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When redistribution makes personalized pricing of externalities useless

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Abstract
We consider a standard optimal taxation framework in which consumers' preferences are separable in consumption and labor and identical over consumption, but are affected by consumption externalities. For every nonlinear, income-dependent pricing of goods there is a linear pricing scheme, combined with an adjusted income tax schedule, that leaves all consumers equally well-off and weakly increases the government's budget. The result depends on whether a linear pricing scheme exists that keeps the aggregate amount of consumption at its initial level observed under nonlinear pricing. We provide sufficient conditions for the assumption to hold. If adjusting the income tax rate is not available, personalized prices for an externality can enhance social welfare if they are redistributive, that is, favor consumers with a larger marginal social value of income.

1 | INTRODUCTION

In a famous result, Atkinson and Stiglitz (1976) showed that, at the second-best optimal allocation, commodity taxation is superfluous when income taxation is present. Laroque (2005) extended this result to suboptimal allocations, showing that if the income tax can be adjusted, every allocation with commodity taxation is weakly dominated by an allocation without commodity taxation.

Recent works modify the Atkinson and Stiglitz (1976) model to include consumption externalities. They show conditions under which optimal prices for externalities follow first-best rules and are unique and linear, that is, do not depend on the pretax income of agents or the quantity of externality-generating goods that is consumed (Gauthier & Laroque, 2009; Jacobs & de Mooij, 2015; Kaplow, 2012). We extend this literature to suboptimal pricing of externalities. Our main finding is that for every nonlinear, income-dependent pricing scheme there is a linear pricing scheme for the externalities, combined with an adjusted income tax schedule, that leaves all consumers equally well-off and weakly increases the government's budget. Personalized prices are useless when distributional concerns are fully handled via income taxation.

Our main finding holds under the following assumptions. Common assumptions include (i) constant returns to scale, (ii) separability of consumers' utility in consumption and labor, and (iii) taste homogeneity in consumption. Extending the model, we allow for multiple externalities that affect productivity and therefore prices and wages, and for consumer preferences to be heterogeneous with respect to labor and the external effects. Our result holds true if it is possible to find a linear pricing scheme for the externalities so that the aggregate amount of consumption remains at its level under the nonlinear, income-dependent pricing scheme, while keeping consumers' consumption subutilities at their initial level. Apart from that, we only require duality between utility maximization and expenditure minimization. For the case of only one good causing an externality, we provide sufficient conditions for the existence of a linear price that keeps the aggregate amount of consumption, and consumers' subutilities, at the original level. The same conditions ensure duality between utility maximization and expenditure minimization.

We extend our result to the setting of climate change and the CO_2-emission externality. In this case, the aggregate consumption of multiple goods causes the same externality. We show that a linear price, equal in all sectors of the economy, is also weakly preferable to a nonlinear, income-dependent and sector-specific price. Furthermore, we consider externalities arising from the income of consumers, and show that personalized prices are again superfluous.

Our general setting extends previous literature in several dimensions. Gauthier and Laroque (2009) and Jacobs and de Mooij (2015) analyze the optimal price of a single good causing an externality. We allow for multiple externalities, whose level may be optimal or nonoptimal. Gauthier and Laroque (2009) further assume that the nonlinear price depends on the income, but not consumption, of consumers; in Jacobs and de Mooij (2015) the nonlinear pricing is continuous and differentiable in individual consumption of the good causing an externality. Our setting includes any functional form of the nonlinear price scheme in income and consumption. Policies of command and control or making exceptions for certain groups of consumers based on their income or consumption are accounted for, which are pervasive in policy-making. In addition, we let the externalities not only affect utilities, but also producer prices and labor productivities. Kaplow (2012) considers tax reforms in case of multiple externalities, and provides sufficient conditions ensuring that moving to, or in the direction of, first-best Pigouvian taxation is Pareto-superior. Kaplow restricts commodity pricing to be linear, whereas we allow for nonlinear, income-dependent pricing. In addition, utility of consumers may not be differentiable in consumption or externalities in our model, an assumption found in Kaplow (2012) and Jacobs and de Mooij (2015).

Our main result shows a possibility: given a nonlinear, income-dependent pricing system for externalities, and given our assumptions, a linear pricing system with an appropriately adjusted income tax schedule performs (weakly) better. Redistribution warrants linear pricing of externalities.

Restribution here means that the income tax schedule can be adjusted freely. For policy makers who want to implement, for example, an environmental tax reform it might be infeasible to redistribute freely via the income tax schedule. Differential pricing could be superior to address distributional concerns, as recently highlighted by Stiglitz (2019). We thus lastly consider the case when income tax rates are fixed, and show that social welfare can be improved with a nonlinear price for an externality that again keeps the externality at its original level. More precisely, we show that such a nonlinear price favors consumers with a higher marginal social value of income. Interestingly, this result holds irrespective of whether the status quo income tax redistributes too much or too little. Even if consumers' utilities are too close together compared to a second-best or third-best (e.g., linear income tax) allocation, a nonlinear price has to be redistributive to enhance social welfare. In case the social welfare function is utilitarian and consumers' marginal utility of income decreases, the nonlinear price impact has to favor the poor.

