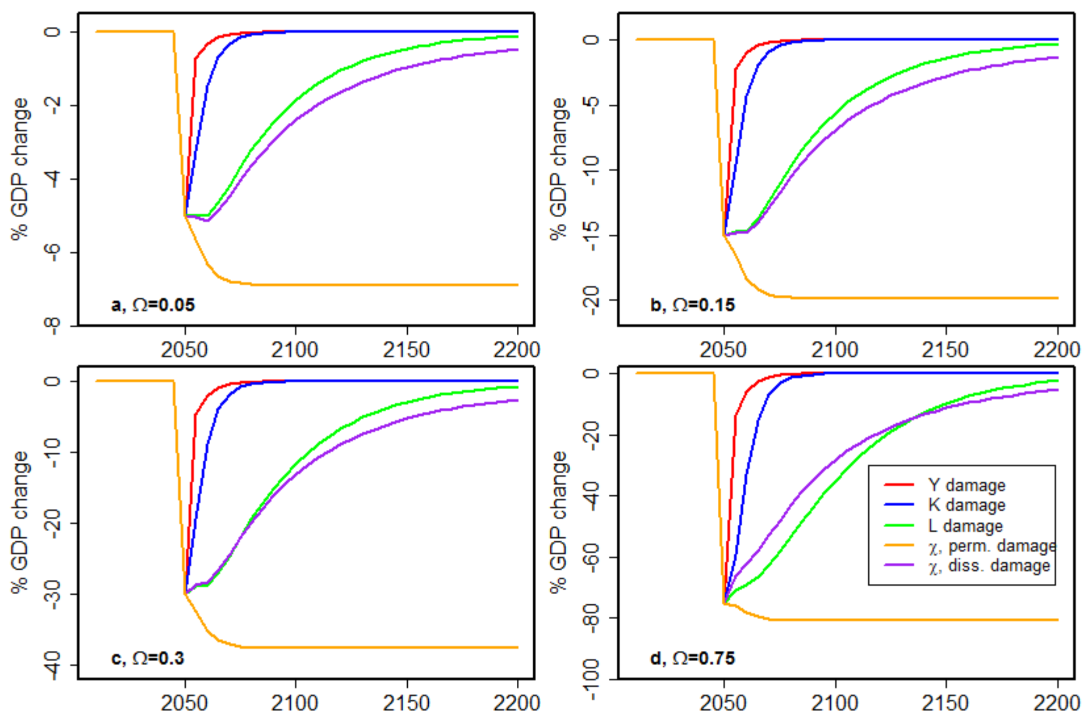

Figure S 4: Long-term GDP effect in the different impact channels (colors) after 4 different shocks between 5% (panel a) and 75% (panel d).

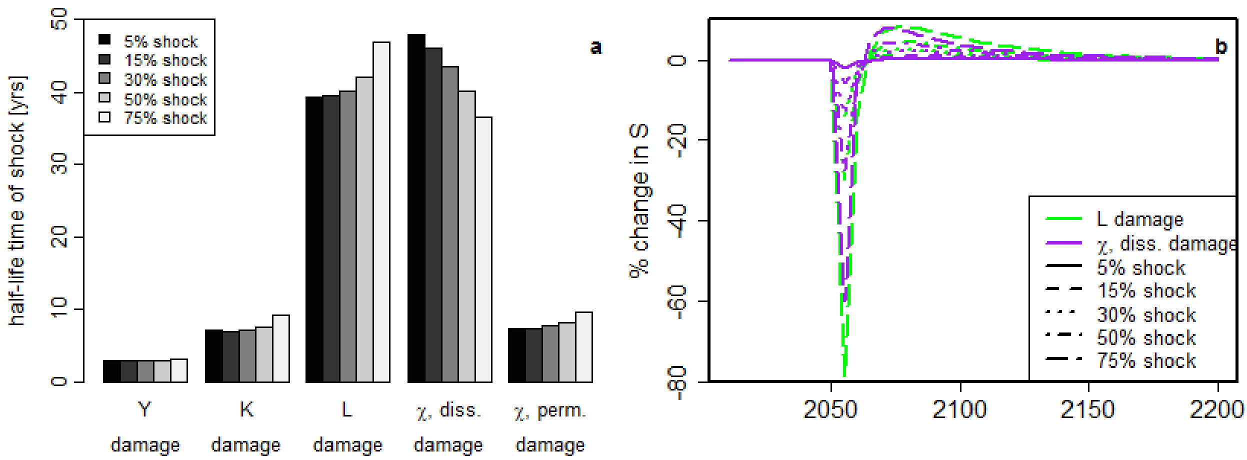

Figure S 5: Panel a - half-life times for the dissipation of the shocks in the different channels for the different shock magnitudes. Panel b – change in the savings rate in response to the shock for the labor (green) and dissipating productivity channel (purple), for the different shock magnitudes.

