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ABSTRACT

During the past few decades, several significant progresses have been made in exploring complex nonlinear dynamics and vibration suppression of conceptual aeroelastic airfoil models. Additionally, some new challenges have arisen. To the best of the author's knowledge, most studies are concerned with the deterministic case; however, the effects of stochasticity encountered in practical flight environments on the nonlinear dynamical behaviors of the airfoil systems are neglected. Crucially, coupling interaction of the structure nonlinearities and uncertainty fluctuations can lead to some difficulties on the airfoil models, including accurate modeling, response solving, and vibration suppression. At the same time, most of the existing studies depend mainly on a mathematical model established by physical mechanisms. Unfortunately, it is challenging and even impossible to obtain an accurate physical model of the complex wing structure in engineering practice. The emergence of data science and machine learning provides new opportunities for understanding the aeroelastic airfoil systems from the data-driven point of view, such as data-driven modeling, prediction, and control from the recorded data. Nevertheless, relevant data-driven problems of the aeroelastic airfoil systems are not addressed well up to now. This survey contributes to conducting a comprehensive overview of recent developments toward understanding complex dynamical behaviors and vibration suppression, especially for stochastic dynamics, early warning, and data-driven problems, of the conceptual two-dimensional airfoil models with different structural nonlinearities. The results on the airfoil models are summarized and discussed. Besides, several potential development directions that are worth further exploration are also highlighted.

Aeroelastic flutter, as a dynamic instability caused by the fluid–structure interaction of inertial, elastic, and aerodynamic forces, has been a significant and fascinating research topic in the aeroelastic scientific community. However, the presence of both nonlinearities and stochasticities, such as atmospheric turbulence, gust, etc., poses a challenge in discerning the underlying mechanisms, which can lead to more complex dynamical behaviors than the deterministic airfoil systems and even induce the occurrence of extreme events. Such vibrations are extremely dangerous and unexpected for engineering practice and can lead to damage or fatigue of the wing structure and even bring catastrophic consequences to an aircraft. Consequently, it is of great significance, but difficulty, to accurately understand, predict, and suppress the complex dynamical behaviors of airfoil models. In recent years, there have been distinguished developments in machine learning, in particular, deep learning, as its powerful capabilities of modeling and characterization. Data-driven techniques for relevant aeroelastic analysis of airfoil models with a complicated nonlinear structure are becoming increasingly fashionable. In the present paper, we give a state-of-the-art overview on complex dynamics and vibration suppression of conceptual airfoil models, which complements the previous results and promotes the rapid development of related fields.

I. INTRODUCTION

Aeroelasticity is a multi-disciplinary research field studying the mutual interaction of inertia, elastic, and aerodynamic forces,[1–9] which is of great significance in aerospace, in particular, the airfoil flutter. As we all know, there are various nonlinear factors in the complex wing structure, which often lead to more complex dynamical behaviors than the linear case, such as limit cycle oscillation (LCO), bifurcation, and even chaotic motion.[2] These nonlinear factors are generally responsible for two typical bifurcations in terms of the aeroelastic airfoil system, namely, supercritical and subcritical Hopf bifurcations,[3–6] as shown in Fig. 1, where stable and unstable responses are marked by solid and dashed lines, respectively. Specifically, the supercritical Hopf bifurcation means that when the flight speed is less than the critical flutter speed of the airfoil system, the system will converge to a stable equilibrium point, while the system will experience LCO motion when the flight speed exceeds the critical flutter speed of the airfoil system. As the flight speed increases, the LCO amplitude grows gradually, with a decreasing slope as the nonlinearities become more powerful. The subcritical one, on the other hand, signifies that the LCO motion takes place before the critical flutter speed of the airfoil system;[3–6] meanwhile, both stable and unstable LCO motion can be observed. In engineering practice, the subcritical Hopf bifurcation is usually more dangerous and unexpected than the subcritical one because the former one can cause structural damage or fatigue and even compromise the flight safety of an aircraft. Therefore, the subcritical Hopf bifurcation is a detrimental bifurcation, whereas the supercritical one can be regarded as a benign bifurcation. To this end, a comprehensive and in-depth understanding of the nonlinear dynamical behaviors of the aeroelastic airfoil systems is extremely crucial for an efficient and safe design of aircraft wings.

Over the past few decades, with the rapid development of aerospace and nonlinear dynamics, there have been tremendous investigations on nonlinear dynamics and vibration suppression of aeroelastic airfoil models.[2–9] In order to limit the complexity of an aeroelastic airfoil system, a two-dimensional airfoil model has been considered. Bisplinghoff and Ashley[1] pointed out that the dynamic behaviors of a wing in a real flight process can be approximately described by selecting a binary section at 70%–75% of the focal line from the root to the tip of the wing structure. It is important to emphasize that the simplified airfoil model does not completely reflect the real situation, but it is commonly used for the studies of aeroelasticity problems in different areas, such as aircraft wing,[1–9] aeroengine turbine blades,[10] and others. A series of research results on conceptual airfoil models have been reported, including bifurcation and stability analysis,[2–34] flutter control,[35–54] airfoil-based energy harvesting,[55–60] etc. Meanwhile, the latest review articles have been given a comprehensive overview on the recent advances of typical aeroelastic airfoil and panel structures.[6–9] However, these studies mainly focus on the deterministic aeroelastic airfoil models but usually neglect effects of inevitable random disturbances.