Our last result echoes the important result of Chichilnisky and Heal (1994), who showed that marginal costs of public good provision are not necessarily equalized at the optimum when redistribution is restricted. Sheshinski (2004) also showed that optimal taxes for an externality should be higher for richer consumers if the income tax system does not sufficiently redistribute. We extend these results by considering suboptimal commodity taxation and introducing labor choices. Stiglitz (2019) also discusses cases in which the benefits of alleviating distributional concerns by imposing greater environmental taxes on rich consumers can outweigh the efficiency costs from differentiated pricing, given that the income tax system cannot be adjusted freely, albeit without invoking a full optimal taxation model.

The rest of the paper is structured as follows. Section 2 introduces the model and notations, followed by Section 3 in which the results are stated and proved. Section 4 discusses the policy implications of our results.

2 | MODEL

Consider an economy with a continuum of consumers $i \in [0, 1]$ that derive utility from the consumption bundle $x_i = \left(x_i^1, ..., x_i^n\right)$, and disutility from labor supply ℓ_i: $U_i(V(x_i, E), \ell_i, E)$. There are N consumption externalities $E = (E_1, ..., E_N)$, with $E_j = E_j(X)$, where X is the vector of the aggregate consumption of those goods that cause the externalities: $X = \left\{..., \int x_i^m, ...\right\}_{m \in S, S \subseteq \{1, ..., n\}}$.[1] Disposable income is given by $R(y_i)$, with the nonlinear tax schedule R only a function of pretax income y_i. We assume constant returns to scale so that prices $p(E)$ and wages $w_i(E)$ do not depend on microeconomic activity levels. Individual wage $w_i(E)$ is private information and pretax income is equal to $y_i = w_i(E)\ell_i$.

Previous literature often considers externalities that only affect consumption preferences V. To keep as much generality as possible, we assume that externalities may influence consumption preferences, the labor-consumption choice, as well as labor and total factor productivity.

Assumption 1. Let utility $V(x, E)$ be such that the indirect utility function

$$\nu(p, R, E) = \max_x \{V(x, E) | px \leq R\}$$

and the expenditure function

[1] The notation $\int x_i^k$ is a short-hand for $\int_0^1 x_i^k di$, and all functions indexed by i are assumed to be Lebesgue integrable in this paper. The results are also valid for a finite set of consumers, provided it is assumed that in their market behavior, consumers neglect the influence of their consumption on the externality.

$$e(p, v, E) = \min_x \{px \,|\, V(x, E) \geq v\}$$

satisfy the following identity $v(p, e(p, v, E), E) \equiv v$.

Assumption 1 reduces our setting to cases where there is duality between utility maximization and expenditure minimization. Duality holds, for example, if utility $V(x, E)$ is continuous and locally nonsatiated in $x, p \gg 0$ and $v \neq V(0, E)$ (see, e.g., Mas-Colell et al., 1995, p. 59).

Further, define the Hicksian demand for commodities as:

$$h(p, v, E) = \operatorname{argmin}_x \{px \,|\, V(x, E) \geq v\}.$$

The government raises funds through income taxes and taxes $t(x_i, y_i)$ on goods that cause external effects. The tax on an individual good may be positive or negative; for the latter the government subsidizes consumption of this good. Government prices generally depend on the consumption of goods and income of consumers. The budget of the government is:

$$G = \int y_i - R(y_i) + \sum_{k \in S} t^k(x_i, y_i) x_i^k.$$

Individual i's budget constraint can then read as $(p(E) + t(x_i, y_i))x_i \leq R(y_i)$.

Consumers maximize their utility U_i given their budget constraint. All variables at the initial equilibrium with nonlinear pricing are denoted with a hat, for example, $\hat{x}_i$, $\hat{\ell}_i$, and $\hat{E}$. Let $\hat{U}_i$ denote consumer i's utility at this equilibrium, and $\hat{V}_i$ the subutility from consumption.

3 | RESULTS

3.1 | With adjustment of income tax schedule

This section first shows that the economy with nonlinear, income-dependent pricing $t(x_i, y_i)$ and initial income tax scheme $R(y)$ is weakly dominated by an economy with linear prices for goods, denoted $\bar{t}$, and an adjusted income tax scheme $\bar{R}(y)$. This result holds true under Assumption 1 and the assumption that, at the linear pricing scheme $\bar{t}$, externalities remain at their original level $\hat{E}$. Based on this main finding, we provide a brief extension showing that a single, linear tax on CO_2-emissions weakly dominates a sector-specific, nonlinear and income-dependent tax. Lastly, we provide sufficient conditions for the two assumptions underlying our main result to hold if only one good causes an externality.

The main result of our paper is summarized in the next proposition.

Proposition 1. *Take a linear pricing scheme $\bar{t}$ levied on goods causing external effects (i.e., $\bar{t}^m = 0 \;\; \forall \;\; m \notin S$). Assume that with this linear pricing scheme and at initial subutilities $\hat{V}_i$, total Hicksian demand for goods with external effects remain at their initial level $\hat{X}^k$ for all $k \in S$. There is an allocation with an alternative income tax schedule $\bar{R}(y)$ and externality pricing at $\bar{t}$ in which each consumer keeps initial utility $\hat{U}_i$ and the government budget is weakly greater than at the initial level.*

Proof. Since utility is separable in consumption and labor, each consumer's optimal consumption bundle only depends on her income. The following function $\hat{v}(y, \hat{E})$ is the utility derived from consumption when before tax income is y:

$$\hat{v}(y,\hat{E}) = \max_x\{V(x,\hat{E})|(p(\hat{E}) + t(x,y))x \le R(y)\}. \quad (1)$$

With $\hat{v}(y,\hat{E})$ given, each consumer chooses labor by maximizing $U_i(\hat{v}(w_i(\hat{E})\ell_i,\hat{E}),\ell_i,\hat{E})$.

