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What do we mean, 'tipping cascade'?

Ann Kristin Klose[1,2], **Nico Wunderling**[1,2,3], **Ricarda Winkelmann**[1,2] **and Jonathan F Donges**[1,3]

[1] FutureLab Earth Resilience in the Anthropocene, Earth System Analysis & Complexity Science, Potsdam Institute for Climate Impact Research (PIK), Member of the Leibniz Association, 14473 Potsdam, Germany
[2] Institute of Physics and Astronomy, University of Potsdam, 14476 Potsdam, Germany
[3] Stockholm Resilience Centre, Stockholm University, Stockholm, SE-10691, Sweden

E-mail: `donges@pik-potsdam.de`, `akklose@pik-potsdam.de`

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Abstract. Based on suggested interactions of potential tipping elements in the Earth's climate and in ecological systems, tipping cascades as possible dynamics are increasingly discussed and studied. The activation of such tipping cascades would impose a considerable risk for human societies and biosphere integrity. However, there are ambiguities in the description of tipping cascades within the literature so far. Here we illustrate how different patterns of multiple tipping dynamics emerge from a very simple coupling of two previously studied idealized tipping elements. In particular, we distinguish between a two phase cascade, a domino cascade and a joint cascade. A mitigation of an unfolding two phase cascade may be possible and common early warning indicators are sensitive to upcoming critical transitions to a certain degree. In contrast, a domino cascade may hardly be stopped once initiated and critical slowing down–based indicators fail to indicate tipping of the following element. These different potentials for intervention and anticipation across the distinct patterns of multiple tipping dynamics should be seen as a call to be more precise in future analyses of cascading dynamics arising from tipping element interactions in the Earth system.

Keywords: tipping cascade, domino effect, tipping interactions, cascading regime shifts, early warning indicators

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1. Introduction

1.1. The concept of tipping cascades

Human–induced impacts on the Earth system increasingly endanger the integrity of the Earth's climate system and some of its most vulnerable components and processes, the so–called tipping elements [1]. Lately, it has been argued that the risk of potential tipping events or even cascading transitions up to a global cascade is rising under ongoing anthropogenic global warming [2, 3]. While this is the case, there is considerable debate about the nature of tipping cascades within the scientific community itself and cascading tipping dynamics have been described rather roughly in the recent literature [2, 3, 4, 5, 6, 7, 8, 9, 10].

The term cascade is used in various fields for a certain class of dynamics possibly exhibited by interacting (sub–)systems. It generally describes the sequential occurrence of similar events (event A is followed by event B which is followed by event C etc.). This sequence of events does not necessarily have to be causal opposed to when event A directly causes event B in a domino effect. The notion of a domino effect is sometimes used synonymously to the term cascade. Examples of cascades comprise cascading failures leading to the collapse of power grids as relevant physical infrastructure networks [11, 12, 13, 14, 15]. Such a cascade may occur as an initial failure increases the likelihood of subsequent failures [11]. In contrast, an initial failure may directly lead to the failure of dependent nodes [12].

Along these lines, cascading tipping events or regime shifts are increasingly discussed following the rising awareness of a highly interconnected world in the Anthropocene [16]. Tipping elements possibly undergoing a transition into a qualitatively different state after the crossing of some critical threshold were identified e.g. in ecology and climate system science [1, 17, 18]. Examples comprise, among others, shallow lakes transitioning from a clear to a turbid state [19, 20], coral reefs [21], the Atlantic Meridional Overturning Circulation [22, 23] and the continental ice sheets on Greenland [24] and Antarctica [25].

In the climate system, multiple interactions between large–scale tipping elements have been identified [26, 27, 28, 29, 30, 31]. For example, the Atlantic Meridional Overturning Circulation may slow down due to increasing meltwater flux originating from the Greenland Ice Sheet [27, 28]. Potential drying over the Amazon rainforest basin may be driven by the Atlantic Meridional Overturning Circulation [30] on the one hand and the El–Niño Southern Oscillation on the other hand [31]. Both can lead to the loss of rainforest resilience. Rocha et al. [8] identified potential links between ecological systems with alternative states such as the interaction of eutrophication and hypoxia or coupled shifts in coral reefs and mangrove systems.

Tipping interactions do not only exist across different large–scale systems, but span various spatial scales as exemplified by spatially extended (and heterogeneous) ecosystems [4, 8]. On a local scale, confined ecosystems such as a shallow lake, in fact, consist of discrete units connected through dispersion or other exchange processes with each unit potentially exhibiting alternative stable states [32, 33, 34]. Regionally, regime shifts may propagate from one ecosystem entity to the other transmitted, among others, via small streams and rivers [35, 36, 37], moisture recycling [4, 38, 39, 40] or biotic exchange through e.g. larvae [10, 34, 41, 42].

Motivated by these and further suggested tipping element interactions, cascading effects arising as potential dynamics have been discussed [2, 3, 4, 5, 6, 7, 8] as a possible mechanism for creating a potential planetary–scale tipping point (of the biosphere) [5, 6, 9, 10]. Lenton et al. [3] stated that we may approach a global cascade of tipping points via the progressive activation of tipping point clusters [43] through the increase of global mean temperature. This could potentially lead to undesirable hothouse climate trajectories [2]. However, it remains unclear whether and how cascade–like dynamics within the Earth system is promoted by the direction and strength of the existing feedbacks [4, 5, 26, 44].