In fact, in complex flight environments, aircrafts are inevitably subject to various random fluctuations, such as atmospheric turbulence, gust, and pressure disturbance.[61–70] Stochasticity has several remarkable impacts on the dynamical behaviors of nonlinear systems due to the coupling between random fluctuations and nonlinear factors. In particular, for the nonlinear systems with bistable or multistable characteristics, random fluctuation can cause rich and interesting behaviors, including critical transition,[71–74] stochastic bifurcation,[75,76] stochastic resonance,[77–79] etc. For the deterministic airfoil systems, if random inputs are considered, the nonlinear airfoil systems will exhibit more complex dynamical behaviors and even induce the occurrence of extreme events, which can cause huge deformation or catastrophic damage to the wing structure and seriously affect the reliability and service life of the wing structure. More importantly, the presence of randomness could reduce the critical flutter speed of the aeroelastic airfoil systems, which is highly unwanted in the real world. For this reason, only taking the random fluctuations into account on the aeroelastic airfoil models can much better reflect the actual operating conditions. As a result, it is very crucial to accurately reveal the complex dynamical responses of the aeroelastic airfoil systems with random fluctuations and their occurrence mechanism. With the introduction of the stochastic concept into aeroelasticity by Lin[80] and the rapid development of nonlinear stochastic dynamics, random flutter of the aeroelastic airfoil systems has gradually arisen much attention. In the 1990s, Poirel and Price[61,62] studied the random flutter of a binary wing in turbulent flow with the help of the power spectral density, the probability density function, and the largest Lyapunov exponent. Since then, several researchers have already made great efforts to the nonlinear stochastic dynamics of the conceptual aeroelastic airfoil models.[64–69] Although plenty of investigations have been worked on nonlinear stochastic dynamics of two-dimensional aeroelastic airfoil systems during the past few decades, they are still limited.

We have already witnessed that some research results have been reported on the nonlinear dynamics and vibration control of typical binary wing models, but there are still many challenges that need to be addressed. It should be noted that, on the one hand, most of the existing review articles are mainly limited to the determinist airfoil systems. On the other hand, the simplified typical two-dimensional airfoil model cannot accurately reflect the mechanism of practical dynamical behaviors in some complex situations. At the same time, the limitations of existing model-based methods have motivated many researchers to develop data-driven methodologies for airfoil systems with the increase of available data in recent years. However, the following two problems naturally arise. Is it possible to perform an analysis on the airfoil models by combining the measured data with the models? Could we identify or reconstruct complex nonlinear systems of the airfoil models from the measured data or time series to overcome the limitation of the simplified airfoil models? To answer these questions, data-driven modeling, prediction, and control of complex nonlinear systems have emerged as a powerful and complementary approach to first-principles modeling due to the development of big data, machine learning, and computational hardware.[81–84] Recent years have seen a growing interest in exploring data-driven problems of aeroelastic systems related to data-driven prediction and control, aerodynamic shape optimization, and others.[81] Nevertheless, it is still in the preliminary exploration stage and several problems remain. To complement the previous works, this paper will report recent progress in complex dynamical responses and vibration suppression of conceptual

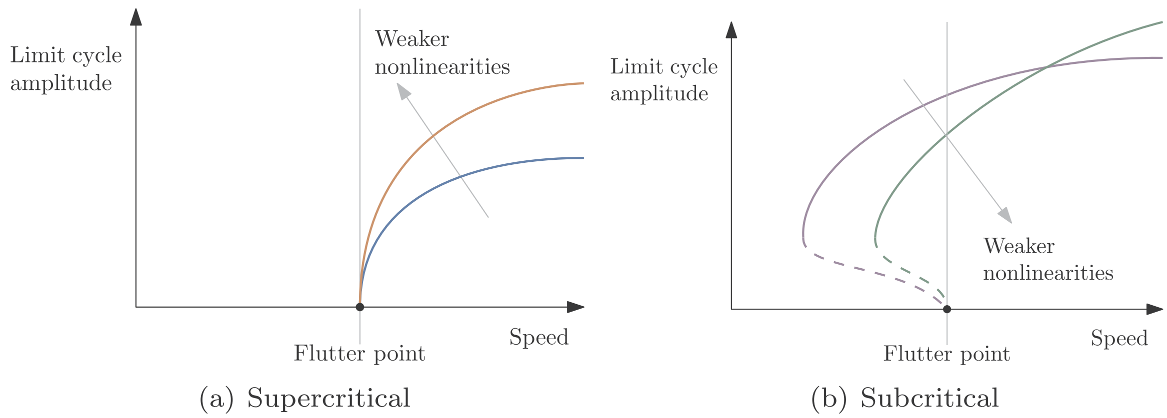

FIG. 1. Schematic of typical bifurcation behaviors in terms of the aeroelastic airfoil system: (a) supercritical Hopf bifurcation and (b) subcritical Hopf bifurcation. Reproduced with permission from Jonsson *et al.*, Prog. Aerosp. Sci. **109**, 100537 (2019). Copyright 2019 Elsevier.

two-dimensional aeroelastic airfoil models, especially in nonlinear stochastic dynamics and data-driven aeroelastic analysis of the airfoil models.

The rest of this paper is organized as follows. We start with giving a general form of the governing equation for the conceptual two-dimensional aeroelastic airfoil models and a brief introduction of typical structural nonlinearity in Sec. II. Then, Secs. III and IV summarize advances in deterministic and stochastic cases of the conceptual two-dimensional airfoil model, respectively. Subsequently, an overview of data-driven studies on the aeroelastic airfoil models is performed in Sec. V. Besides, potential development directions from the past inspiration are presented, and some challenges are also highlighted in Sec. VI. Finally, a summary is presented in Sec. VII. We hope that this review will provide new insights and ideas for further in-depth studies of related problems in the future.

II. GOVERNING EQUATION OF THE AIRFOIL MODEL

The generic governing equations of the conceptual two-dimensional aeroelastic airfoil models with n degrees of freedom can be written in a matrix formulation as follows:

$$\mathbf{M}(\boldsymbol{\theta})\,\ddot{\mathbf{q}} + \mathbf{C}(\boldsymbol{\theta})\,\dot{\mathbf{q}} + \mathbf{K}(\boldsymbol{\theta})\,\mathbf{q} + \mathbf{G}\left(\mathbf{q},\dot{\mathbf{q}};\boldsymbol{\theta}\right) = \mathbf{F}\left(t,\mathbf{q},\dot{\mathbf{q}};\boldsymbol{\theta}\right), \quad (1)$$