Now, fix the pricing scheme at the linear $\bar{t}$. Let $\bar{x}_i = h(p(\hat{E}) + \bar{t}, \hat{v}(w_i(\hat{E})\hat{\ell}_i,\hat{E}),\hat{E})$. By assumption of the proof, aggregate consumption remains the same with the linear pricing scheme as before: $\int\bar{x}_i^k = \int\hat{x}_i^k = \hat{X}^k, \forall\ k \in S$. We next construct a different income tax schedule $\bar{R}(y)$ that keeps the function $\hat{v}(y,\hat{E})$ unchanged, so that labor choices remain the same with linear as with nonlinear pricing.

Let $\bar{R}(y) = e(p(\hat{E}) + \bar{t}, \hat{v}(y,\hat{E}),\hat{E})$. The utility maximization problem defines $\bar{v}(y,\hat{E}) = \max_x\{V(x,\hat{E})|(p(\hat{E}) + \bar{t})x \le \bar{R}(y)\} = v(p(\hat{E}) + \bar{t}, \bar{R}(y),\hat{E})$. By Assumption 1, $\bar{v}(y,\hat{E}) = \hat{v}(y,\hat{E})$.

By definition of Hicksian demand, one has, for all i,

$$(p(\hat{E}) + \bar{t})\bar{x}_i \le (p(\hat{E}) + \bar{t})\hat{x}_i.$$

Therefore, $p(\hat{E})\bar{x}_i \le p(\hat{E})\hat{x}_i + \bar{t}(\hat{x}_i - \bar{x}_i)$, and summing up over the agents,

$$\int p(\hat{E})\bar{x}_i \le \int p(\hat{E})\hat{x}_i + \sum_{k\in S}\bar{t}^k\underbrace{\int\left(\hat{x}_i^k - \bar{x}_i^k\right)}_{=0}. \quad (2)$$

The result then follows since:

$$\hat{G} = \int w_i(\hat{E})\hat{\ell}_i - R(w_i(\hat{E})\hat{\ell}_i) + t(\hat{x}_i, w_i(\hat{E})\hat{\ell}_i)\hat{x}_i = \int w_i(\hat{E})\hat{\ell}_i - \int p(\hat{E})\hat{x}_i$$
$$\le \int w_i(\hat{E})\hat{\ell}_i - \int p(\hat{E})\bar{x}_i = \int w_i(\hat{E})\hat{\ell}_i - \bar{R}(w_i(\hat{E})\hat{\ell}_i) + \bar{t}\bar{x}_i = \bar{G}$$

□

Nonlinear, income-dependent pricing of the externalities is hence superfluous, irrespective of whether the income tax schedule is optimal and the externality is fully internalized in the adjusted economy. Given the assumptions of the proposition, it is always possible to move to the tax system with linear pricing while keeping all consumers at their original utilities.

The government experiences a surplus in tax revenue through the tax reform. The government budget is weakly larger with linear pricing for a similar reason that, in the absence of externalities, commodity taxation is not needed when income taxation is only restricted by incentive constraints (Atkinson & Stiglitz, 1976; Kaplow, 2006; Laroque, 2005). Labor taxation distorts the economy for good reason: income taxes raise revenue and redistribute between consumers. Commodity taxation is needed due to the external effects. Differentiated commodity taxation would only be needed if it were better able to raise revenue, redistribute or regulate the externality. However, leisure is separable from the consumption choice and consumption choices are homogenous across consumers, making all goods equal substitutes or complements for leisure. In addition, the externalities are caused by aggregate consumption with no particular consumer contributing more or less at the margin. Nonlinear and income-dependent commodity taxation cannot provide further benefits if the income tax can be flexibly adjusted, and at worst creates further distortions in the consumption choice.

The government may use the surplus in its budget to increase the utility of all consumers. Assume that utility of consumers has multiplicative or additive separability between subutility $V(x)$, labor choices, and external effects. This is the case, for example, if $U_i = U_i(V(x_i)\cdot g_i(\ell_i)\cdot d_i(E))$ or $U_i = U_i(V(x_i) + g_i(\ell_i) + d_i(E))$. A Pareto-superior point can be reached if the surplus finances a reduction in the harm (in case of negative external effects) or an increase in the benefits (in case of positive external effects) from the externalities, for example, change function $d_i(E)$ to some $\bar{d}_i(E)$ with $U_i(\ldots d_i(E)) < U_i(\ldots \bar{d}_i(E))$. In the context of pollution from CO_2 emissions, the additional resource would finance adaptation to climate change, thereby weakly increasing the utility of all consumers.

Assuming that the surplus in the government's budget can be redistributed to consumers so that everyone is better off, consumers would also absorb the benefits of the tax reform. Laroque (2005), Remark 1, discusses conditions under which this is possible in the absence of externalities: each consumer receives the same lump-sum transfer. In our setup, redistribution to consumers may change the level of externalities, and thereby could make some, or all, consumers worse off. Kaplow (2012) shows that a Pareto-superior point can be reached even when the levels of externalities change, however only under the assumption that the marginal harm of all externalities is constant. As in Kaplow (2012) one may just assume that "more is better," so that the government can and will use the surplus at hand to improve the situation of consumers, for example, by providing more of a public good that does not adversely interfere with the level of externalities. Investigating more precise circumstances under which a Pareto-improvement is possible in our setup is beyond the scope of this paper but should be considered in future research.

