Emergent inequality and business cycles in a simple behavioral macroeconomic model

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Standard macroeconomic models assume that households are rational in the sense that they are perfect utility maximizers and explain economic dynamics in terms of shocks that drive the economy away from the steady state. Here we build on a standard macroeconomic model in which a single rational representative household makes a savings decision of how much to consume or invest. In our model, households are myopic boundedly rational heterogeneous agents embedded in a social network. From time to time each household updates its savings rate by copying the savings rate of its neighbor with the highest consumption. If the updating time is short, the economy is stuck in a poverty trap, but for longer updating times economic output approaches its optimal value, and we observe a critical transition to an economy with irregular endogenous oscillations in economic output, resembling a business cycle. In this regime households divide into two groups: poor households with low savings rates and rich households with high savings rates. Thus, inequality and economic dynamics both occur spontaneously as a consequence of imperfect household decision-making. Adding a few "rational" agents with a fixed savings rate equal to the long-term optimum allows us to match business cycle timescales. Our work here supports an alternative program of research that substitutes utility maximization for behaviorally grounded decision-making.

endogenous business cycles | social learning | computational simulation

Economic growth and inequality are important problems in economics (1, 2). Standard macroeconomic models are based on the assumption of a single representative rational utility-maximizing agent and assume that the dynamics of business cycles are driven by exogenous shocks. However, empirical evidence from behavioral economics indicates that real households are heterogeneous and make substantial deviations from rationality. This has led to new directions of research, including the incorporation of heterogeneous or boundedly rational agents into macroeconomic models. This is typically done by allowing the agents to differ in terms of factors such as education while preserving the assumption of rationality (3–5) or alternatively allowing for bounded rationality but maintaining utility maximization (6). Realism is injected through imposing frictions, such as sticky wages. These models require shocks to generate economic dynamics.

However, it has long been known that endogenous dynamics are possible in economic models (7–12). Furthermore, ref. 13 shows that network structure can result in fluctuations, and Beaudry et al. (14, 15) have shown how limit cycles can emerge in a standard framework where agents are perfect utility maximizers. An alternative approach bases household decision-making on simple heuristics rather than rationality (16, 17). This leads to waves of optimism and pessimism, generating irregular business cycles and giving fat-tailed distributions for economic outcomes such as gross domestic product (GDP). We further develop this line of research by demonstrating how a very simple heterogeneous behavioral macroeconomic model leads to an endogenous business cycle that is not driven by externally imposed shocks. We do this by demonstrating that rich emergent behavior can occur even under very simple behavioral assumptions.

Here we extend the Ramsey–Cass–Koopmans (RCK) model, which is one of the foundational models of economic growth theory. In this model a representative agent rationally chooses a savings rate to maximize discounted consumption. However, there is ample evidence that households do not act as intertemporal optimizing agents and often respond myopically (18–20). Evidence from laboratory experiments suggests that individuals perform poorly in finding optimal consumption paths. In ref. 21 subjects deviated from optimal consumption choices by roughly 30% on average, which increased to roughly 50% when subjects were shown the average consumption level in the previous period. Learning from past generations' consumption paths is somewhat more successful, but the deviations from long-term optimal consumption levels are still substantial (22, 23).

We take the opposite approach and assume a strong form of bounded rationality. Following work that incorporates imitation of behavior under limited information, such as refs. 24 and 25, in our model households are embedded in a social network and make their savings decisions by simply copying their apparently most successful network neighbor. They do this episodically and myopically: From time to time they check all their neighbors and adopt the savings rate of the neighbor that currently has the highest consumption rate. The rationale for choosing current consumption as a proxy for the performance of savings behavior

Significance

Macroeconomic models play an important role in guiding fiscal and monetary policy. At the core of many of these models is the assumption of a single representative, rational agent. We develop a model that loosens these constraints and explicitly models the social dynamics of households embedded in a social network. In some circumstances this leads to endogenously oscillating economic output resembling business cycles, where households spontaneously become rich or poor. Nonetheless, the production of the economy is close to optimal, even though each household acts myopically. This presents an alternative approach to macroeconomics without using rationality assumptions on households.

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is that consumption is also the optimization target in the classical model and that one can argue that people will typically be able to estimate their acquaintances' consumption expenses much better than their capital, since they can observe many consumption behaviors directly. The assumption that agents can copy their acquaintances' savings rates can be justified by assuming that they can estimate not only their consumption but also their income, e.g., from their profession, working hours, and seniority or even directly from their publicly accessible tax declaration.

Although we do not claim that this behavior is fully realistic, there is empirical justification for considering a simple rule of this type. Our agents can be viewed as short-sighted, profligate "conspicuous consumers," and the tendency of households to copy one another has been well documented since the time of Thorstein Veblen (26). Imitate the best is one of the decision-making heuristics often applied in settings of high uncertainty and variability (27) and is observed in economic experiments (28). Savings behavior is highly dependent on social interaction with peers (29–32) and comparing consumption levels incorporates the visibility bias and selection neglect observed in savings-rate decisions (33).