in which $\mathbf{q} = \left[q_1, q_2, \ldots, q_n\right]^T$ represents the generalized coordinate vector that depend on the number of structural degrees of freedom of a specific airfoil model. The dots over the variable indicate the derivatives with respective to time t. The matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the generalized mass, structural damping, and structural stiffness of aeroelastic airfoil models, respectively. The vector $\mathbf{G}\left(\mathbf{q},\dot{\mathbf{q}};\boldsymbol{\theta}\right)$ denotes the nonlinear term, and the vector $\mathbf{F}\left(t,\mathbf{q},\dot{\mathbf{q}};\boldsymbol{\theta}\right)$ represents the generalized force collecting all the forces and moments affecting the wing structure. The vector $\boldsymbol{\theta}$ denotes all the parameters in the aeroelastic airfoil system (1), including airstream velocity, material properties, etc. Meanwhile, different types of random fluctuations, including parameter and external load uncertainties, could be included in the airfoil system (1) depending on the considered specific problem. Most of the studies are particularly interested in typical two or three degrees of freedom two-dimensional aeroelastic airfoil models, which are a very popular and important type of models describing the dynamics of the wing structure and will be considered in Secs. III–V. As an example, when considering a typical two-dimensional airfoil section with plunge (h) and pitch (α) degrees of freedom and the effects of structural nonlinearity, the governing equation of the airfoil motion is given by[2,34]

$$\begin{bmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} C_h & 0 \\ 0 & C_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} G_h(h) \\ G_\alpha(\alpha) \end{bmatrix} = \begin{bmatrix} -L_{aero} \\ M_{aero} \end{bmatrix},$$

in which m, S_α, I_α, C_h, and C_α are the airfoil mass, the static moment about the elastic axis, the moment of inertia about the elastic axis, the plunge damping coefficient, and the pitch damping coefficient, respectively. $G_h(h)$ and $G_\alpha(\alpha)$ are the nonlinear plunge and pitch stiffness terms, and L_{aero} and M_{aero} are the aerodynamic lift force and moment acting on the airfoil structure, respectively.

Generally, nonlinear factors that arise from a complex airfoil structure include structural and aerodynamic nonlinearities. Three common types of structural nonlinearities are usually considered in the system (1), namely, cubic,[2] freeplay,[2,7,9] and hysteresis nonlinearities.[2] Specifically, we show the expressions of structural nonlinearity in the pitch degree of freedom $G_\alpha(\alpha)$ as follows.

(1) *Cubic nonlinearity*
The cubic nonlinearity is expressed as[2]

$$G_\alpha(\alpha) = \beta_0 + \beta_1\alpha + \beta_2\alpha^2 + \beta_3\alpha^3,$$

where β_0, β_1, β_2, and β_3 are constants.

(2) *Freeplay nonlinearity*
The freeplay nonlinearity is determined by[2]

$$G_\alpha(\alpha) = \begin{cases} M_0 + \alpha - \alpha_f, & \alpha < \alpha_f, \\ M_0 + M_f\left(\alpha - \alpha_f\right), & \alpha_f \le \alpha \le \alpha_f + \delta, \\ M_0 + \alpha - \alpha_f + \delta\left(M_f - 1\right), & \alpha_f + \delta < \alpha, \end{cases}$$

in which M_0, M_f, α_f, and δ indicate, respectively, the preload magnitude, the stiffness in the freeplay range, the beginning of the freeplay, and the freeplay magnitude.

(3) *Hysteresis nonlinearity*
The hysteresis nonlinearity is given by[2]

$$G_\alpha(\alpha) = \begin{cases} \alpha - \alpha_f + M_0, & \alpha < \alpha_f;\ \dot{\alpha} > 0, \\ \alpha + \alpha_f - M_0, & \alpha > -\alpha_f;\ \dot{\alpha} < 0, \\ M_0, & \alpha_f \le \alpha \le \alpha_f + \delta;\ \dot{\alpha} > 0, \\ -M_0, & -\alpha_f \le \alpha \le -\alpha_f - \delta;\ \dot{\alpha} < 0, \\ \alpha - \alpha_f - \delta + M_0, & \alpha > \alpha_f + \delta;\ \dot{\alpha} > 0, \\ \alpha + \alpha_f + \delta - M_0, & \alpha < -\alpha_f - \delta;\ \dot{\alpha} < 0, \end{cases}$$

in which the meanings of each symbol are the same as in the case of the freeplay nonlinearity.

For the sake of convenience, the governing equation (1) can be rewritten in a state-space form as follows:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), t), \tag{2}$$

in which $\mathbf{x}(t) = \left[\mathbf{q}(t), \dot{\mathbf{q}}(t)\right]^T$ denotes the state vector of the aeroelastic airfoil system (1) and $\mathbf{f}(\mathbf{x}(t), t)$ represents the nonlinear vector fields. In Secs. III–V, the problems of relevant complex dynamics and vibration suppression of conceptual aeroelastic airfoil models will be discussed based on Eqs. (1) and (2). It should be emphasized that some slight differences, such as external or parametric (deterministic or stochastic) excitations, control force terms, and others, could be included into Eqs. (1) and (2).

III. DETERMINISTIC CASE OF THE AIRFOIL MODEL

A. Flutter analysis

Flutter analysis is very necessary to provide a reliable basis for the design of nonlinear aeroelasticity of an aircraft. However, exact analytical solutions in nonlinear aeroelastic problems are only possible for a few special cases. For this consideration, approximated analytical or numerical results are desired for the two-dimensional airfoil system (1).

On the one hand, to understand well the nonlinear behaviors of the aeroelastic airfoil model, several approximated analytical techniques have been developed to theoretical analysis of the airfoil system (1), including the harmonic balance method, the incremental harmonic balance method, multiple scales method, etc.[11–19,34,37] Dai *et al.*[11] presented a time-domain collocation method for studying flutter behaviors of a two-dimensional airfoil model with hardening cubic structural nonlinearity. Wei and Mottershead[12] addressed complex dynamical behaviors of a two degrees of freedom rigid rectangular wing through combining a describing function method with a Sherman–Morrison formula, in which softening cubic stiffness nonlinearity in the pitch degree of freedom was considered. Liu *et al.*[14] investigated LCO and quasiperiodic regimes of a nonlinear aeroelastic airfoil system with an external store via an incremental harmonic balance method. Sanches *et al.*[17] studied aeroelastic tailoring of a typical airfoil section with hardening cubic stiffness nonlinearity, and a multiple scales method was employed to obtain the approximated analytical solutions. Recently, Zheng *et al.*[15] proposed a modified incremental harmonic balance method combined with Tikhonov regularization to get a semi-analytical solution for an airfoil model. Furthermore, Zheng *et al.*[19] proposed a time-domain minimum residual method to predict LCO responses of a two-dimensional airfoil system with non-smooth hysteresis nonlinearity. Compared with the existing semi-analytical methods, e.g., the incremental harmonic balance method, the proposed one can lead to the approximated solutions with a higher accuracy.[19] Additionally, Liu *et al.*[37] studied nonlinear behaviors of a typical two degrees of freedom airfoil model with a fractional viscoelastic property and a harmonic external force. Therefore, to characterize the viscoelasticity of the wing material, the following stress–strain constitutive relationship of a simple fractional Scott-Blair's model was employed,[37]