Our proof is an extension of Laroque (2005). The key addition, to deal with the externality, is the assumption that there is a linear pricing scheme $\bar{t}$ that keeps the aggregate levels of goods that cause externalities at their original levels with nonlinear pricing $t(x, y)$. In fact, any linear pricing scheme with this property does the trick. This additional assumption drives our possibility result. Such a linear pricing scheme should exist in common economic circumstances, and we below provide sufficient conditions for the case of only one good that causes the externality.

Remark 1. Proposition 1 can be reformulated using different assumptions about linear pricing. Assume that a linear pricing scheme $\bar{t}$, that generally allows $\bar{t}^k \neq 0 \;\; \forall \;\; k$, exists that (i) at initial subutilities $\hat{V}_i$ leads to Hicksian demand which keeps externalities at their original level $\hat{E}$ (while aggregate levels of consumption may change) (ii) does not increase the producer value of consumption compared to the original level $\int p(\hat{E})\hat{x}_i$. Again, with linear pricing at $\bar{t}$ and adjusted income taxation, given by the proof to Proposition 1, the government budget weakly increases.

Remark 2. Following up on Remark 1, we may actually search for the pricing scheme that minimizes the producer value of consumption.[2] The program

$$\bar{x} = \min_x \left(\int p x_i \,\middle|\, \hat{E}_j = E_j(X) \; \forall \, j \quad \text{and} \quad \hat{V}_i = V(x_i, \hat{E}) \; \forall \, i \right)$$

defines the allocation with maximum government budget while keeping externalities fixed and utilities at their original value (ensuring that incentive constraints are fulfilled). Assuming that every function is differentiable, one can show that commodity prices with maximum government budget are linear.[3]

[2] We are grateful to an anonymous referee who pointed out this approach.

[3] The first-order-conditions are $\forall \;\; k, i$: $p^k = \sum_j^N \frac{\partial E_j}{\partial X^k} \cdot \zeta^j + \lambda_i \frac{\partial V_i}{\partial x_i^k}$. From these we have: $\frac{\partial V_i / \partial x_i^k}{\partial V_i / \partial x_i^m} = \frac{p^k - \sum_j^N \partial E_j / \partial X^k}{p^m - \sum_j^N \partial E_j / \partial X^m}$, showing that the marginal rate of substitution between goods is the same for all consumers. They can hence not face differentiated after-tax prices.

Based on Proposition 1 further extensions can be derived.

Remark 3. Checking the proof, our main result in Proposition 1 extends to externalities derived from pretax income of consumers $E = E(X, \{y_i\}_{i\in[0,1]})$.[4]

For the case of CO_2-emissions, an additional specification can be made: the production of almost all consumption goods entails CO_2-emissions that cause the same externality—global warming. The following result is a straightforward extension of Proposition 1. We show that a move from a nonlinear pricing scheme to a different income tax and a linear tax on emissions s (a scalar)—the same in all sectors—is weakly preferable.

Denote the emission intensity of each good k as α^k (amount of CO_2-emissions per amount of good). The climate change externality is determined by the aggregate amount of emissions: $E = \sum_{k=1}^{n} \alpha^k X^k$. We now consider a nonlinear tax on emissions $t(x, y)$. The tax may differ between sectors of the economy so that generally $t^m(x, y) \neq t^k(x, y),\ \ m \neq k$.

Proposition 2. *Assume that it is possible to find a single linear tax on emissions, $\bar{s}$, such that, at initial subutilities $\hat{V}_i$, the total Hicksian demand for all goods is such that total emissions are equal to the initial $\hat{E} = \sum_{k=1}^{n} \alpha^k \hat{X}^k$. There is an allocation with an alternative income tax schedule $\bar{R}(y)$ and an emission tax at $\bar{s}$ in which each consumer keeps initial utility $\hat{U}_i$ and the government budget is weakly greater than at the initial level.*

Proof. The proof is almost exactly equivalent to that of Proposition 1. We only need to adjust summing over expenditures at Equation (2) to

$$\int p(\hat{E})\bar{x}_i \leq \int p(\hat{E})\hat{x}_i + \bar{s}\int \underbrace{\sum_{k=1}^{n}(\alpha^k \hat{x}_i^k - \alpha^k \bar{x}_i^k)}_{=\hat{E}-\bar{E}=0}.$$

□

The same tax rate on CO_2 emissions in all sectors of the economy is weakly superior to differentiated taxation. As a potentially relevant example in which taxes are differentiated, consider the case of EU carbon pricing on emissions by power stations, which entails a lower marginal cost of abatement in the power sector than in road transportation, where regulation is done through performance standards and national taxes (Heinrichs et al., 2014). Introducing the same CO_2 price on EU-wide emissions can generate a surplus in the EU's budget, assuming that income taxation between consumers can be flexibly adjusted. However, since the EU is a monetary but not a fiscal union (yet), such an adjustment might actually not be feasible. We revisit this point in Section 3.2.

So far, we had to rely on two assumptions to show our main possibility result in Proposition 1, and its extension in Proposition 2. First, there is a linear pricing scheme that keeps aggregate consumption of goods at their original levels. Second, we assume duality between utility maximization and expenditure minimization (Assumption 1). We next present sufficient conditions for both assumptions to hold in the case when only good 1 causes an externality.