Recently, first conceptual steps based on Brummitt et al. [45] and Abraham et al. [46] have been undertaken to determine whether the network of Earth system tipping elements is capable to produce global tipping cascades [47, 48]. Note that the proposed system capturing idealized interacting tipping elements is related to the double cusp catastrophe, which has been studied mathematically by, among others, Godwin [49] and Callahan [50]. More generally, coupled cell systems have been considered previously (e.g., Golubitsky et al. [51]). Using still conceptual, but process–based models, Dekker et al. [52] demonstrated a possible sequence of tipping events in a coupled system of the Atlantic Meridional Overturning Circulation and El–Niño Southern Oscillation. Social costs of future climate damages caused by carbon emissions originating from domino effects of interacting tipping elements were studied using an integrated assessment model [53, 54]. Earlier, the propagation of critical transitions in lake chains as an ecological example was analyzed, coupling established models of shallow lakes by a unidirectional stream or via diffusion processes [32, 35]. The effect of spatial heterogeneity and connectivity of bistable patches on the overall ecosystem response was further studied by the application of simple models for eutrophication and grazing of a (logistically–growing) resource [32, 33]. In addition, examples beyond the biogeophysical Earth system possibly giving rise to the propagation of critical transitions were proposed such as coupled subsystems in the fields of economics and finance [4, 45].

1.2. Descriptions of tipping cascades vary across the literature

However, tipping cascades or, more generally, patterns of multiple tipping dynamics discussed to arise from the interaction of tipping elements are often loosely described suffering a similar fate as the ancestral 'tipping point' concept [55]. We encountered important differences across the description of tipping cascades in the recent literature. These differences are in particular related to whether causality is a necessary ingredient for a cascade or not. For example, the pattern where tipping of one system causes the tipping of another system is described as domino dynamics or tipping cascade by Lenton et al. [4]. The propagation of regime shifts by an initial critical transition causing a following one is underpinned by generalized tipping element interactions and termed a cascade by Brummitt et al. [45]. By comparison, the term cascading tipping is used for a sequence of abrupt transitions in Dekker et al. [52] that may not necessarily be causal. This notion of cascading tipping is exemplary applied to the Atlantic Meridional

Overturning Circulation and El–Nino Southern Oscillation as climatic tipping elements [52]. Furthermore, and not restricted to causal events, an effect of one regime shift on the occurrence of another regime shift is suggested as cascading in Rocha et al. [8]. It is confirmed to connect ecological regime shifts such as fisheries collapse and transitions of kelp, mangrove and seagrass ecosystems [8].

Here we systematically identify, characterize and name patterns of multiple tipping dynamics as a domino cascade, a two phase cascade and a joint cascade, which arise in a previously studied system of idealized interacting tipping elements (section 2 and 3). In particular, these patterns of multiple tipping dynamics differ in the way of how the critical transition propagates from one tipping element to another. The domino cascade, the two phase cascade and the joint cascade are subsequently related to the varying descriptions of tipping cascades in the literature and examples of multiple tipping events with comparable characteristics in the Earth system are given. Furthermore, we address the potential for intervention and anticipation by common early warning indicators based on critical slowing down. Implications of the distinct patterns of multiple tipping for the resilience of the Earth system, limitations of studying idealized interacting tipping elements and necessary future research are discussed (section 4).

2. Methods

2.1. Model of idealized interacting tipping elements

Distinct patterns of multiple tipping dynamics emerge from the linear bidirectional coupling of two idealized tipping elements (figure 1). In this model of idealized interacting tipping elements based on Brummitt et al. [45] and Abraham et al. [46], each tipping element depends on its control parameter (or driver) c_i, where $i = 1, 2$, the variation of which may induce a critical transition from a normal to an alternative state with the crossing of a critical control parameter threshold $c_{i_{\text{crit}}}$, where $i = 1, 2$. We consider homogeneous tipping elements, i.e. both tipping elements undergo a critical transition at the same control parameter threshold and on the same intrinsic tipping time scales. A linear coupling term with a coupling strength d_{ij} captures the interaction of the tipping elements following Wunderling et al. [47], where the state of one tipping element is added linearly to the control parameter of another, coupled tipping element. We refer to Wunderling et al. [47] and Klose et al. [56] for a detailed description of the model of idealized interacting tipping elements.

2.2. Evolution of tipping elements in control parameter space

Different pathways through the control parameter space of both tipping elements are applied to the model of idealized interacting tipping elements (as sketched in figure 1(c)). These pathways give rise to distinct patterns of multiple tipping dynamics as described in section 3 and illustrated in figure 2. More specifically and as indicated by the (purple) arrows in figure 1(c), the control parameter c_1 is increased (corresponding to going