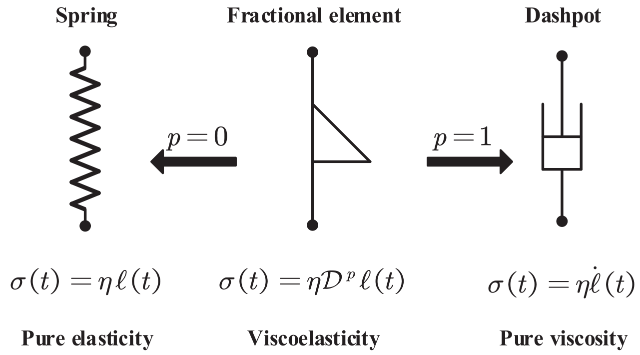

FIG. 2. A fractional Scott-Blair's model. Reproduced with permission from Liu *et al.*, J. Sound Vib. **432**, 50–64 (2018). Copyright 2018 Elsevier.

$$\sigma(t) = \eta\mathcal{D}^p\ell(t), \quad 0 \le p \le 1, \tag{3}$$

in which $\sigma(t)$ and $\ell(t)$ are the stress and strain, respectively. The material-dependent constants η and p represent the viscosity coefficient and the order of a fractional derivative, respectively. The fractional differential operator $\mathcal{D}^p$ in Eq. (3) is considered via the well-known Caputo definition; that is,[37]

$$\mathcal{D}^p f(t) = \frac{1}{\Gamma(1-p)}\int_0^t \frac{f'(s)}{(t-s)^p}\,ds, \quad t > 0,\ 0 < p < 1,$$

in which $f(t)$ is a continuously differentiable function and $\Gamma(t)$ is the Gamma function. As the order p varies from $p = 0$ to $p = 1$, the property changes from pure elasticity to pure viscosity, as shown in Fig. 2. Then, an averaging technique was developed to derive the amplitude–frequency relations of the established viscoelastic airfoil system, and its correctness was verified by numerical simulations. The results indicated that the obtained approximated analytical solutions have a good agreement with the numerical ones. More recently, Martini *et al.*[34] exploited a describing function method to detect subcritical Hopf and fold bifurcations in an aeroelastic airfoil system with pitch and plunge degrees of freedom.

On the other hand, for the case that the analytical solutions are hard to obtain, many researchers have also investigated numerically and experimentally the effects of structural nonlinearities on the two-dimensional airfoil system (1).[5,20–33] As an illustration, He *et al.*[20] proposed an extended Hénon's technique to perform influences of friction and asymmetric freeplay on the nonlinear dynamical behaviors of an aeroelastic airfoil system, in particular, the LCO motion. In practical applications, aeroelastic airfoil systems usually are affected by different types of structural nonlinearities. Effects of combined hardening cubic and freeplay nonlinearities were explored on the nonlinear dynamical responses of a typical aeroelastic section with three degrees of freedom numerically and experimentally.[21] da Silva and Marques[23] studied dynamical responses of a typical airfoil section with a trailing-edge control surface, in which different combinations of concentrated structural nonlinearities were considered. Besides, viscoelastic materials, with viscosity and elasticity, have already been investigated in several vibration-related engineering problems in recent years.[24,26,27,37] For instance, Sales *et al.*[26,27] successfully attached viscoelastic materials into a typical airfoil model with freeplay structural nonlinearity for mitigating undesired aeroelastic responses. Besides, Corrêa and Marques[28] first explored nonlinear dynamics of a typical airfoil section with a bistable spring at plunge degree of freedom. More interestingly, Nitti *et al.*[10] recently investigated the problem of vibration localization in a rotor constituted by multiple coupled airfoils with two degrees of freedom. At the same time, the concept of basin stability introduced by Menck *et al.*,[85] as a global stability concept, was exploited to ascertain the likelihood of the system converging to a certain localized state. The following localization coefficient was defined:[10]

$$L(\hat{\mathbf{z}}) = \left(\frac{\max(\hat{\mathbf{z}})}{\sum_{i=1}^{N} \hat{z}_i} - \frac{1}{N} \right) \frac{N}{N-1}, \quad L \in [0, 1], \tag{4}$$

in which $\hat{\mathbf{z}} = \left[\hat{z}_1, \ldots, \hat{z}_N\right]^T$, N is the number of blades, and $\hat{z}_i$ is the root mean square of a steady-state amplitude of the ith state. Subsequently, the basin stability value $\mathcal{S}_{\mathcal{B}}$ of four different vibration patterns defined according to Eq. (4), including homogeneous vibrations ($L_{0.0-0.05}$), slightly localized vibrations ($L_{0.05-0.15}$), moderately localized vibrations ($L_{0.15-0.45}$), and strongly localized vibrations ($L_{0.45-1.0}$), along the airstream velocity V, were calculated, as displayed in Fig. 3. It can be seen clearly that both the homogeneous and the slightly localized vibrations are more likely to occur in comparison with the strongly localized vibrations.