Consider the case in which externalities stem only from total consumption of good 1, $E = X^1$. In what follows we study possible variations of demand in a hypothetical world in which the externality $\hat{X}^1$ is fixed (even if total demand for good 1 does not add up to this level).

[4] We thank an anonymous referee for pointing this out to us.

From the utility function $V(x, X^1)$, one derives the utility $V^R(x^1, z, \hat{X}^1)$, where $z = \sum_{k\geq 2}^{n} p^k(\hat{X}^1)x^k$. Since $p^k(\hat{X}^1), k \geq 2$ are fixed when $\hat{X}^1$ is fixed, we will treat the expenditure on these goods as a composite commodity (the price of which is 1).[5] Let $\chi = \sup\{V^R(x^1, 0, \hat{X}^1) | 0 \leq x^1 \leq \hat{X}^1\}$.

Lemma *Assume that $V^R(x^1, z, \hat{X}^1)$ is continuous, locally nonsatiated and strictly quasi-concave in (x^1, z). Further assume that $V^R(x^1, z, \hat{X}^1) > \chi$ for all x^1, z such that $z > 0$.*

Given utility levels $V(\hat{x}_i, \hat{X}^1) > \chi \ \forall\ i$, Assumption 1 holds true (for $X^1 = \hat{X}^1$) and there exists a linear price for the externality $\bar{t}^1$ so that the aggregate Hicksian demand of good 1 is $\hat{X}^1$.

Proof. See Appendix A. □

The Lemma adds rather mild assumptions about subutility V^R and the initial allocation to show that Proposition 1 holds true and Assumption 1 is fulfilled. Standard assumptions about utility V^R are: continuity, local nonsatiation and strict quasi-concavity. In addition, we assume that initial subutilities $\hat{V}_i$ are not attainable by consuming good 1 only.

The assumptions in the Lemma show why we choose to work with V^R instead of V. The Lemma can also be formulated using the original $V(x, X^1)$; however one needs strict quasi-concavity of V in x. We believe it is less restrictive to assume strict quasi-concavity of V^R instead of V.

3.2 | With income tax rate fixed

Superiority of linear pricing in the presence of externalities critically depends on a well-designed adjustment of the income tax system. As highlighted by Stiglitz (2019), a change in the income tax might not be attainable when reforming commodity pricing, and differentiated pricing should be used to alleviate distributional concerns. This difficulty prominently features for global warming and the taxation of CO_2 emissions. When attempting to equalize international carbon prices, policy-makers cannot rely on income taxation to deal with distributional issues as it does not exist between countries.

To accommodate such policy-relevant settings, this section illustrates that if the income tax rate is fixed at a suboptimal level compared to the second-best optimum, a nonlinear price for the externality, diffentiated by income,[6] can improve social welfare. To that end, we restrict attention to the model with only good 1 causing a consumption externality. The analysis is inspired by Chichilnisky and Heal (1994), who showed that in the absence of redistribution, marginal costs of public good provision are generally not equalized between agents in the optimum. Sheshinski (2004) and Stiglitz (2019) discuss similar results. Here, we retain the incentive constraints for labor supply in the present optimal taxation model.

The next proposition shows that whenever a social improvement is possible with a differentiated price, it has to be redistributive, that is, display a negative correlation between the agents' personal tax reform impact and their marginal social value of money, echoing the results of

[5]The construction of V^R follows Varian (1992) p. 148. Denote p^0 as the price vector of all other goods but good 1. Hence we have $z = p^0 \cdot (x^2, ..., x^n)$. We can construct the indirect utility $v(p_1, P, R) = \max_x V(x),\ s.t.\quad p^1x^1 + Pp^0 \cdot (x^2, ..., x^n) = R$. Utility V^R is $V^R(x^1, z) = \min_{p^1, P} v(p_1, P, R),\ s.t.\quad p^1x^1 + Pz = R$, with which we have $v(p^1, P, R) = \max_{x^1, z} V^R(x^1, z),\ s.t.\quad p^1x^1 + Pz = R$.

[6]Since the consumption of goods depends on income only, due to identical consumption preferences in the population, there is no need in this section to make the price for the externalities depend on consumption in addition to income. The function $t(x_i, y_i)$ was introduced in previous sections only to allow for full generality in the tax function.

Chichilnisky and Heal (1994) and Sheshinski (2004) and the intuition in Stiglitz (2019). What is noteworthy about this result is that it does not depend on whether the status quo income tax is too little redistributive or not. Even if the status quo income tax is too redistributive according to the social welfare function, the reform still has to enhance redistribution to improve social welfare.

Here is a simple example of this paradox. Assume that

$$U_i = \sqrt{\sqrt{x_i^1 x_i^2} - 0.5\ell_i^2 - 0.01X^1},$$

and consider an economy with two agents, with wage rates $w_1 = 1$, $w_2 = 2$. Producer prices are 1 for both goods. The government has no expenditure, and the income tax schedule takes the simple linear form $R(y) = b + (1 - a)y$, where b is adjusted to balance the budget. The status quo has a 10% tax on x^1. In this example, for a utilitarian social welfare function, the optimal linear tax rate a is then at 10.4%, leaving utilities of consumers 1 and 2 at 0.4 and 0.66, respectively. But suppose that the status quo has a tax rate a of 50%, leaving utilities of consumers 1 and 2 at 0.42 and 0.51, respectively, which decreases utilitarian welfare as there is too much redistribution. However, still at this point, social welfare can be improved by a reform that reduces the tax on x^1 to 9% below an income threshold of 0.5 and raises it to 10.4% above that threshold, while keeping $a = 50\%$ and adjusting b to balance the budget. The tax reform keeps the aggregate consumption of good 1 at the initial level. This reform advantages individual 1 and penalizes individual 2, and in spite of their utilities being too close together, social welfare increases (see Table B1 in the appendix).