Finally, we also looked at the influence of capital adjustment costs on the damages. Capital adjustment costs are affecting economic decisions on investments but are often not included in growth models, including DICE. As they constitute a cost markup for changing investments, i.e. making replacement of destroyed capital more expensive, they may increase overall long-term damages. We tested this, implementing them, following the literature (e.g. Ortigueira and Santos 1997), as part of the budget equation as $C_t = Y_t^N - I_t\,(1 + b\,I_t/K_t)$ (where C is consumption). The constant b was set to 0.2 which fits with the literature for aggregated economies (Shapiro 1986; Bond et al. 2011). However, as this parameterization results in a very small markup only, the adjustment costs turn out to be a (fairly small, on the order of 0.2%) baseline effect with no influence on the magnitude of the damages.

A.3.3 Dynamics of the system with recurring cumulative shocks

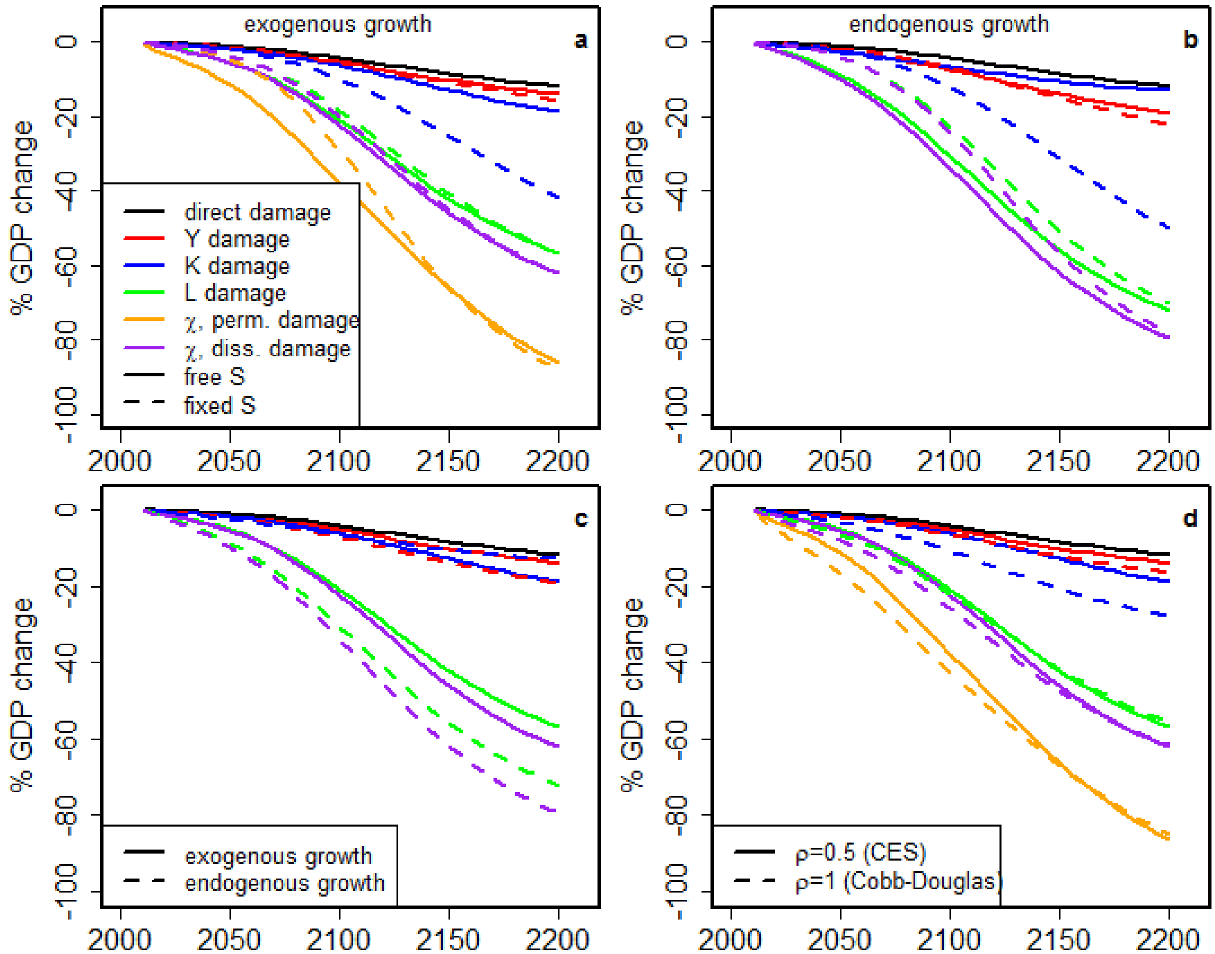