B. Flutter suppression

Flutter is a typical self-excited vibration, which is unwanted in practice and can lead to a reduction in aircraft performance or even catastrophic failure to the wing structure in the worst case. To this end, several passive or active control strategies have been presented to suppress the flutter instability of the airfoil system (1), including time-delay feedback control, sliding mode control (SMC), nonlinear energy sink (NES), and others.[35–54] Yuan *et al.*[36] employed a radial basis function (RBF) neural network to design an adaptive RBF observer-sliding mode controller for suppressing the catastrophic large-amplitude vibration of a two-dimensional airfoil system with a single trailing-edge control surface. Liu *et al.*[37] carried out an investigation on the vibration suppression of a two degrees of freedom airfoil model with a harmonic excitation and fractional viscoelastic damping terms. A fractional integral sliding surface was employed to design a sliding model controller, and numerical simulations were implemented to illustrate the performance of the proposed control scheme. The obtained results showed that the proposed SMC has good performance in vibration suppression. Moreover, shape memory alloys, as an intelligent material, have been employed as thermomechanical actuators for active vibration control of aeroelastic airfoil systems. de Sousa *et al.*[43–45] provided numerical and experimental investigations on dynamical responses of typical airfoil section with shape memory alloy springs.

Active control methods can achieve effective suppression of the undesired flutter responses for nonlinear aeroelastic airfoil systems. However, the design of an active controller usually requires the attachment of sensors and actuators to the wing structure, which increases the weight of the aircraft. Therefore, passive control is preferred over the active one in the design of practical controllers. In recent years, the NES technique as a passive control strategy has been applied to nonlinear aeroelastic systems to effectively suppress the flutter behavior of the airfoil systems.[38–42,46–49] Bichiou *et al.*[40] studied the effectiveness of the NES in suppression of the LCOs of an aeroelastic airfoil system. Zhang *et al.*[42] numerically explored the flow-induced vibration of a two-dimensional airfoil model coupled with two NESs located at the leading and trailing edges. Pidaparthi and Missoum[41] investigated flutter mitigation of a typical two degrees of freedom airfoil model with nonlinear stiffness in pitch and heave employing an optimally designed NES based on a stochastic optimization approach. Kassem *et al.*[38,39] proposed a novel technique for flutter suppression of a conceptual two-dimensional airfoil system using an active dynamic vibration absorber. Theoretical and experimental results showed that the proposed technique is very effective and feasible for vibration suppression of the airfoil model. Recently, to improve the performance of the flutter suppression, multiple NESs were considered in aeroelastic airfoil systems. For example, Bergeot and Bellizzi[47] studied flutter mitigation in a two degrees of freedom airfoil system coupled to a set of NESs. Basta *et al.*[49] examined flutter suppression of a wing model via distributed vibration absorbers. They found that a proper selection for the vibration absorber can lead to a 23.4% increase in the flutter speed when one single absorber is attached. However, an 84% increase in the flutter speed can be achieved when employing an array of distributed vibration absorbers.[49]

IV. STOCHASTIC CASE OF THE AIRFOIL MODEL

Several random fluctuations, such as atmospheric turbulence, gusts, etc., always exist in the practical flight environment, which will affect the performance of an aircraft.[61–70] As a consequence, it is of extreme importance to consider stochasticity in the design process of an aircraft. The development of random dynamic theory provides us an opportunity to understand the complex dynamic behaviors of the conceptual airfoil system (1) with different random fluctuations. In general, for a nonlinear dynamics problem, three aspects are usually of interest, i.e., input–system–output. When we consider the effects of random fluctuations on the aeroelastic airfoil system (1), the following three problems arise. The first one is how to describe complex stochastic excitations in the aeroelastic airfoil models? The second one is how to obtain the solution of the established stochastic airfoil systems? The third one is how to suppress the undesired random vibrations in the aeroelastic airfoil

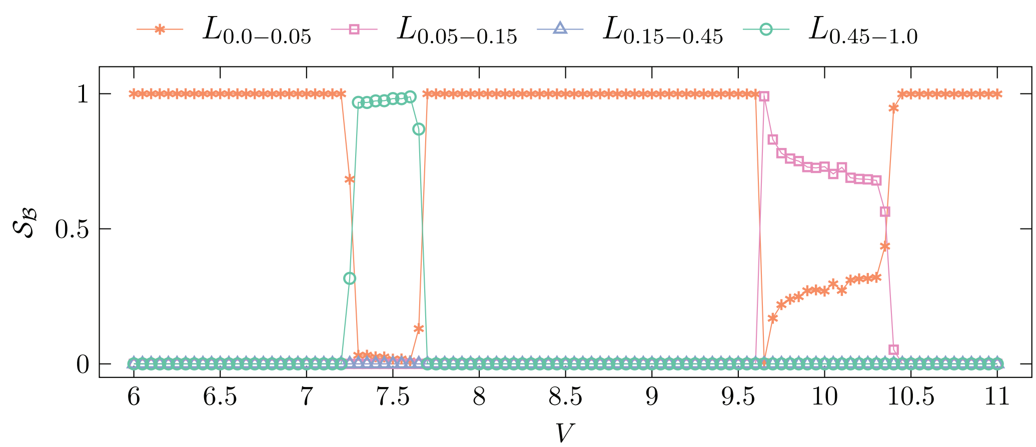

FIG. 3. Basin stability value for four different vibration patterns of the first blade against the airstream velocity. Reproduced from Nitti *et al.*, Nonlinear Dyn. **103**, 309–325 (2021). Copyright 2021 Author(s), licensed under a Creative Commons Attribution (CC BY) License.

systems? In recent years, the development of stochastic dynamics exhibits the following characteristics. The excitation models develop from simple Gaussian, white, and stationary noises to non-Gaussian, non-white, and non-stationary noises and other complex situations. The systems expand from simple low-dimensional, smooth nonlinear systems to high-dimensional, non-smooth, and other complex cases. These facts promote the development of the stochastic flutter of the conceptual airfoil models.