We find that a key parameter governing economic behavior is the average time interval τ at which households update their savings rate, which we call the "social interaction time." When τ is small, meaning that households update frequently, the savings rate is low, and the performance of the economy is suboptimal in terms of aggregate consumption. When τ is sufficiently large, in contrast, the economy-wide aggregate savings rate, which equals the income-weighted average household savings rate, becomes close to the optimal rate. For small τ the population of households remains homogeneous, but as τ increases there is a sharp phase transition at a critical value τ_c where the population becomes strongly bimodal, dividing into rich households with high savings rates and poor households with low savings rates. Correspondingly, for low values of τ the GDP and other economic indicators are constant with only small fluctuations, whereas above the critical transition there is an endogenous aperiodic oscillation, resembling a business cycle, in which the aggregate savings rate fluctuates, the population of households alternately becomes richer or poorer, and economic output varies substantially over time.

Our model shows that the use of heterogeneous agents following explicit behavioral rules can produce aggregate behavior that is qualitatively different from that of rational agents. Our model is mostly qualitative, but our results suggest that an approach that explicitly incorporates empirical behavioral knowledge into household decision-making may naturally lead to an explanation of business cycles in terms of endogenous dynamics.

The Standard RCK Model

The RCK model considers a closed economy in which a representative household provides both labor and capital for the production of a single good by a representative firm. The household receives wages w for labor and a nominal rate of return r on its investment. It spends a fraction $1-s$ of its income on consumption and invests the remaining fraction s (*Materials and Methods*).

Given the current value of per-capita capital k, the household chooses a current value of per-capita consumption $c(s)$ determined by intertemporal optimization, leading to an optimal consumption path that maximizes the household's long-term discounted aggregate utility. This determines the time evolution of k and c toward a steady state at (k^*, c^*).

An Agent-Based Version of RCK

Economic Model. We introduce a heterogeneous agent model in the tradition of agent-based modeling (34–40), using agents that follow a very simple behavioral rule. Our model contains N households labeled by i with heterogenous capital K_i. For simplicity, all households supply the same labor $L_i = L/N$. (Introducing heterogeneous labor has little effect on the results.) As in the original RCK model, total economic production is given by the Cobb–Douglas production function, in this case applied to the aggregate input factors $K = \sum_{i=1}^{N} K_i$ and $L = NL_i$. As in the original model, capital returns r and wages w equal marginal returns (*Materials and Methods* and Eq. **7**), but incomes I_i now differ between households,

$$I_i = rK_i + wL/N. \qquad \textbf{[1]}$$

Our key assumption is that each household individually and dynamically sets its time-dependent savings rate $0 \leq s_i(t) \leq 0$ according to a behavioral decision rule introduced below, leading to household capital dynamics

$$\dot{K}_i = s_i I_i - \delta K_i = (rs_i - \delta)K_i + ws_i L/N. \qquad \textbf{[2]}$$

At the steady state where $\dot{K}_i = 0$, the steady-state value K_i^* for household i's capital is a function of the aggregate capital K via its dependence on w and r, nonlinearly interconnecting all of the agents' savings rates and consumption levels.

Household Decision-Making. While the standard RCK model is a one-dimensional dynamical system in which consumption is a deterministic function of the total capital, the agent-based version is $2N$ dimensional, and aggregate consumption depends on all households. We assume that each household updates its savings rate at random times according to a Poisson process with rate $1/\tau$.* We will see that τ plays a crucial role for the model's behavior.

Households are embedded in a social network in which each household i has neighbors $\mathcal{N}(i)$. Whenever household i updates its savings rate, it compares the consumption rates of its neighbors and applies the "imitate-the-best" heuristic, copying the savings rate of the neighbor with the highest current consumption with a small deviation that can be interpreted either as an error or as an exploration (43). More precisely, when the consumption of a neighbor is higher, it adopts a new savings rate of

$$s_i^{\text{new}} = s_{\substack{\arg\max \\ j \in \mathcal{N}(i)}}(C_j) + \epsilon, \qquad \textbf{[3]}$$

where ϵ is distributed uniformly in the interval of $\pm 1\%$. (The behavior is insensitive to this as long as there is some diversity.)

Results

We simulate the model for a variety of different parameters such as the average social interaction time τ and the network topology. In Fig. 1 we show the distribution of the final savings rates as a function of the social interaction time τ for a complete network with the other parameters fixed. Fig. 1 compares this to the optimal, "golden rule" savings rate s_{gold}, corresponding to the rational expectations equilibrium where the consumption of the representative agent is maximized.