Figure S 6: Panels a and b: GDP loss in the case of recurring cumulative shocks, comparing the standard setup with an endogenous savings rate to the sensitivity case with a fixed exogenous savings rate (solid vs. dashed lines) for the different impact channels (colors). Panel c: Direct comparison of GDP loss for the cases of exogenous (solid lines) and endogenous (dashed lines) growth (endogenous savings). Panel d: Comparison of the GDP loss for the standard case with a CES production function (solid lines) and the variation with a Cobb-Douglas specification (dashed lines).

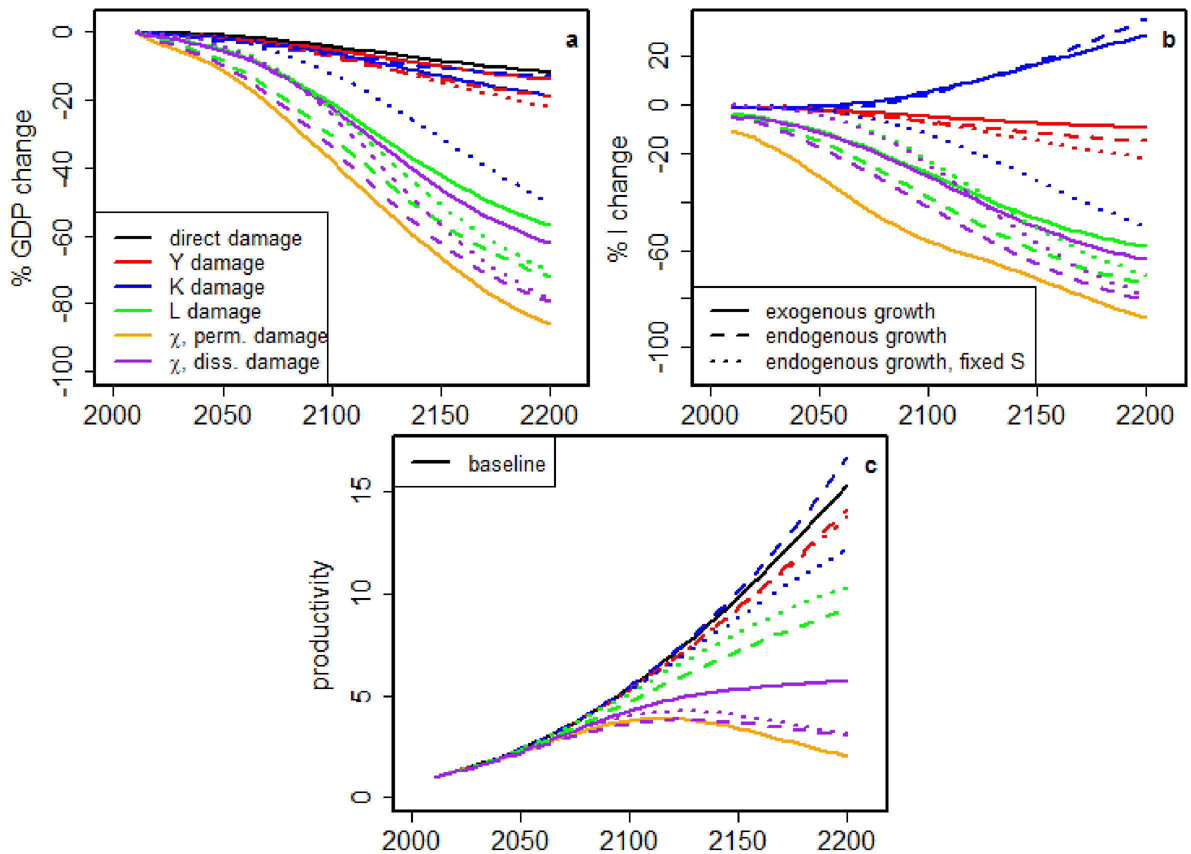

Figure S 7: GDP loss (panel a), change in investment (panel b) and productivity (panel c) in the case of recurring cumulative shocks, for the standard setup (solid lines), with endogenous growth (dashed lines) and with endogenous growth and fixed savings rate (dotted lines). A productivity effect is only visible for the endogenous growth cases (dashed and dotted lines) and for the two productivity channels with exogenous growth (purple and orange solid line).