A. Stochasticity quantification

Stochasticity quantification is of extreme significance and the key step to subsequent research studies and has attracted much attention from scholars in the aeroelastic fields.[66,86–91] It should be emphasized that appropriate models to quantify the uncertainties are of considerable importance to flutter analysis of the aeroelastic systems. In general, probabilistic, non-probabilistic, and fuzzy methods are three main approaches employed for modeling different uncertainties.[92] If enough data are available to determine the statistical distribution of uncertainty parameters, probabilistic methods are preferred. For example, Xu *et al.*[93] and Hu *et al.*[94] employed Gaussian white noise to model the external random fluctuation acting on a typical two degrees of freedom airfoil model. The idealized Gaussian white noise usually cannot well describe the complex flight environment. On the contrary, the noises with non-zero correlation time are more desired in practice. In the past few decades, Poirel and Price[61,62] exploited colored noise models to characterize stochasticity of a two-dimensional airfoil model in turbulence flow. They considered the Dryden turbulence model and modeled the vertical and longitudinal velocity components as[61,62]

$$du_T + \frac{V_m}{\overline{L}} u_T\, dt = \sigma_T \left(\frac{2V_m}{\pi \overline{L}}\right)^{\frac{1}{2}} dW_1, \tag{5a}$$

$$dw_T + \frac{2V_m}{\overline{L}} w_T\, dt + \frac{V_m^2}{\overline{L}^2}\left(\int_0^t w_T\, ds\right) dt = \sigma_T \left(\frac{V_m^3}{\pi \overline{L}^3}\right)^{\frac{1}{2}} \left(\int_0^t dW_2\right) dt + \sigma_T \left(\frac{3V_m}{\pi \overline{L}}\right)^{\frac{1}{2}} dW_2, \tag{5b}$$

in which u_T and w_T are the longitudinal and vertical components, respectively, V_m is the mean freestream velocity, $\overline{L}$ is the total lift force, σ_T is the standard deviation, and W_1 and W_2 are two independent Wiener processes. In addition, Liu *et al.*[95] employed an exponentially correlated, colored Gaussian noise to establish a new stochastic airfoil model with harmonic excitation. Specifically, the considered colored noise satisfies[95]

$$\mathbb{E}\left[\xi\,(t)\right] = 0, \quad \mathbb{E}\left[\xi\,(t)\,\xi\,(s)\right] = \frac{\nu}{\tau}\exp\left[-\frac{|t-s|}{\tau}\right], \tag{6}$$

in which $\mathbb{E}[\cdot]$ denotes the mathematical expectation and ν and τ are, respectively, the noise intensity and the correlation time. Furthermore, they also used a narrow-band process with the following expression to characterize random disturbances of the external flight environment:[96]

$$\xi\,(t) = A\cos\left[\Omega t + \nu W\,(t)\right], \tag{7}$$

in which A, Ω, $W(t)$, and ν are, respectively, the amplitude, the central frequency, the standard Wiener process, and the noise intensity. Subsequently, Chassaing *et al.*[66] employed an adaptive stochastic spectral projection method to quantify stochasticity on a two-dimensional elastically mounted lifting surface in supersonic flow. They described the stochasticities in structural damping of both plunge and pitch degrees of freedom as uniform random variables. Besides, other important excitation models with different statistical properties were also presented to quantify the stochasticity on the two-dimensional aeroelastic airfoil models, such as non-Gaussian colored noise,[97,98] randomly fluctuation flow,[99,100] etc.[101–103]

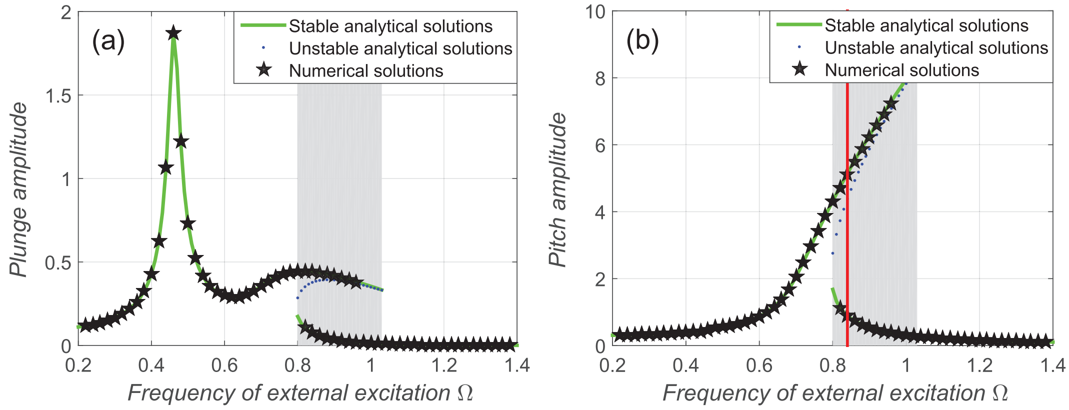

FIG. 4. Steady-state amplitude–frequency responses of the airfoil system (1) with the narrow-band random fluctuation (7) when the noise intensity $\nu = 0$ and the flow speed less than the critical flutter speed: (a) plunge motion and (b) pitch motion. "–," stable analytical solutions; "·," unstable analytical solutions; and "★," numerical solutions. Reproduced with permission from Liu *et al.*, Commun. Nonlinear Sci. Numer. Simul. **84**, 105184 (2020). Copyright 2020 Elsevier.

However, in practice, available information on uncertainty is usually limited and insufficient, which leads to difficulties in determining their statistical characteristics.[92,104] For this case, interval analysis, as a non-probabilistic method, has been preferred and performed in the aeroelastic airfoil models with non-probabilistic interval uncertainties.[92,104–113] For instance, Zheng and Qiu[105] employed an interval uncertain model to quantify the stochasticity of the airfoil system's parameters. Both aerodynamic and structural parameters were considered to be dependent on an n-dimensional interval uncertain parameter vector $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_n]^T$ that satisfies[105]

$$\boldsymbol{\gamma} \in \boldsymbol{\gamma}^{\mathrm{I}} = \left[\underline{\boldsymbol{\gamma}}, \overline{\boldsymbol{\gamma}}\right] = [\boldsymbol{\gamma}^{\mathrm{c}} - \Delta\boldsymbol{\gamma}, \boldsymbol{\gamma}^{\mathrm{c}} + \Delta\boldsymbol{\gamma}]\,. \tag{8}$$

It follows that

$$\gamma_j \in \gamma_j^{\mathrm{I}} = \left[\underline{\gamma_j}, \overline{\gamma_j}\right] = \left[\gamma_j^{\mathrm{c}} - \Delta\gamma_j, \gamma_j^{\mathrm{c}} + \Delta\gamma_j\right], \quad j = 1, 2, \ldots, n,$$

in which $\underline{\boldsymbol{\gamma}} = (\underline{\gamma_j})$ and $\overline{\boldsymbol{\gamma}} = (\overline{\gamma_j})$ denote, respectively, the lower and upper bounds of the vector $\boldsymbol{\gamma}$, and $\boldsymbol{\gamma}^{\mathrm{c}} = (\gamma_j^{\mathrm{c}})$ and $\Delta\boldsymbol{\gamma} = (\Delta\gamma_j)$ represent the nominal value and the interval radius of the vector $\boldsymbol{\gamma}^{\mathrm{I}}$, respectively.[105]