There are two distinct regimes, separated by a critical social interaction time τ_c ($\tau_c \approx 250$ y for $\alpha = 0.5, \delta = 0.05$ and a fully connected network—see *SI Appendix* for all parameters). In the stable regime, corresponding to $\tau < \tau_c$, the savings rates of the households are unimodally distributed around a low savings rate. For very small values of τ the savings rates are close to zero, and the economy is stuck in a poverty trap in which its output is very low. As τ increases, the savings rate and output increase, but

*This leads to smoother transitions than synchronous updates (41, 42).

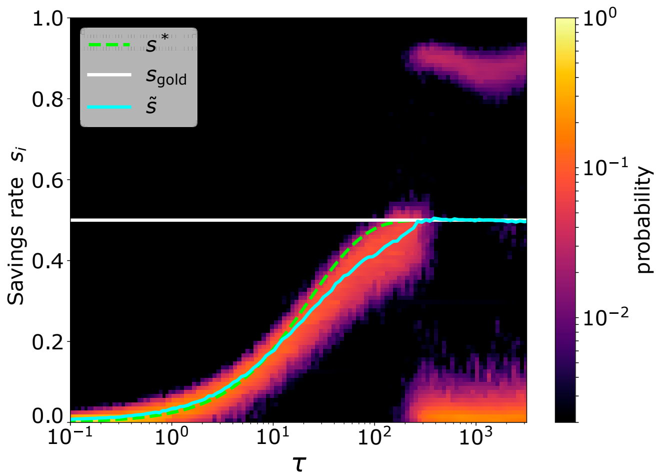

Fig. 1. The critical transition from the stable regime to the oscillatory regime. We perform an ensemble of simulations at different values of the social interaction time τ, with other parameters held fixed such as $\delta = 0.1$ and network topology (see *SI Appendix* for details). We show a heatmap indicating the probability density of the distribution of individual household's savings rates for each value of τ, along with the aggregate savings rate $\tilde{s}$. We compare it to the golden rule savings rate $s_{gold} = 0.5$ and the savings rate s^* predicted by Eq. **4**.

the distribution remains unimodal, with a suboptimal aggregate saving rate.

For $\tau > \tau_c$ we enter what we call the oscillatory regime, where the behavior is dramatically different. In this regime the savings rate distribution is bimodal—some households have high savings rates and are quite wealthy, while others have low savings rates and are very poor. We thus observe the spontaneous emergence of extreme inequality, with a lower class and an upper class.†

Strikingly, as long as $\tau > \tau_c$, the ensemble average of the aggregate savings rate $\tilde{s}$ is within 1% of the optimal value $s_{gold} = 0.5$, given $\alpha = 0.5$ even when the individual distributions are bimodal. Furthermore, the time averages of total economic output $Y(t) = 10.15$ and consumption $C = 4.99$ are close to their optimal values $Y^* = s_{gold}L/\delta = 10$ and $C^* = (1 - s_{gold})Y^* = 5$ in the standard RCK model. It seems surprising that such a simple, near zero-intelligence learning rule can maintain the system this close to its optimal behavior.

The system dynamics become clearer when we look at the economy as a function of time, as illustrated in Fig. 2. For $\tau > \tau_c$ there is an endogenous oscillation in many of the aggregate properties of the economy, including the aggregate savings rate $\tilde{s}(t)$ and output $Y(t)$. This oscillation is also visible in the behavior of individual households. If we follow any single household, it goes through epochs with a high savings rate, near $s_i \approx 0.90$, and a low savings rate, near $s_i \approx 0.05$. At any point in time there is typically an imbalance between rich households and poor households, so that the aggregate savings rate and the aggregate output fluctuate. We loosely refer to this endogenous oscillation as a "business cycle."

Understanding the Stable Regime ($\tau < \tau_c$). Although our behavioral rule requires minimal intelligence, the selection process of copying the household with the highest consumption provides a simple mechanism of collective search that becomes more effective as the social updating time τ increases. This is perhaps counterintuitive, as it means that inattention results in superior collective outcomes. The underlying explanation is as follows: The savings rate of the household that is copied has on average been fixed for a time interval of order τ. When τ is small, planning is too myopic, "short-term thinking" dominates, and the households cannot escape using low savings rates with high consumption. As τ gets bigger, however, the time between updates becomes long enough that there is more time to accrue an advantage by saving, which drives the savings rate up and increases economic output. The competitive selection process guarantees that for a sufficiently large population and large τ the savings rates get closer to optimality.

We use this intuition to derive an approximate formula for the aggregate savings rate s^* as a function of τ. We take advantage of the fact that in the stable regime the distribution is unimodal and assume that all households have essentially the same savings rate and derive the optimal savings rate for time horizon τ. As explained in detail in *SI Appendix*, for capital elasticity $\alpha = 0.5$ the optimal savings rate under these conditions is

$$s^*(\tau) = \frac{1 - e^{-\delta\tau/2}}{2 - e^{-\delta\tau/2}}. \quad [4]$$

This approximation is shown in green in Fig. 1 and provides a good fit throughout the stable regime.