A redistributive nonlinear commodity tax improves welfare despite the linear income tax being too redistributive. The reason is the following. Restricting the income tax to being linear entails a large efficiency cost when using this instrument to decrease inequality between the two consumers (so that social welfare is lower at 50% income tax compared to 10.4%). Still, the motive to reduce inequality is large coming from the utilitarian social welfare function: even at a linear income tax of 50%, the marginal social value is greater for individual 1. While further distributive benefits cannot be generated through raising the linear income tax because the costs are too large, the nonlinear commodity tax is better able to redistribute. The tax break individual 1 enjoys in the reform looms larger than the penalty endured by the other individual.

In the same setting, consider the optimal combination of a linear income tax and a uniform tax on good 1. The income tax rate is then 12.8% and the tax on good 1 is at 4%. But this is not the optimal second-best allocation, because of the linearity of the income tax. One can then improve on this allocation by lowering the tax rate on good 1 at 3% below income 0.5 and raising it to 4.4% above that threshold, while keeping $\bar{X}^1$ unchanged and the budget balanced through a small adjustment of b (see Table B1 in the appendix).

Our more general result is the following.

Proposition 3. *Assume that the income tax schedule is fixed at $\bar{R}$ and the status quo price for the externality is the linear $\bar{t}^1$. Assume further that an increase in social welfare is feasible through an infinitesimally small change from $\bar{t}^1$ to $\bar{t}^1 + \delta(\bar{y}_i)$ that does not alter the externality level $\bar{X}_1$, and that the budget is kept in balance by a uniform lump-sum transfer δ_0. Then one must have*

$$\int \beta_i(\delta_0 - \bar{x}_i^1 \delta(y_i)) > 0,$$

where $\beta_i = \frac{\partial W}{\partial U_i}\frac{\partial U_i}{\partial v}\frac{\partial v_i}{\partial R(y)}$ denotes i's marginal social value of income.

Proof. In the following we omit the dependence on the aggregate level of the external good X^1 as it does not change. The subutility V of consumers can be represented by the indirect utility function $v_i = v\left(p + t_i^1, z_i\right)$ with after-tax income $z_i = R(y_i)$ and individually specific prices for the externality t_i^1 for each consumer i. The change in social welfare when adjusting the price $\bar{t}^1$ to $\tilde{t}_i^1 = \bar{t}^1 + \delta(y_i)$, and introducing the lump-sum transfer δ_0, is:

$$\delta W = \int \frac{\partial W}{\partial U_i}\left[\frac{\partial U_i}{\partial v}\left(\frac{\partial v_i}{\partial p^1}\delta(y_i) + \frac{\partial v_i}{\partial z}(\delta_0 + \bar{R}' w_i d\ell_i)\right) + \frac{\partial U_i}{\partial \ell} d\ell_i\right] \tag{3}$$

$$= \int \frac{\partial W}{\partial U_i}\frac{\partial U_i}{\partial v}\frac{\partial v_i}{\partial z}\left(\underbrace{\frac{\frac{\partial v_i}{\partial p^1}}{\frac{\partial v_i}{\partial z}}}_{=-x_i^1}\delta(y_i) + \delta_0 + \bar{R}' w_i d\ell_i + \underbrace{\frac{\frac{\partial U_i}{\partial \ell}}{\frac{\partial U_i}{\partial v}\frac{\partial v_i}{\partial z}}}_{=0} d\ell_i\right) \tag{4}$$

The first under-braced equality follows from Roy's identity, the second from the envelope theorem. Hence,

$$dW = \int \beta_i\left(\delta_0 - \bar{x}_i^1 \delta(y_i)\right).$$

□

The condition $\int \beta_i(\delta_0 - \bar{x}_i^1\delta(y_i)) > 0$ means that agents with a positive fiscal impact $\delta_0 - \bar{x}_i^1\delta(y_i)$ must on average have a greater marginal social value of money, and conversely. Under a typical income tax, with a utilitarian social welfare function and diminishing marginal value of money, the worse-off individuals have greater marginal social welfare function even if redistribution is excessive compared to the second-best or third-best optimum. In such a setting, when reforming the income tax rate is not possible, differentiated commodity taxation that is socially beneficial has to favor the poorer agents.

Note that we do not provide sufficient conditions for social welfare to improve with differentiated prices. An improvement might actually be hard to attain along the second-best Pareto-frontier.

Proposition 3 shows the direction in which the linear price needs to be adjusted to obtain an increase in welfare, for small changes to the price. Welfare can however also increase for substantial changes to the linear price. Reconsider the example above. If the status quo has an income tax rate at 30% (which is excessively large) and a tax on x^1 at 10%, social welfare increases if the tax on x^1 is canceled for the low-income group and raised to 13.8% for the high-income group. (See Table B1 in the Appendix for a comparison of all numerical examples.)

4 | CONCLUSIONS FOR POLICY DESIGN

For policy-makers our results point to two quite different recommendations. First, an externality tax reform that replaces a nonlinear, sector-specific pricing scheme with a single linear price, the same in all sectors, can save resources. A successful reform depends on a well-designed adjustment of the income tax schedule. A policy-maker should therefore introduce linear pricing whenever it can be combined with adjusted income taxation.