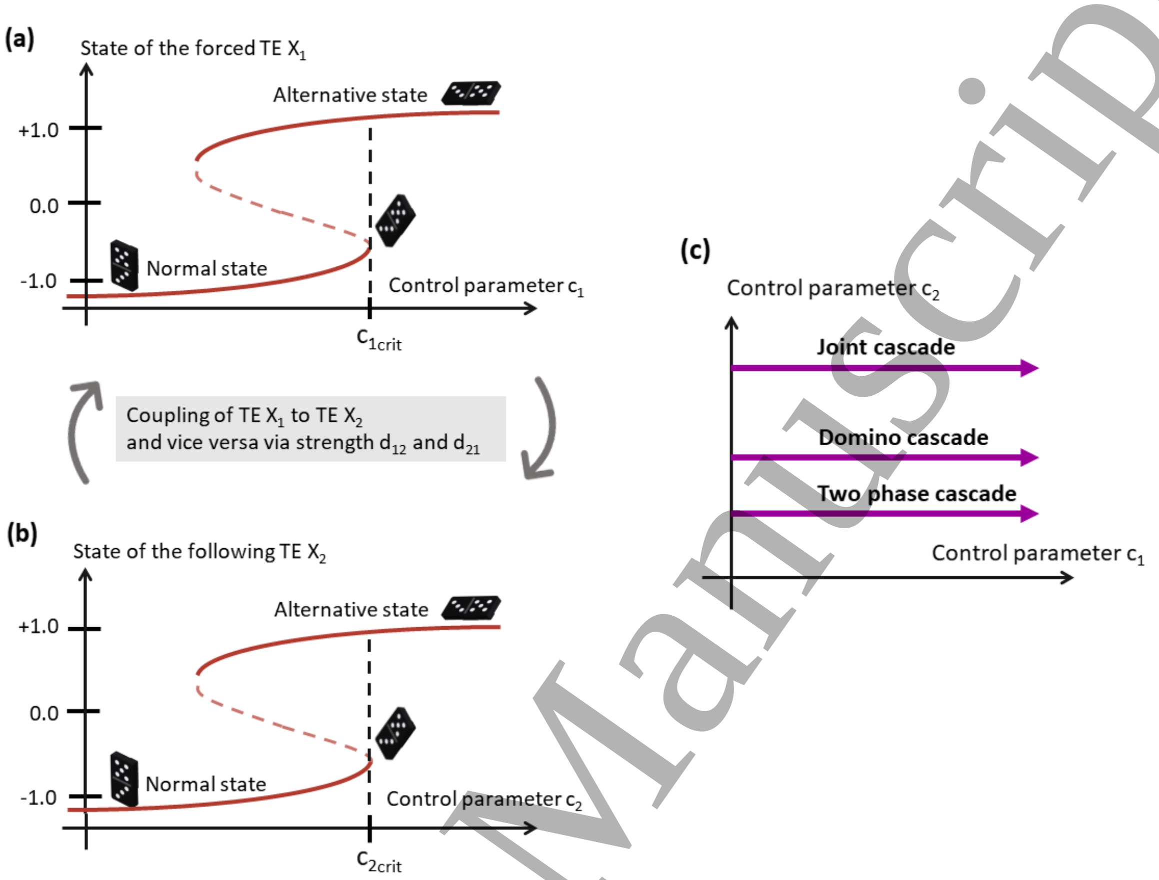

Figure 1. (a) & (b): Long–term behavior of the idealized tipping elements (TE) X_1 (a) and X_2 (b) captured by the respective differential equation of the form $\frac{\mathrm{d}x_1}{\mathrm{d}t} = -x_1^3+x_1+c_1+\frac{1}{2}d_{21}(x_2+1)+\sigma dW$ for subsystem X_1 and $\frac{\mathrm{d}x_2}{\mathrm{d}t} = -x_2^3+x_2+c_2+\frac{1}{2}d_{12}(x_1+1)+\sigma dW$ for subsystem X_2. σ is the noise level of Gaussian white noise which is applied to the system of idealized interacting tipping elements when determining early warning signals. Note that for determining the fixed points (given in red) of the idealized tipping elements X_1 and X_2 the coupling term is not taken into account, i.e. the uncoupled case with $d_{21} = 0$ and $d_{12} = 0$ is shown here. Below the critical threshold $c_{i_{\mathrm{crit}}}$, $i = 1, 2$, there exist two stable fixed points within a certain range of the control parameter c_i, $i = 1, 2$. As soon as the control parameter transgresses its critical value $c_{i_{\mathrm{crit}}}$, the system may tip from the lower (normal) state x_{i-}^* to the upper (alternative) state x_{i+}^*. (c) Sketch of the different scenarios of the control parameter evolution (indicated by purple arrows), which are applied to the model of idealized interacting tipping elements. The control parameter c_1 of the driven tipping element X_1 is increased, while the control parameter c_2 of the following tipping element X_2 is kept constant at distinct levels, giving rise to distinct patterns of multiple tipping dynamics.

from left to right along the outer x–axis in figure 2) sufficiently slowly such that the respective subsystem X_1 can follow its (moving) equilibrium. In other words, by a separation of the intrinsic system time scale and the time scale of the forcing, the system can be regarded as a fast–slow system [57], where the change in the forcing of the system is slow compared to the intrinsic system time scale. The control parameter c_2 of subsystem X_2 is kept constant for simplicity and comparable to Dekker et al. [52]. Distinct levels of the control parameter c_2 are applied (indicated by distinct purple arrows in figure 1(c)), extending Dekker et al. [52] and eventually bringing about qualitatively different patterns of multiple tipping (corresponding to going from top to bottom along outer y–axis in figure 2). In the following, subsystem X_1 is called the *driven* tipping element, being externally driven (towards a critical transition) by the change in the corresponding control parameter c_1. Subsystem X_2 is named the *following* tipping element, only following the change in the external conditions mediated by the coupling on the other hand. Phase space portraits in figure 2 illustrate the loss and gain of fixed point as well as the flow in the phase space along the pathway in the control parameter space. Based on these phase space portraits, possible critical transitions arising from the loss of stable fixed points in a bifurcation can be identified and the dynamics of the patterns of multiple tipping are characterized.