On the other hand, for many practical engineering problems, knowledge is usually incomplete or unavailable. Hence, the fuzzy method is more suitable to describe the uncertain behavior.[92,114–116] In this work,[92] the fuzzy uncertainty and the reliability analysis of an airfoil system were investigated utilizing a fuzzy interval approach. Therefore, the uncertainties were modeled as fuzzy triangular membership functions.[92]

B. Noise-induced complex dynamics

1. Stochastic bifurcation

The coupling interaction between the nonlinearity and the stochasticity can lead to complex vibration behaviors in comparison with a deterministic model.[64–66,96,99,117–127] Thus, it is important to understand the resulting dynamics of the aeroelastic airfoil system (1) with uncertain disturbances via analytical, numerical, or experimental methods. Several effective techniques, such as the center manifold reduction method,[64] the incremental harmonic balance method,[65] and the adaptive spectral method,[66] have been developed to analyze the nonlinear behaviors of the two-dimensional airfoil systems with gust-load or parameter uncertainty. Besides, Liu *et al.*[93,95,96] revealed analytically the influence mechanisms of different types of random fluctuations, including Gaussian white noise,[93] Gaussian colored noise,[95] and narrow-band noise,[96] on a two degrees of freedom aeroelastic airfoil model. They developed some approximated analytical techniques to achieve theoretical analysis for the established airfoil systems, including a stochastic averaging technique[93,95] and a method of multiple scales.[96] In Refs. 93 and 95, a perturbation technique, together with the stochastic averaging method, was examined to achieve the reducing and decoupling of a coupled two degrees of freedom typical airfoil system. Then, the amplitude–frequency relation and the time-averaged mean square response of the amplitude for the stochastic airfoil models were deduced and verified. What's more, in Ref. 96, the effects of both viscoelasticity and narrow-band random fluctuations defined by Eq. (7) on the airfoil model were further studied in detail. Therefore, the method of multiple scales was employed to derive approximated analytical solutions of the proposed airfoil model and the correctness of which was verified through numerical simulations. They found that the narrow-band random fluctuation has some remarkable influences on the nonlinear dynamics of the airfoil model. For the case of the noise intensity $\nu = 0$, the airfoil systems show different dynamic behaviors when the flow speed is less than and beyond the critical flutter speed. Single-periodic responses are only in certain ranges of the frequency Ω or amplitude A when the flow speed is beyond the critical flutter speed. Such behaviors are different from the case of the flow speed less than the critical flutter speed, where the airfoil system always shows periodic motions for a larger periodic excitation.[96] Figure 4 demonstrates the steady-state amplitude–frequency responses of the airfoil model when the noise

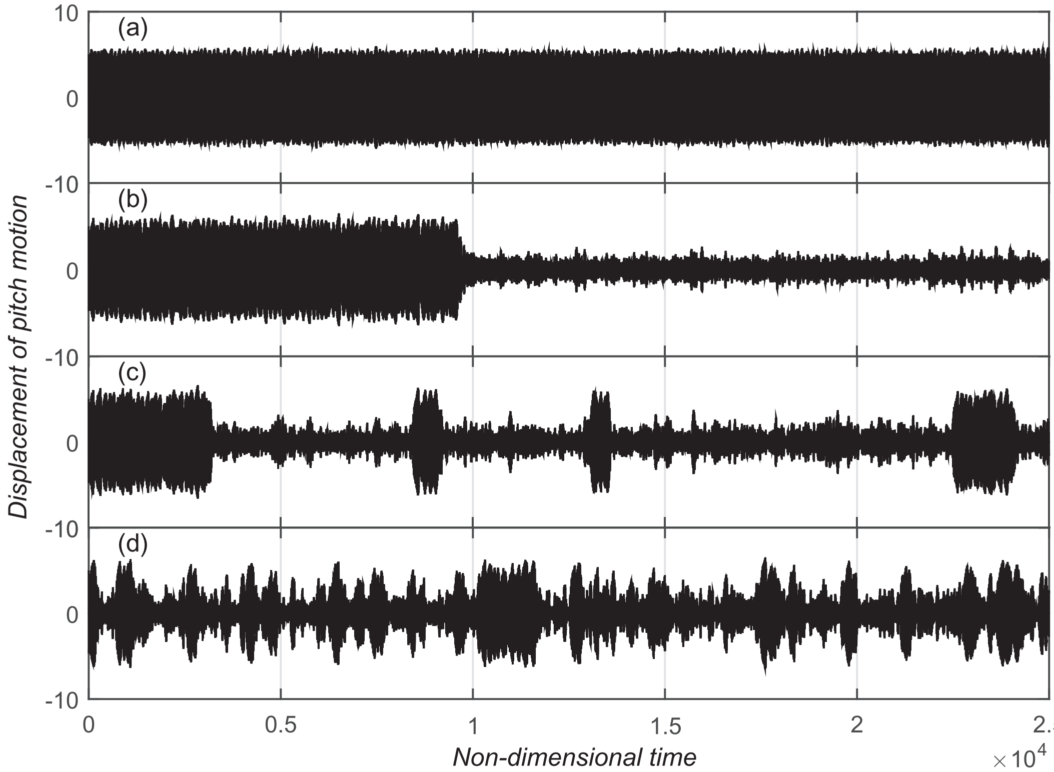

FIG. 5. Time history of the pitch motion of the airfoil system (1) with the narrow-band random fluctuation (7) under different noise intensities when the frequency $\Omega = 0.84$ [marked by the red solid-line in Fig. 4(b)]: (a) $\nu = 0.05$, (b) $\nu = 0.10$, (c) $\nu = 0.15$, and (d) $\nu = 0.25$. Reproduced with permission from Liu *et al.*, Commun. Nonlinear Sci. Numer. Simul. **84**, 105184 (2020). Copyright 2020 Elsevier.

intensity $\nu = 0$ and the flow speed is less than the critical flutter speed. We can observe a good consistency between the approximated analytical solutions and the numerical ones and a bistable behavior as marked by the gray regions in Fig. 4. If the frequency Ω is beyond a certain critical value, a saddle-node bifurcation will occur where jump phenomena appear in the amplitude–frequency response curve. Within the bistable region, two probable vibration patterns, called low-amplitude and high-amplitude oscillation states, coexist. When the noise intensity $\nu = 0$, the steady-state responses of the airfoil model are strongly determined by the initial conditions of the system. However, an interesting phenomenon, i.e., a stochastic jump phenomenon between high-amplitude and low-amplitude oscillations, can be induced with the increase of the noise intensity ν gradually, as plotted in Fig. 5.