In many situations, however, the governmental body in charge of changing commodity pricing is not in charge of income taxation (e.g., an environmental agency wants to regulate pollution but does not have the rights of the fiscal agency). A simultaneous change in both tax systems might not be feasible. In the second line of reasoning, our last result shows that if income tax rates are fixed, a reform that introduces an income-dependent price for the externality can enhance social welfare only by being redistributive. For a utilitarian social welfare function, the tax reform has to favor poorer agents, leaving them better off on average. Interestingly, this result does not depend on whether the income tax redistributes too little or not. Even if redistribution is too large, we show that a price reform has to be redistributive to enhance social welfare.

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REFERENCES

Atkinson, A., & Stiglitz, J. (1976). The design of tax structure: Direct versus indirect taxation. *Journal of Public Economics*, *6*(1–2), 55–75.

Chichilnisky, G., & Heal, G. (1994). Who should abate carbon emissions? *Economics Letters*, *44*(4), 443–449.

Gauthier, S., & Laroque, G. (2009). Separability and public finance. *Journal of Public Economics*, *93*(11–12), 1168–1174.

Heinrichs, H., Jochem, P., & Fichtner, W. (2014). Including road transport in the EU ETS (european emissions trading system): A model-based analysis of the german electricity and transport sector. *Energy*, *69*, 708–720.

Jacobs, B., & de Mooij, R. A. (2015). Pigou meets mirrlees: On the irrelevance of tax distortions for the second-best pigouvian tax. *Journal of Environmental Economics and Management*, *71*, 90–108.

Kaplow, B. L. (2012). Optimal control of externalities in the presence of income taxation. *International Economic Review*, *53*(2), 487–509.

Kaplow, L. (2006). On the undesirability of commodity taxation even when income taxation is not optimal. *Journal of Public Economics*, *90*(6–7), 1235–1250.

Laroque, G. R. (2005). Indirect taxation is superfluous under separability and taste homogeneity: A simple proof. *Economics Letters*, *87*(1), 141–144.

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic theory*. Oxford University Press.

Sheshinski, E. (2004). On atmosphere externality and corrective taxes. *Journal of Public Economics*, *88*(3–4), 727–734.

Stiglitz, J. E. (2019). Addressing climate change through price and non-price interventions. *European Economic Review*, *119*, 594–612.

Varian, H. R. (1992). *Microeconomic analysis*. W. W. Norton & Company, Inc.

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APPENDIX A: PROOF OF LEMMA 1

First, we show that Assumption 1 holds true. The Hicksian and Walrasian demands are defined as follows:

$$h(p^1, v, \hat{X}^1) = \text{argmin}_{x^1,z}\{p^1x^1 + z \,|\, V^R(x^1, z, \hat{X}^1) \geq v\},$$
$$x(p^1, R, \hat{X}^1) = \text{argmax}_{x^1,z}\{V^R(x^1, z, \hat{X}^1) \,|\, p^1x^1 + z \leq R\}.$$

By continuity of V^R, we have $V^R(\bar{x}^1, \bar{z}, \hat{X}^1) = v$ for all $(\bar{x}^1, \bar{z}) \in h(p^1, v, \hat{X}^1)$.

We now show that $V^R(\hat{x}^1, \hat{z}, \hat{X}^1) = v$ for all $(\hat{x}^1, \hat{z}) \in x(p^1, R, \hat{X}^1)$ if the budget is equal to the expenditure resulting from Hicksian demand $R = (p^1, 1)\cdot(\bar{x}^1, \bar{z})$, $(\bar{x}^1, \bar{z}) \in h(p^1, v, \hat{X}^1)$. (Note that $R = e(p^1, v, \hat{X}^1)$.) Assume this does not hold. Then the solutions to the Walrasian demand $(\hat{x}^1, \hat{z}) \in x(p^1, R, \hat{X}^1)$ lead to utility $V^R(\hat{x}^1, \hat{z}, \hat{X}^1) > v$ and $(p^1, 1)\cdot(\hat{x}^1, \hat{z}) \leq R$. Since $v > \chi$ so that $\hat{z} > 0$, we can reduce expenditure in $\hat{z}$ while, by continuity of V^R, keeping utility above v, contradicting that $(\bar{x}, \bar{z})$ leads to minimal expenditure. Assumption 1 holds.

Second, we show that there exists a linear price for the externality that fixes aggregate demand at the original level $\hat{X}^1$.

To that end, we first show that the Hicksian demand for good 1, denoted $h^1(p^1, v, \hat{X}^1)$, is a continuous function in its first argument.

h^1 is a function. Suppose $(\bar{x}^1, \bar{z}), (\hat{x}^1, \hat{z}) \in h(p^1, v, \hat{X}^1)$ and $\bar{x}^1 \neq \hat{x}^1$. Hence, $p^1\bar{x}^1 + \bar{z} = p^1\hat{x}^1 + \hat{z}$ and $V(\bar{x}^1, \bar{z}, \hat{X}^1), V(\hat{x}^1, \hat{z}, \hat{X}^1) = v$. Since V^R is strictly quasi-concave, allocation $(\tilde{x}^1, \tilde{z}) = \lambda(\bar{x}^1, \bar{z}) + (1 - \lambda)(\hat{x}^1, \hat{z})$, for $\lambda \in (0, 1)$, leads to $V^R(\tilde{x}^1, \tilde{z}, \hat{X}^1) > v$ and $p^1\tilde{x}^1 + \tilde{z} = p^1\bar{x}^1 + \bar{z} = p^1\hat{x}^1 + \hat{z}$. We can reduce expenditure in $\tilde{z}$ while, by continuity of V^R, keeping utility above v, contradicting that $(\bar{x}, \bar{z})$ and $(\hat{x}, \hat{z})$ lead to minimal expenditure.