2.3. Critical slowing down and statistical properties of a system of interacting tipping elements

We derive insights on critical slowing down and hence the potential for the anticipation of emerging multiple tipping patterns by the assessment of the corresponding eigenvectors and eigenvalues and their change along the pathway in the control parameter space. The importance of the orientation of the dominant eigenvector for critical slowing down in multi–component system was recognized by Boerlijst et al. [58] and Dakos [59]: It was found that critical slowing down occurs in the direction of the eigenvector corresponding to the dominant eigenvalue. The system component closest to the dominant eigenvector exhibits the slowest exponential recovery rate compared to the other components. We refer to the Supplementary Material for further details on the assessment of the eigenvectors and eigenvalues to gain an understanding of critical slowing down in systems of (idealized) interacting tipping elements.

To relate the insights on critical slowing down gained by the assessment of the eigenvectors and eigenvalues to the statistical time series properties of the different multiple tipping patterns, we estimate autocorrelation and variance as prominent statistical indicators within a sliding window [60, 61] (figure 3). We hereby complement the specific case of multiple tipping dynamics considered by Dekker et al. [52]. Time series are generated by the simulation of the system of interacting tipping elements illustrated in figure 1 under a relatively low noise level in an ensemble of 100 members, using sdeint [44, 62]. Starting from equilibrium, the control parameter c_1 is slowly increased following the sketched pathways in control parameter space (figure 1(c)). We only determine autocorrelation and variance for sliding windows which do not include any critical transition. Otherwise, the estimates of the statistical indicators would be biased [61]. The trend in the statistical indicators is quantified by Kendall's τ coefficient, where a value of $\tau = +1$ (-1) reflects a monotonically increasing (decreasing) statistical indicator with time.

3. Patterns of multiple tipping in a model of idealized interacting tipping elements

In the following, we present three qualitatively different dynamic patterns of multiple tipping and their characteristics, which are relevant for the potential for intervention and anticipation (figure 2).

3.1. Two phase cascade (figure 2(a))

For a relatively low level of the constant control parameter c_2, an increase of the control parameter c_1 across its threshold and the resulting critical transition of subsystem X_1 is not sufficient to directly trigger a critical transition in subsystem X_2. The system converges intermediately to a stable fixed point (compare phase space portraits in figure 2(a), going from $c_1 = 0.0$ to $c_1 = 0.3$ and $c_1 = 0.6$; corresponding to the first domino as subsystem X_1 being tipped while the second domino as subsystem X_2 is not affected). Only a further increase of the control parameter c_1 can initiate the critical transition in subsystem X_2 by the loss of the intermediately occupied stable fixed point (compare phase space portraits in figure 2(a), going from $c_1 = 0.6$ to $c_1 = 1.15$; corresponding to the first, tipped domino being driven towards the second domino which consequently topples). Thus, by limiting the further increase in the control parameter c_1 after the first tipping event of subsystem X_1, a full two phase cascade can be mitigated.

We can identify the two phase cascade with the properties of the cascade described and simulated in Dekker et al. [52] using a comparable model of idealized tipping element interactions. Within the climate system, a stepwise change in the oxygen isotopic ratio at the Eocene–Oligocene transition may be interpreted as a two phase cascade of the Atlantic Meridional Overturning Circulation as the driven tipping element and the Antarctic Ice Sheet as the following tipping element in response to a slowly decreasing atmospheric carbon dioxide concentration [52, 63].

An increase in common statistical indicators of critical slowing down such as autocorrelation and variance (figure 3(a) and (d), black) based on an increasingly slower recovery from perturbations (Supplementary Material, figure S1–S2) are observed for subsystem X_1 on the approach of the two phase cascade in a *pre–tipping time span* before the critical transition of subsystem X_1 (marked in light red in figure 3(a) and (d)). In contrast, for subsystem X_2, an increasingly slower recovery from perturbations (Supplementary Material, figure S1–S2) as well as increasing autocorrelation and variance (figure 3(a) and (d), turquoise) cannot be detected in the pre–tipping time span prior to the critical transition of subsystem X_1. However, given the intermediate convergence to a stable fixed point after the critical transition of subsystem X_1 and prior to the critical transition of subsystem X_2 (see phase space portrait in figure 2(a), for $c_1 = 0.6$), an *intermediate time span* (marked in dark red in figure 3(a) and (d)) offers the possibility to indicate the upcoming critical transition of subsystem X_2 in the two phase cascade. A step–like change to a relatively higher level of the statistical indicators for subsystem X_2 compared to the respective level in the pre–tipping time span is observed (figure 3(a) and (d), turquoise, compare also [52]), indicating an increased vulnerability of subsystem X_2 to a critical transition. The height of the step–like change in the statistical indicators varies with the magnitude of the constant control parameter c_2 as a consequence of an increasingly slower recovery from perturbations in the intermediate time span with increasing magnitude of the constant control parameter c_2. This observation corresponds to the rotation of the eigenvectors and the change in the eigenvalue magnitude of the system of interacting tipping elements, which determine the magnitude and direction of the recovery to perturbations and hence critical slowing down prior to a bifurcation–induced critical transition ([58, 59], Supplementary Material, figure S2). However, no threshold, i.e. a height of the step–like change above which this following tipping occurs, can be observed but it rather is a continuous and relative quantity. In other words, a step–like change of the statistical indicators (though comparably smaller) may also be present after the critical transition of subsystem X_1 even if a critical transition of subsystem X_2 does not follow. Thus, to use this height of the step–like change to clearly indicate an upcoming following transition may be difficult in practice.