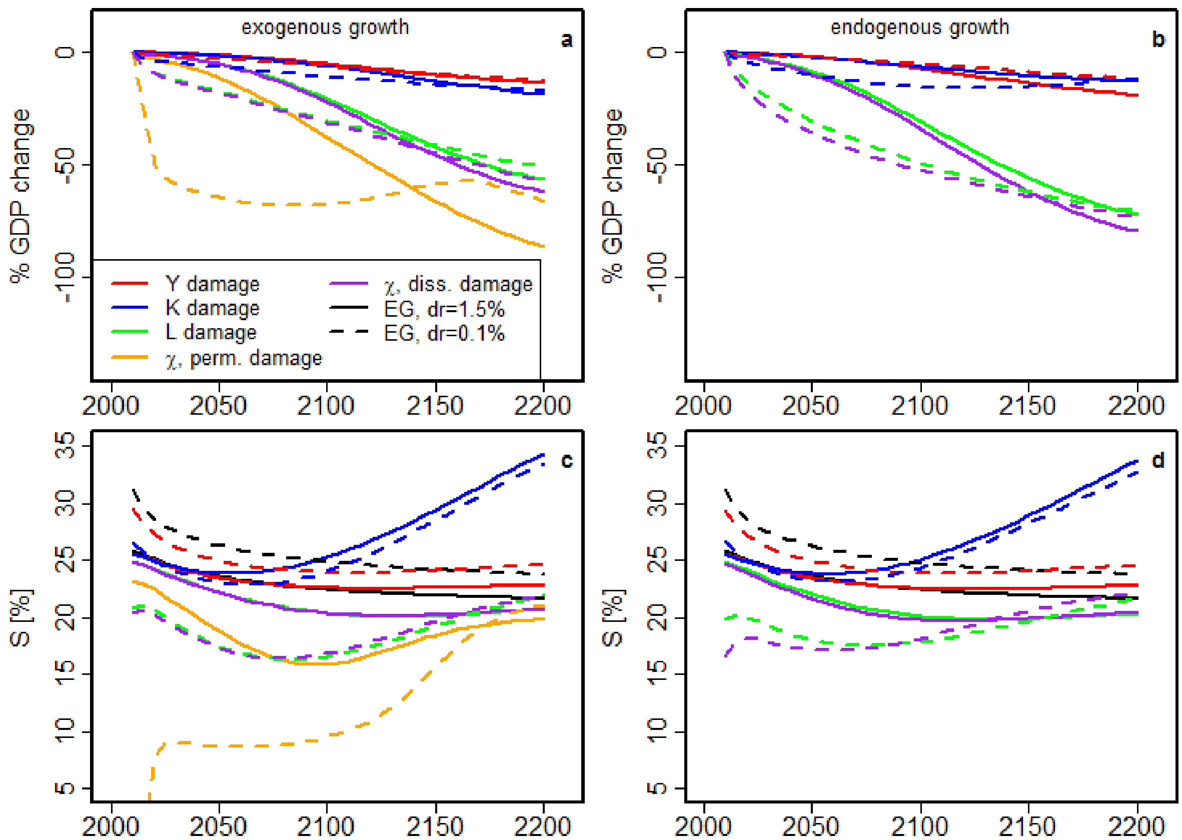

Figure S8: The effect of a lower discount rate in the case of recurring cumulative shocks with exogenous (panels a,c) and endogenous (panels b,d) growth, on GPD loss (top panels) and savings dynamics (bottom panels) in comparison to the standard case (dashed vs solid lines) for the different impact channels colors). The lower discount rate is implemented following the Stern settings as described in Nordhaus (2014), with an initial rate of social time preference per year of 0.1% and an elasticity of marginal utility of consumption of 1.01, lowering the discount rate from 3.6 to 1.5% in 2100.

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