The optimal savings rate in the classical RCK model depends on the discount rate ρ, which is a free parameter. As shown in *SI Appendix*, substituting s^* from Eq. **4** into the relation for the classical RCK model gives an effective discounting rate for our model in terms of the social interaction time τ and the depreciation rate δ,

$$\rho(\tau) = \frac{\delta/2}{e^{\delta\tau/2} - 1}. \quad [5]$$

In the limit as $\tau \to 0$, the discount rate $\rho \to \infty$, consistent with the observed collectively myopic behavior. But for $\tau \to \infty$, $\rho \to 0$. Thus in this case the individually myopic households act collectively "as if" they were farsighted, with an emergent effective

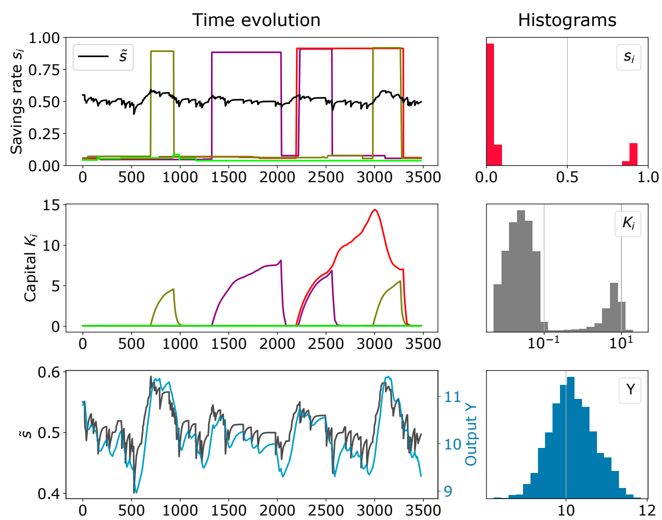

Fig. 2. The endogenous business cycle in the oscillatory regime. We show several time series when $\tau > \tau_c$. *Top Left* shows the savings rates $s_i(t)$ for four randomly chosen households as a function of time, as well as the aggregate savings rate $\tilde{s}$. *Middle Left* shows the capital $K_i(t)$ of the same four households as a function of time. *Bottom Left* shows the cyclic behavior of the aggregate output superimposed on the aggregate savings rate. *Top Right*, *Middle Right*, and *Bottom Right* are histograms of the indicated variables, accumulated over a longer interval.

†Very near τ_c the distribution in Fig. 1 is trimodal. This is due to intermittent oscillations between the unimodal and bimodal regimes. Thus, the system exhibits either a middle class or a lower class and an upper class, but never all three at once.

discounting rate $\rho(\tau)$ that is not a free parameter but is rather a function of the social interaction time τ.

Understanding the Oscillatory Regime ($\tau > \tau_c$). To get a deeper understanding of what is happening in the oscillatory regime, where $\tau > \tau_c$, in Fig. 3 we illustrate the collective and individual dynamics. In Fig. 3*A* we show the average per-capita consumption rate c as a function of the average capital k. This illustrates how the aggregate consumption and capital orbit around the optimal steady state (k^*, c^*) of the standard RCK model, generating a business cycle. In relation to the optimal savings rate $s^* = 0.5$ of the RCK model, the effective aggregate savings rate $\tilde{s}$ is typically greater than s^* when the system is below the optimality curve and less than s^* when it is above the optimality curve. This is interesting as the optimality curve is obtained via optimizing household consumption for an infinite horizon, whereas our model has no explicit optimization.

To understand what is going on at the individual level, in Fig. 3*B* we plot a snapshot of the capital vs. the savings rate for all households at two different times, t_1 and t_2. At time t_1 the economy is just beginning to recover from a recession. There are two clusters of households, corresponding to rich households ("the capitalists") in the upper right corner and poor households ("the workers") in the lower left corner. More households are poor, and because the return r is inversely proportional to total capital according to $r \propto K^{-1+\alpha}$, this means that returns to investment are high. Thus, when the household shown in blue at t_1 gets its chance to update its savings rate, it copies the higher savings rate of one of the capitalists that enjoy high returns on their savings, transitions to the right as indicated by the arrow, and begins accumulating capital by saving more. Other workers follow, and eventually the economy reaches the state shown in Fig. 3 *B*, *Bottom* at time t_2, where many households have high savings rates and are rich. The resulting excess capital makes the returns on savings low, which, when combined with the capitalists' high savings rates, drives their consumption down. As a result, when one of the capitalists gets its turn to update, it copies a worker household with a low savings rate and goes on a spending spree. At this point its consumption rate becomes very high, as is shown by the high iso-consumption curve that the newly transitioned green household is enjoying at t_2. With this household's high consumption, its neighbors soon start copying it, thus creating a boom in consumption while decreasing the aggregate savings rate. A majority of households eventually become impoverished and the cycle repeats itself. In addition, we confirm the working of this return on capital $(r - \delta)$ vs. wage w seesaw mechanism in another graphic in *SI Appendix*.