3.2. Domino cascade (figure 2(b))

For a slightly elevated constant level of the control parameter c_2, the increase of the control parameter c_1 across its threshold and the corresponding critical transition of subsystem X_1 towards its alternative state is sufficient to trigger a critical transition of subsystem X_2. Note that, in contrast to the two phase cascade, no further increase of the control parameter c_1 is necessary to observe the domino cascade. Instead the tipping of one subsystem (the driven tipping element; the first domino) directly causes and initiates the tipping of another (the following tipping element; the second domino, which is tipped by the toppling of the first domino). This corresponds to the description of a tipping cascade given in Lenton et al. [4] and Brummitt et al. [45] and the general notion of a domino effect including causality [64]. A notable feature is the expected path of the system in the phase space: The intermediately occupied stable fixed point involved in the two phase cascade is lost in a collision with an unstable fixed point with the initiation of the domino cascade (corresponding to leaving the phase space portrait for $c_1 = 0.3$ and comparing the phase space portraits for $c_1 = 0.6$ in figure 2(a) and (b)). Nevertheless, it still influences the dynamics (as indicated by the flow in the phase space portrait in figure 2(b) for $c_1 = 0.6$) as a 'ghost' (e.g. [65, 66, 67, 68]), such that the pathways of a possible trajectory of the system in the phase space are comparable for the two phase cascade and the domino cascade.

As demonstrated recently in a conceptual model, domino cascades may propagate through tipping elements in the Earth system, such as the large ice sheets on Greenland and West Antarctica and the Atlantic Meridional Overturning Circulation [47, 69].

A domino cascade may not be preceded clearly by the increase of the common early warning indicators and relying on these indicators may lead to an unexpected following critical transition of the following tipping element. Increasing autocorrelation and variance as common statistical indicators (figure 3(b) and (e), black) as a consequence of an increasingly slower recovery from perturbations (Supplementary Material, figure S2) are observed for subsystem X_1 on the approach of the domino cascade in the pre–tipping time span (marked in light red in figure 3(b) and (e)). The statistical indicators for subsystem X_2 remain constant but on a relatively higher level than for the two phase cascade in the pre–tipping time span (figure 3(b) and (e), turquoise, compared to figure 3(a) and (d)). However, no clear intermediate time span prior to the critical transition of subsystem X_2 exists allowing for an additional detection of early warning signals as for the two phase cascade.

3.3. Joint cascade (figure 2(c))

Subsystem X_1 and subsystem X_2 may tip jointly (as indicated by the dominoes) with a possible trajectory evolving close to the phase space diagonal for an increase of the control parameter c_1 across its threshold (phase space portrait for $c_1 \leq 0.3$ in figure 2(c)) as opposed to the other two multiple tipping patterns. Such a joint cascade is observed with a strongly elevated level of the constant control parameter c_2. The critical transitions of the respective subsystems cannot be clearly distinguished with regard to their order of tipping. This is in contrast to the domino cascade with subsystem X_2 tipping after the critical transition of subsystem X_1 and the two phase cascade with its intermediatly occupied stable fixed point.

Though the case of a joint cascades has not been treated explicitly in the recent literature on interacting tipping elements, a similar behaviour may be observed in spatially extended bistable ecosystems subject to regime shifts [32, 33].

For both subsystems, a slower recovery from perturbations is expected prior to their joint tipping (Supplementary Material, figure S1–S2). For subsystem X_1, autocorrelation and variance increase on the approach of the joint cascade with increasing control parameter c_1 (figure 3(c) and (f), black). Subsystem X_2 exhibits a relatively high constant level of these statistical indicators prior to the joint cascade (figure 3(c) and (f), turquoise) corresponding to the level of the constant control parameter c_2 (Supplementary Material, figure S2) and indicating the vulnerability of this subsystem to critical transitions.