h^1 is continuous. Assume the opposite: a case in which $h^1(p^1, v, X^1) > h^1(\hat{p}^1, v, X^1) + \varepsilon, \varepsilon > 0$, for all $\hat{p}^1 > p^1$. By continuity of the indifference curve in the plane (x^1, z), there is a bundle $(\bar{x}^1, \bar{z})$ on this indifference curve such that $\bar{x}^1 = h^1(p^1, v, \hat{X}^1) - \varepsilon/2$. There is a supporting price $\bar{p}^1$ for this bundle, and by the law of demand (i.e., $h^1(p^1, v, \hat{X}^1)$ is nonincreasing in p^1), one must have $\bar{p}^1 \in [p^1, \hat{p}^1]$ for all $\hat{p}^1 > p^1$. This implies $\bar{p}^1 = p^1$, which contradicts the fact that h^1 is a function since $\bar{x}^1 \neq h^1(p^1, v, \hat{X}^1)$.

Considering the original allocation with nonlinear pricing, there is a linear price $\hat{t}_i^1$ for every consumer i such that the original bundle $\hat{x}_i$ satisfies:

$$\hat{x}_i = h\left(p^1(\hat{X}^1) + \hat{t}_i^1, \hat{v}(w_i(\hat{X}^1)\hat{e}_i, \hat{X}^1), \hat{X}^1\right).$$

$\left(\text{If the tax function } t^1(x^1, y) \text{ is differentiable, one has } \hat{t}_i^1 = t^1\left(x_i^1, y_i\right) + x_i^1 \frac{\partial}{\partial x^1} t^1\left(x_i^1, y_i\right).\right)$

By the law of demand, for every i, one has

$$h^1\left(p^1(\hat{X}^1) + \sup_j \hat{t}_j^1, \hat{v}(w_i(\hat{X}^1)\hat{e}_i, \hat{X}^1), \hat{X}^1\right) \leq \hat{x}_i^1 \leq h^1\left(p^1(\hat{X}^1) + \inf_j \hat{t}_j^1, \hat{v}(w_i(\hat{X}^1)\hat{e}_i, \hat{X}^1), \hat{X}^1\right). \tag{A1}$$

By continuity, the Hicksian demand varies continuously between the lower and upper bounds for each consumer by changing the price. Consider now two cases. First, assume that the upper bound $h^1\left(p^1(\hat{X}^1) + \inf_j \hat{t}_j^1, \hat{v}(w_i(\hat{X}^1)\hat{e}_i, \hat{X}^1)\right)$ is finite for all consumers. Then we can sum (A1) over the whole population to derive

$$\int h^1\left(p^1(\hat{X}^1) + \sup_j \hat{t}_j^1, \hat{v}(w_i(\hat{X}^1)\hat{e}_i, \hat{X}^1), \hat{X}^1\right) \leq \hat{X}^1 \leq \int h^1\left(p^1(\hat{X}^1) + \inf_j \hat{t}_j^1, \hat{v}(w_i(\hat{X}^1)\hat{e}_i, \hat{X}^1), \hat{X}^1\right).$$

Second, if the upper bound is infinite for some consumers, there is an alternative price $t_{\min}^1 \geq \inf_j \hat{t}_j^1$ so that the aggregate demand for good 1 is infinite for all $t^1 \leq t_{\min}^1$.

In both cases, by continuity of the Hicksian demand, there is a $\bar{t}^1$, either in $\left[\inf_j \hat{t}_j^1, \sup_j \hat{t}_j^1\right]$ or in $\left[t_{\min}^1, \sup_j \hat{t}_j^1\right]$, such that

$$\hat{X}^1 = \int h^1(p^1(\hat{X}^1) + \bar{t}^1, \hat{v}(w_i(\hat{X}^1)\hat{e}_i, \hat{X}^1)).$$

APPENDIX B: NUMERICAL EXAMPLES

TABLE B1 Numerical examples in Section 3.2

	t_1^1	t_2^1	a	b	U_1	U_2	W
Example 1	0.1	0.1	0.5	0.33	0.42	0.51	0.94
Changes	−0.01	+0.004	0	-5×10^{-4}	$+9 \times 10^{-4}$	-9×10^{-4}	$+10^{-5}$
Example 2	0.04	0.04	0.128	0.16	0.40	0.66	1.06
Changes	−0.01	+0.004	0	-2×10^{-4}	$+1.6 \times 10^{-3}$	-10^{-3}	$+6 \times 10^{-4}$
Example 3	0.1	0.1	0.3	0.29	0.43	0.59	1.03
Changes	−0.1	+0.04	0	-4×10^{-3}	+0.01	−0.01	$+2 \times 10^{-3}$

Abbreviations: a, linear income tax rate; b, lump-sum anonymous transfer; t_i^1, government price for good 1 for each consumer i; U_i, utility level of each consumer i; W, utilitarian sum of consumer utilities.