Critical Social Interaction Time. What determines the critical social interaction time τ_c? There are two key factors: the size of potential deviations from s^* and the social network's parameters. For the first part, we approximate the imitate-the-best heuristic by modeling it as behavior that is optimal over a time horizon τ. To understand the instability driving the transition, suppose that an external shock of size Δ perturbs the aggregate savings rate $\tilde{s}$ away from s^*, and suppose that household i is allowed to optimize its savings rate s_i while the others hold theirs constant. A numerical investigation shows that when $\tau \ll \tau_c$, the optimal savings rate s_i computed remains close to $\tilde{s}$. In contrast, when $\tau \gg \tau_c$, if $\Delta > 0$, then the optimal savings rate is very small, with s_i approaching 0, and if $\Delta < 0$, the optimal savings rate is large, with s_i approaching 1 (*SI Appendix*, Fig. S3). This happens because when $\Delta > 0$, the aggregate savings rate is high, so the returns on investment are low, which discourages saving. Similarly, when $\Delta < 0$, the aggregate savings rate is low, so returns on investment are high, which encourages saving. This destabilizes the unimodal solution around s^*. The transition occurs sharply at a parameter value near τ_c, although the precise value depends on Δ. The size of the shock Δ perturbing the aggregate savings rate, in turn, depends inversely on the number of agents (as shown in *SI Appendix*, Fig. S6).

Network Size and Structure. We have so far used complete networks in the simulations, but in general the behavior depends on the network size and structure. For example, we investigate Erdős–Rényi networks with average degree $\langle k \rangle = Np$, where p is the probability that any two nodes are connected. The critical social interaction time τ_c depends on both $\langle k \rangle$ and N (*SI Appendix*, Figs. S4 and S5). Starting at any given node and moving one link at a time, the number of neighbors that are reached grows exponentially with time at rate $\langle k \rangle$. The typical distance required for a disturbance to propagate across the network is the average shortest path length χ, defined as the average number of nodes that must be traversed to go from any given

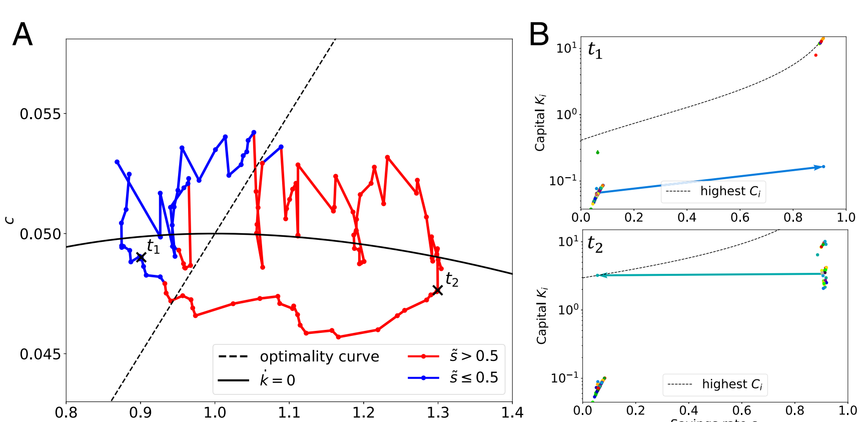

Fig. 3. Endogenous dynamics in the oscillatory regime. (*A*) We plot the average per-capita consumption c against the average per-capita capital k and show the aggregate saving rate $\tilde{s}$ as red when it is greater than 0.5 and blue when it is less than 0.5. The trajectory orbits around the optimal steady state (k^*, c^*) of the standard RCK model, which is at the intersection of the dashed optimality curve and the solid black $\dot{k} = 0$ line. Each dot corresponds to one timestep; the orbit is counterclockwise. (*B*) An illustration of the cause of the oscillatory dynamics. *Top* and *Bottom* show snapshots at two different times as indicated in *A*. At time t_1 the aggregate savings rate is low, aggregate capital is low, and the economy is in a depression; at time t_2 the opposite is true. The capital and savings rates of individual households are shown as dots with different colors. There are two clusters, corresponding to rich and poor households. The household that is currently switching its savings rate is indicated by an arrow connecting its previous state to its current state. The dashed black curve indicates the iso-consumption curve for the household i with the highest consumption.