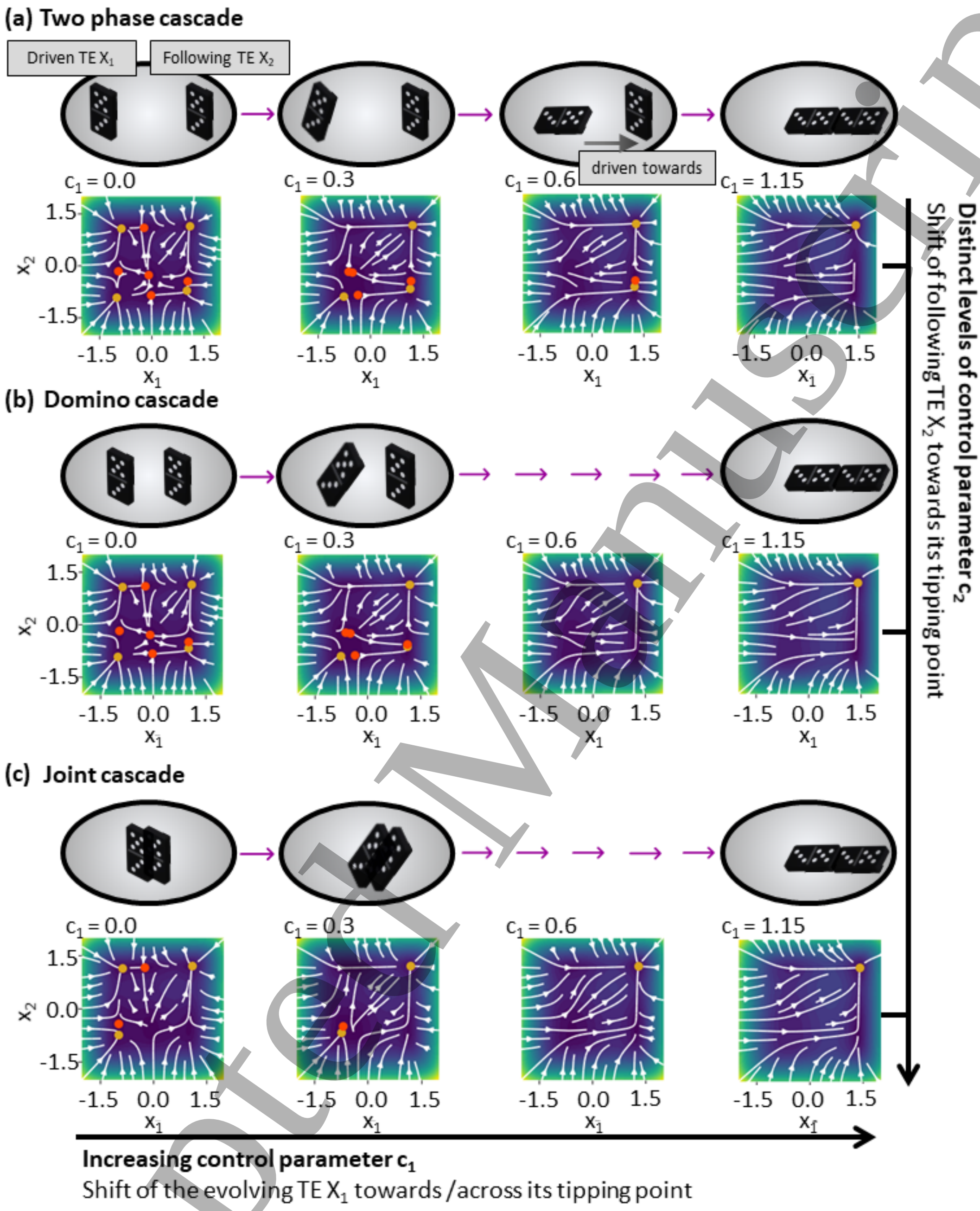

Figure 2. Different patterns of multiple tipping dynamics as identified in the model of idealized interacting tipping elements (compare figure 1(a) and (b)), illustrated in terms of dominoes and by phase space portraits. Within the phase space portraits, orange dots represent stable fixed points, while unstable fixed points are given by red dots. The background colour indicates the normalized speed $v = \sqrt{\dot{x}_1^2 + \dot{x}_2^2}/v_{max}$ going from close to zero (purple) to fast (yellow–green). The patterns of multiple tipping arise by applying specific scenarios of control parameter evolution (sketched in figure 1(c)): The control parameter c_1 of the driven tipping element (TE) X_1 is increased, i.e. the subsystem is driven closer to and across its tipping point (going from left to right). The control parameter c_2 of the following tipping element (TE) X_2 is kept constant for each pattern, while its level differs between the multiple tipping patterns (comparing top to bottom). (a) Two phase cascade, (b) Domino cascade, (c) Joint cascade