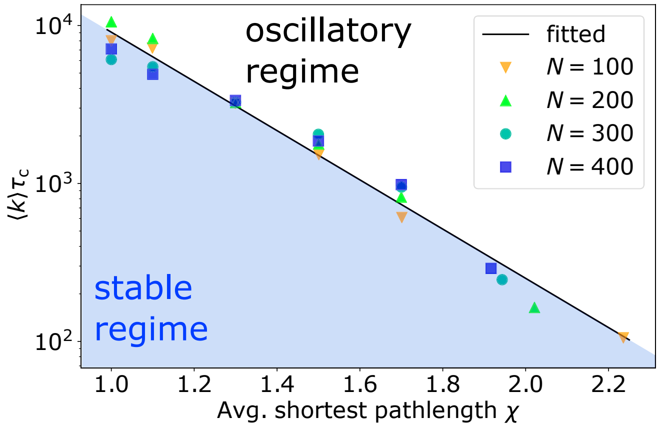

Fig. 4. The critical social interaction time depends on network properties. The logarithm of the mean number of neighbors $\langle k \rangle$ times the critical interaction time τ_c is plotted vs. the average shortest path length χ for various values of N and p, confirming Eq. **6**. The stable regime ($\tau < \tau_c$) is shaded in blue.

node to any other node. Motivated by this logic, we investigate the empirical relationship between τ_c, χ, and $\langle k \rangle$, finding the proportionality

$$\tau_c \sim e^{-\chi}/\langle k \rangle. \quad [6]$$

Fig. 4 shows that this makes a good prediction of τ_c. Because χ increases with N, in the large N limit the system is always in the oscillatory regime. Varying the network size and structure parameters also results in qualitative changes in the nature of the oscillation, affecting its frequency, amplitude, and variability. (See a few examples in *SI Appendix*.)

Rationality and Realistic Business Cycle Timescales. Our model generates oscillations, but does this occur on timescales that match those of business cycles? Since World War II business cycles in G7 countries have periods that are in the range 9 to 20 y for different countries (15), although there are large uncertainties because there are typically fewer than eight observed cycles and the period of cycles varies considerably. The timescale of the model is determined by the depreciation parameter δ, but the average timescale of its endogenous dynamics depends on several parameters, including network size and network topology. For most parameter values that we have investigated the timescale for a cycle is longer than that of a typical business cycle, but some sensible parameter values yield cycles that are reasonably close; e.g., as shown in Fig. 5*A*, the dominant period of $Y(t)$ for a Barabási–Albert network with $N = 4{,}000$, mean degree of 40, $\delta = 0.2$, and $\tau = 1.5$ is around 28 y.

The period becomes even shorter if we assume that a small percentage of the households behave like the representative household in the standard RCK model. This can be interpreted as households who are "rational" and believe the other households are rational as well, so that they use the fixed savings rate s^*_{RCK} given by Eq. **9**; alternatively, these can be thought of as patient households who ignore their neighbors and maintain a fixed rate equal to the long-term optimal rate. Adding a 5% share of such households, the dominant period goes down to 10.5 y, as shown in Fig. 5*B*.

The oscillations also become much more regular, although they still show clearly distinguishable phases of different cycle length. In the shorter phases with slower oscillations, most agents still switch between rather low and rather high s, while in phases with faster oscillations, most agents instead switch between a low s and an intermediate value s^*_{RCK}. When comparing to business cycles, it is important to note that our model retains these lower-frequency cycles. While in business cycles the aggregate capital remains almost constant, it varies substantially in our case ($Y \propto K^\alpha$). Therefore, while the frequencies can match those of real business cycles, the proposed model can be better understood as a theory of interest and savings rates oscillations.

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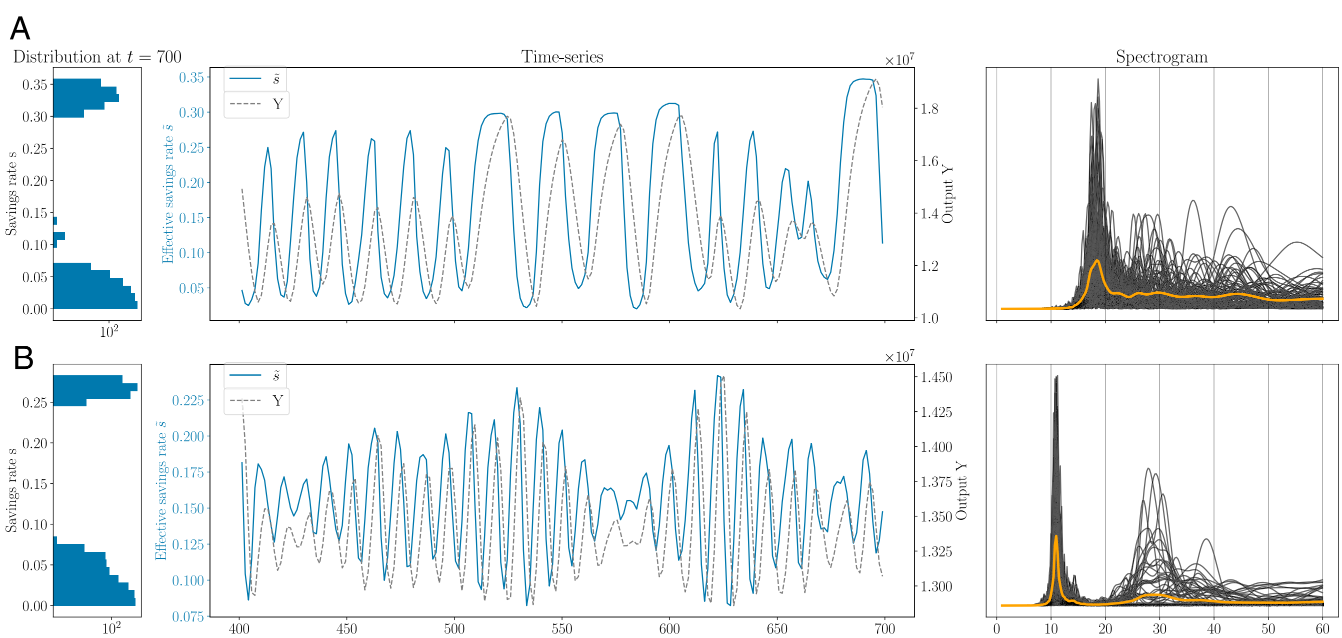

Fig. 5. Business-cycle–like periodicities can emerge from our model. (*A* and *B*) We show from *Left* to *Right* the histogram of the final savings rates of the households, time series of the effective savings rate $\tilde{s}$ and capital return r, and Lomb–Scargle spectrograms averaged over 100 runs. We use a scale-free Barabási–Albert network (44) with $N = 4{,}000$, $\delta = 0.2$, $\tau = 1.5$, $\langle k \rangle = 40$, where $\langle k \rangle$ is the mean degree. In *A* all households change their savings rate, while in *B*, 5% of households keep it fixed at a long-term optimal rate $s^* = 2/9$, corresponding to a discount rate $\rho = 0.05$.

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Homophily in Network Structure. Many agent-based models of social dynamics also assume a process of homophilic change to the network topology by which over time links between agents displaying different behavior are replaced by links between agents with similar behavior. Adding such a process to our model influences the frequency of oscillations. We simulated variants of a homophilic change process and found that, depending on parameters, the dominant peak in Fig. 5*B* can move from around 11 y to between 8 and 17 y. If new links tend to be formed between agents that already have a common neighbor, this tends to lead to clustering of similar agents and slows down the oscillations, while if new links tend to be formed between any two agents, this tends to decrease the diameter of the network and accelerates the oscillations. Both effects appear to be larger when the criterion for similar behavior is the absolute level of consumption and smaller if the criterion is the savings rate. We provide the figures and numerical details in *SI Appendix*.

Discussion

Our primary purpose here is to make a conceptual point by demonstrating how emergent inequality and endogenous dynamics can naturally emerge from a heterogeneous behavioral model. Nonetheless, our model makes the prediction that during recessions savings rates increase before output rises (Fig. 2). This has been observed for private savings in 19 Organization for Economic Cooperation and Development (OECD) countries (45).

Although our model has two random inputs, they are small and very different in character from the shocks that drive the dynamics of standard models. The first random input determines the time at which individual households update their savings rates under the Poisson process. This must be random to ensure that the order in which households update their savings rates varies. (A fixed order leads to a static economy.) The second random input is the copying error for the savings rate. This is small (1%) and its value makes little difference to the behavior. In contrast to standard shocks, which affect the economy as a whole, both of these inputs are at the level of individual households and affect each household differently. For a large number of households the copying errors cancel out but the endogenous dynamics nonetheless persist. Thus, while random inputs are necessary in our model, they do not directly drive booms and recessions as the shocks of standard models do. This is why we say that the economic dynamics in our model are endogenous.

To illustrate the conceptual difference between our model and standard macroeconomic models it is useful to draw an analogy to a simple physical system. Consider the problem of pole balancing, in which a man attempts to move his hand to maintain a pole in a vertical position, as shown in Fig. 6.

Fig. 6. The problem of pole balancing is analogous to the problem of optimizing savings in an otherwise unstable economy. A man attempts to maintain a pole in a vertical position. This is possible if the pole is long enough, but small errors in the control process drive endogenous oscillations in the angle of the pole.

Short poles tip over more quickly than long poles, making it impossible to maintain a vertical position because the pole will tip over before the man can react. If the pole is long enough, however, the man can move his hand to compensate and maintain the pole in a roughly vertical position (46). There is a sharp critical transition between stability and instability that occurs when the pole is about 1 m in length.[‡] Nonetheless, even when the pole is very long, it is not possible to maintain a perfectly vertical position, and the pole oscillates substantially.