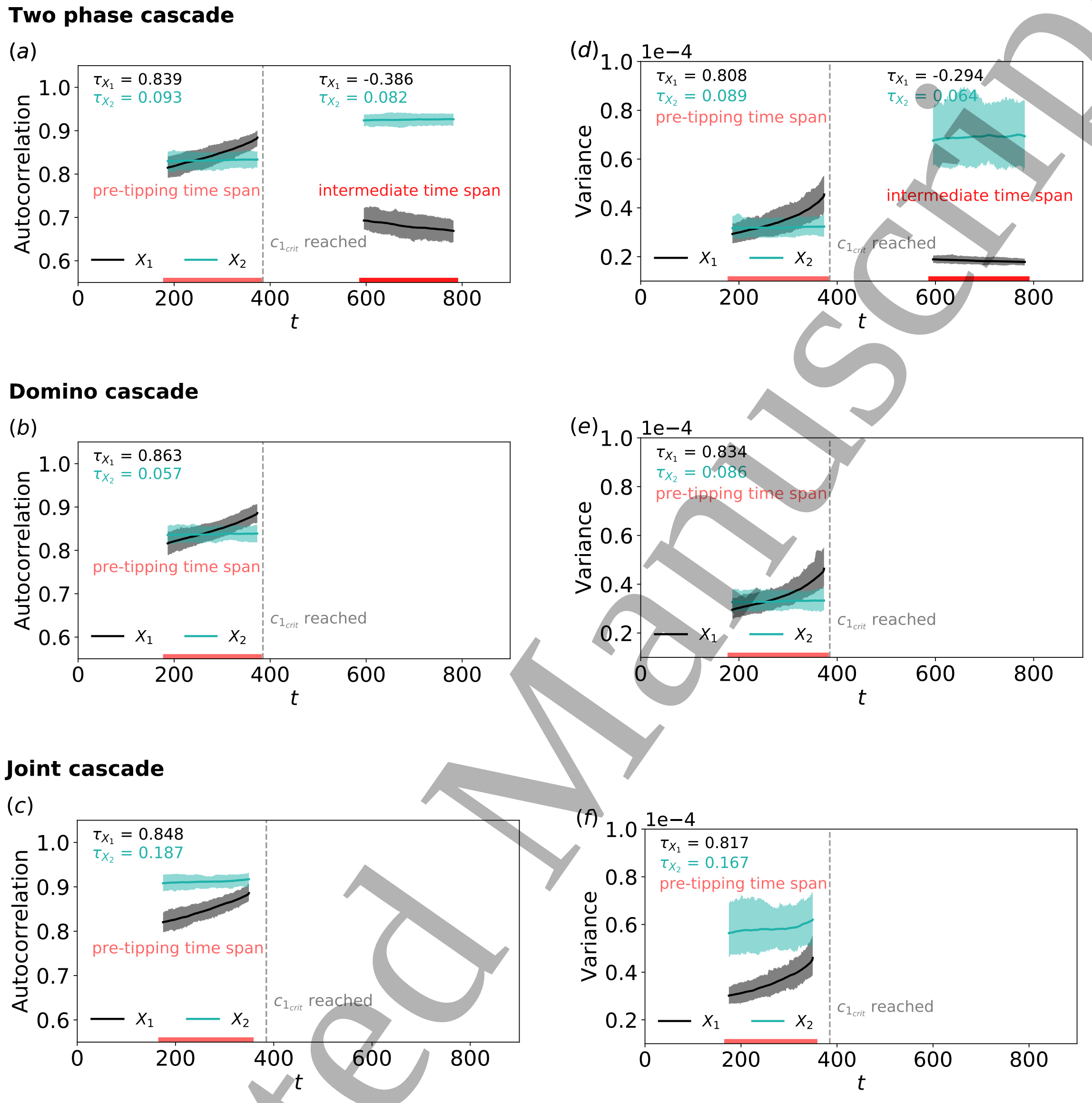

Figure 3. Evolution of autocorrelation (left column) and variance (right column) along different paths within control parameter space for two bidirectionally coupled tipping elements under a relatively low noise level with $d_{21} = 0.2 > 0$ and $d_{12} = 0.2 > 0$. The different pathways within the control parameter space correspond to the patterns of multiple tipping emerging by a slow linear increase of the control parameter c_1 of subsystem X_1 from $c_1 = 0$ while keeping the control parameter c_2 of subsystem X_2 constant ($c_2 = \text{const.}$), compare figure 1(c) for sketch of evolution in control parameter space (with (a) & (d): $c_2 = 0.15$, (b) & (e): $c_2 = 0.16846$, (c) & (f): $c_2 = 0.344$). The dashed grey line indicates the point in time where the critical control parameter threshold $c_{1_{\text{crit}}}$ of subsystem X_1 is reached. Note that a critical transition of subsystem X_1 may occur before $c_{1_{\text{crit}}}$ is reached due to its interaction with subsystem X_2. The pre–tipping time span and the intermediate time span (in case of the two phase cascade) are marked in light and dark red, respectively.

4. Discussion

Studying a system of idealized interacting tipping elements, qualitatively different dynamic patterns of multiple tipping were identified as a two phase cascade, a domino cascade and a joint cascade. We characterize these patterns of multiple tipping dynamics, highlight their differences and derive the related potential for intervention and their anticipation through early warning signals as discussed below. Thereby, we bring together and extend previous work on specific cases of modelled multiple tipping dynamics [52, 47] as well as the general and rather rough description of potentially emerging cascading dynamics due to tipping element interactions (e.g. [4, 9, 10]).

The various patterns of multiple tipping are associated with different, though simplified pathways through control parameter space. In the end, the control parameter evolution determines the emergence of the specific system behavior, which may be a domino cascade, a two phase cascade or a joint cascade. In other words, the control parameter evolution, i.e., the evolution of the drivers, can therefore determine the characteristics of multiple tipping that are observed. However, other factors such as the strength and the sign of coupling are as well decisive for the emergence of tipping cascades. Moreover, in more complex systems, control parameters can not be treated separately for each tipping element and drivers may be shared [8].

The different observed patterns of multiple tipping may have implications for the *mitigation* of tipping by controlling the respective drivers. A limitation of the forcing can prevent the two phase cascade to unfold since a critical transition of the driven tipping element is not sufficient for the spread of a tipping event to a following subsystem. Instead, the critical transition needs to be followed by a further evolution of the respective subsystem's state before a following critical transition is initiated. However, in a domino cascade an initial critical transition of the driven tipping element is sufficient to trigger a slightly delayed but inevitable following critical transition of another tipping element.

In addition, the potential success of *anticipating* the emergence of tipping cascades through early warning indicators based on critical slowing down [70, 71, 72] was assessed using insights of Boerlijst et al. [58] and Dakos [59] on critical slowing down in multi–component systems in relation to the eigenvector orientation. It is demonstrated that the potential for anticipation differs across the patterns of multiple tipping. Thereby, the analysis of statistical properties of the two phase cascade in Dekker et al. [52] is extended to other patterns of multiple tipping dynamics. In particular, we find that common statistical indicators based on critical slowing down may fail for upcoming domino cascades in a system of idealized interacting tipping elements. While increasing autocorrelation and variance are observed for the driven tipping element on the approach of the domino cascade, constant levels of these statistical indicators were determined for the following tipping element. In the case of a two phase cascade or a joint cascade, the critical slowing down based indicators express some degree of vulnerability (or resilience) in the system of interacting tipping elements. However, their application may be unfeasible in practice. More specifically, for the two phase cascade, the critical transition of the driven tipping element is preceded by increasing autocorrelation and variance of the respective subsystem, while a step–like change towards a relatively higher level of the statistical indicators in the intermediate time span is found for the following tipping element. The joint cascade may be conceivable with a raised but constant level of autocorrelation and variance for the following tipping element accompanied by an increase of statistical indicators for the driven tipping element. With the slower recovery from perturbations for both tipping elements, correlations between the subsystems' time series comparable to the application of spatial early warning signals [33, 73, 74, 75, 76] may unfold.