An argument in the style of a standard macroeconomic model would posit that the man is a perfect pole balancer, and any deviations in the angle must be driven by external shocks, such as sharp gusts of wind, that suddenly cause the pole to deviate from vertical. Under this view, after each shock the man moves his hand perfectly to make the pole vertical again as fast as possible, but before he can achieve this, another shock strikes it, making the pole oscillate around its vertical position. For pole balancing it is clear that this explanation is wrong. Instead, theories that assume that oscillations are endogenously caused by imperfect control provide a better explanation (46).

We show how myopically imitating high-consuming acquaintances' savings rates can induce fluctuations for the whole economy without diverging from a roughly optimal aggregate state, even if there are no exogenous shocks at all, whether aggregate or individual. In our model, the ultimate source driving the fluctuations is the boundedly rational decision-making of households, namely the random timing of each household's updating and the small random error made when copying savings rates. This is in contrast to other possible causes of economic fluctuations, such as aggregate shocks to total factor productivity, propagation of idiosyncratic local shocks through a supply network (13), or the discrete nature of investment decisions (47). It is also in contrast to models that assume other effects of comparing one's consumption level to others', such as the rational optimization of an individual utility function that has an additional dependency on the average consumption level of the economy (48).

Compared to these previous works, our model is blatantly simple—yet we observe endogenous business-cycle–like fluctuations. This minimal demonstration suggests that we should revisit the conceptual explanation for business cycles and look for the minimal set of requirements, as opposed to adding further modeling assumptions. In the pole-balancing analogy above, the position of the man's hand is like the collection of household savings decisions, and the pole/gravity system is like the economy. Our model adds weight to the idea that at least part of the variation in savings and investment that occurs during business cycles emerges endogenously due to imperfect reasoning. Our model also suggests that models with heterogeneity might help illuminate the interaction between business cycles and inequality. The fact that such rich behavior emerges from such a simple model supports a research agenda for macroeconomics based on minimal, empirically derived behavioral rules.

Materials and Methods

The Standard RCK Model. In our formulation of the standard RCK model, we follow ref. 1, pp. 287–317, and ref. 49, pp. 85–135, and use continuous time, as in the original model (50). We ignore labor growth for brevity.

Using a fixed amount of labor L and the varying amount of capital K, the economy produces a single numeraire good Y, assuming a Cobb–Douglas production function $Y = K^{\alpha} L^{1-\alpha}$ with capital and labor

[‡]This is trivial to confirm empirically—simply attempt to balance a pole of 60 cm vs. 130 cm.

elasticities $\alpha, 1-\alpha \in (0, 1)$. Per-capita production, $y = Y/L$, is thus a function of per-capita capital, $k = K/L$, only, $y = k^\alpha$. The model assumes fully competitive factor markets and thus the two factors are compensated according to their marginal products, giving wages and capital rents

$$w = \partial_L Y = (1-\alpha)y, \quad r = \partial_K Y = \alpha y/k, \qquad [7]$$

thereby fully redistributing the numeraire good to households and leaving the representative firm with no profits. The main model parameters of interest are the savings rate $s \leq 1$ and capital depreciation rate $\delta > 0$ that govern aggregate and per-capita capital growth,

$$\dot{K} = s(rK + wL) - \delta K, \quad \dot{k} = \bar{r}k + w - c, \qquad [8]$$

where $\bar{r} = r - \delta$ is the real return rate and $c = (1-s)(rk + w)$ is per-capita consumption. The household aims at maximizing its discounted aggregate utility $\int_0^\infty dt\, e^{-\rho t} u(c(t))$, by choosing an optimal path $s(t)$ for the savings rate, where $\rho > 0$ is its discount rate.

For the instantaneous utility, one assumes a constant relative risk aversion (CRRA) function parameterized by $\theta \geq 0, \theta \neq 1$, $u(c) = (c^{1-\theta} - 1)/(1-\theta)$. The solution to this problem fulfills the Ramsey–Keynes equation that gives the relative consumption growth rate as $\dot{c}/c = (\bar{r} - \rho)/\theta$. In particular, this system has two steady states with $\dot{c} = 0$, a trivial one in which $c = k = 0$ and another in which the real return rate equals the discount rate, $\bar{r} = \rho$, corresponding to a modified "golden rule" (ref. 1, p. 300), with capital, consumption, and savings rate given by

$$k^* = \left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}, \quad c^* = k^{*\alpha} - \delta k^*, \quad s^*_{\mathrm{RCK}} = \frac{\alpha\delta}{\rho+\delta}. \qquad [9]$$

For the limit case $\rho \to 0$, this reproduces the Solow model's golden rule (ref. 49, p. 35), $s^*_{\mathrm{RCK}} = s_{\mathrm{gold}} = \alpha$, leading to the largest possible sustainable consumption, $c^* = (1-\alpha)(\alpha/\delta)^{\alpha/(1-\alpha)}$. For $\rho > 0$, the discount rate pushes the households to save less and shift consumption toward the present.

Data Availability. Code has been deposited in GitHub (https://github.com/yukimasano/rck_abm).

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