These very specific and simplified scenarios of control parameter evolution demonstrate that an increase of autocorrelation and variance prior to multiple tipping events cannot necessarily be expected. Hence, common early warning indicators should not be relied on as the only way of anticipating cascading critical transitions in systems of interacting tipping elements. In addition, often referenced limitations, false alarms and false positives complicate the application of critical slowing down based indicators to individual tipping elements and the anticipation of upcoming critical transitions [77, 78, 79]. It thus seems to be necessary to invoke a combination of process–based modelling accompanied by monitoring the system under investigation as well as data–driven techniques [61, 78, 79] to detect upcoming multiple transitions and, in particular, the domino cascade.

Note that the presented discussion is restricted to bifurcation–induced tipping with a relatively weak noise. Furthermore, a sufficiently slow change of the tipping element driver is applied. Hence, our examination of tipping cascades excludes early tipping [80] and flickering [81] due to noise as well as rate–induced effects. These ingredients will further influence the presented patterns of multiple tipping, their characteristics such as the intermediate time span of the two phase cascade and hence the potential for anticipation and mitigation. In a related stochastic system, similar patterns were demonstrated as fast and slow domino effects [82]. The patterns of multiple tipping are expected to change in response to a fast change of the tipping element driver with respect to the intrinsic response time scales. Such relative time scale differences between driver and system response cannot be ruled out given the current unprecedented anthropogenic forcing of the biogeophysical Earth system [83, 84]. In addition, rate–induced transitions may occur [85, 86] as suspected based on modelling studies for the Atlantic Meridional Overturning Circulation [87, 88, 89], predator–prey systems [90, 91, 92] and for the release of soil carbon in the form of the compost–bomb instability [86, 93]. These may further complicate the early warning of cascading tipping [80, 94]. Heterogeneity across the response of tipping elements to the same control parameter level [10, 41] and in the intrinsic time scales of tipping [47, 95, 96] was neglected in our study.

Finally, it is assumed that the long–term behaviour of many real–world systems in terms of the system's state such as the overturning strength of the Atlantic Meridional Overturning Circulation [23, 97], the ice volume of the Greenland Ice Sheet [98] and the algae density in shallow lakes [19, 20] can be qualitatively captured by the studied idealized tipping elements featuring a fold bifurcation as tipping mechanism. However, biogeophysical and biogeochemical processes involved in the behaviour of these real—world systems and included in some more complex climate models may either give rise to further types of cascading tipping or may dampen the overall possibilities of tipping behavior [47, 99].

5. Conclusion

Qualitatively different patterns of multiple tipping dynamics in interacting nonlinear subsystems of the climate and ecosystems have been identified in this work. These multiple tipping patterns may emerge as illustrated in a system of idealized interacting tipping elements and include the cases of joint cascades, domino cascades and two phase cascades. As described in Lenton et al. [4] and Brummitt et al. [45] as well as corresponding to the general notion of a domino effect [64], tipping of one subsystem causes or triggers the tipping of another subsystem in a domino cascade. In addition, we find a two phase cascade corresponding to the tipping pattern presented in Dekker et al. [52]. While we reveal that it may be possible to find critical slowing down based early warning indicators for the two phase cascade, such indicators can fail in the case of a domino cascade.

However, our results are limited by the conceptual nature of the system investigated here. In particular, in more complex and process–detailed models of tipping elements the respective nonlinear properties might be smeared out and the presented characteristics of the emerging multiple tipping patterns might be altered due to processes such as strong noise, interactions to other system components or further biogeophysical processes that are not modelled here.

Cascading tipping dynamics have been described rather roughly in the recent literature. As discussed above, the presented patterns of multiple tipping dynamics differ in the potential of their mitigation and anticipation. Given these differences, establishing the notion that multiple tipping dynamics may come about in distinct forms as illustrated in our study is important for further studying interacting tipping elements. We therefore suggest to be more precise in future discussions on potential dynamics arising from the interaction of tipping elements and, in particular, on tipping cascades and to go beyond a loose description of some cascading tipping. For example, in terms of real–world applications, mathematical mechanisms (e.g. rate–induced cascades [80]) as well as related biophysical processes and the evolution of corresponding (and possibly shared [8]) tipping element drivers that may contribute to multiple tipping events should be evaluated carefully.

In the future, a quantitative assessment of interacting tipping elements with an ongoing improvement of their representation in complex (climate) models e.g. by including interactive evolving ice sheets into Earth system models [100] as well as the additional use of paleoclimate data [101, 102] may help to reduce uncertainties on the preconditions for the emergence of tipping cascades and possible early warning indicators based on process–understanding. To the end, these insights may contribute to reflections on the boundaries of the safe–operating space for humanity, and to a better understanding of Earth system resilience with respect to anthropogenic perturbations more generally.

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