Recently, Raaj *et al.*[99,118] explored the synchronization phenomenon in a two degrees of freedom aeroelastic airfoil system during an intermittency route to flutter. Furthermore, coupling nonlinear dynamical systems are also studied, which can lead to some complex behaviors.[120,121,128] Amplitude death, as one of the most interesting phenomena, has been explored in the airfoil systems.[119–121] Raaj *et al.*[120,121] further considered the nonlinear dynamics of two coupled nonlinear aeroelastic systems with identical structures subjected to axial flows and uncovered an interesting phenomenon, i.e., amplitude death. Riley *et al.*[88] proposed a methodology for uncertainty quantification in an aeroelastic airfoil model, and the modeling uncertainty was used to drive the necessity of further experimental data points. More recently, Iannelli *et al.*[91] explored the influences of model uncertainty on the nonlinear dynamics of an aeroelastic system. Besides, Zheng and Qiu[105] proposed a novel numerical method to study the flutter stability of airfoil systems with interval uncertain parameters. They considered interval uncertain parameter vector $\boldsymbol{\gamma}$ described by Eq. (8) into both aerodynamic and structural parameters of an aeroelastic airfoil system (1) and then expressed the governing equations with interval uncertainty as follows:[105]

$$\mathbf{M}(\boldsymbol{\gamma})\ddot{\mathbf{q}} + \left(\mathbf{C}(\boldsymbol{\gamma}) + \mathbf{C}_{\Delta p}(\boldsymbol{\gamma})\right)\dot{\mathbf{q}} + \left(\mathbf{K}(\boldsymbol{\gamma}) + \mathbf{K}_{\Delta p}(\boldsymbol{\gamma})\right)\mathbf{q} = \mathbf{0}, \quad (9)$$

in which $\mathbf{C}_{\Delta p}$ and $\mathbf{K}_{\Delta p}$ represent the aerodynamic damping and stiffness matrices, respectively. Subsequently, two effective approaches, including the sensitivity-analysis-based vertex method and the Bernstein polynomial method, are implemented to solve a generalized eigenvalue problem associated with Eq. (9) to obtain interval bounds of the uncertain parameters. At the same time, some comparisons were performed among the proposed two approaches, the interval perturbation method, and the Monte Carlo simulation method, in terms of a two degrees of freedom aeroelastic airfoil model.[105]

2. Stochastic stability

The stability of aeroelastic systems with random fluctuations is very crucial to engineering applications. Singh *et al.*[129] explored stochastic stability of a typical two-dimensional airfoil aeroelastic model in turbulence flow. The Dryden turbulence models (5a)–(5b) were exploited to describe the vertical and longitudinal velocity components, and the Hopf bifurcation phenomenon was analyzed via a stochastic dimensional reduction method. Liu *et al.*[94,97,101,130] studied stochastic stability of a binary airfoil model under different random fluctuations, including colored, narrow-band, and other noises, through calculating the moment Lyapunov exponent, respectively. In general, the pth moment Lyapunov exponent is defined as[94,97,101,130]

$$\Lambda\left(p, \mathbf{x}_0\right) = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}\left[\|\mathbf{x}\left(t, \mathbf{x}_0\right)\|\right]^p, \quad p > 0, \tag{10}$$

in which $\mathbf{x}\left(t, \mathbf{x}_0\right)$ denotes the solution process. If $\Lambda\left(p, \mathbf{x}_0\right) < 0$, the pth moment is stable; otherwise, it is unstable almost surely. Under some specified conditions, the limit of Eq. (10) exists and is independent of the initial value $\mathbf{x}_0$. Consequently, $\Lambda\left(p, \mathbf{x}_0\right)$ can be expressed as $\Lambda\left(p\right)$, and then the largest Lyapunov exponent is given by

$$\lambda = \left.\frac{d\Lambda\left(p\right)}{dp}\right|_{p=0} = \lim_{t\to+\infty} \frac{1}{t} \log\|\mathbf{x}\left(t, \mathbf{x}_0\right)\|.$$

It should be noted that it is of great difficulty to determine the moment Lyapunov exponent in general. As a result, approximated analytical or numerical solutions are expected. In these works of literature,[94,97,101,130] utilizing the singular perturbation theory, approximated analytical solutions of the moment Lyapunov exponent were obtained, which agree well with the Monte Carlo simulation results.

Besides, for some real-world problems, the desired system state may be mathematically unstable in the sense of Lyapunov stability but oscillate sufficiently near the desired state. For this moment, its performance is still acceptable because such a system state is not able to damage the structure of an aircraft.[131] Thus, the concept of practical stability was proposed by LaSalle and Lefschetz[132] and then developed by Laksmikantham *et al.*[131,133] to study the stability of nonlinear systems. Generally speaking, the practical stability is more appropriate to some practical applications than the Lyapunov one because it can characterize both qualitative and quantitative behaviors of the systems.[132] Under the above consideration, Liu *et al.*[95] exploited the concept of practical stability to analyze the stochastic stability of a two degrees of freedom aeroelastic airfoil system with a combined excitation of harmonic external force and colored Gaussian noise.

3. Early warning of aeroelastic flutter

It should be emphasized that the presence of both nonlinearity and stochasticity can induce some complex dynamical behaviors, including intermittency[134–139] and undesired critical transition behavior from the low-amplitude oscillation state to the high-amplitude one[93,95,96] induced by random fluctuations in aeroelastic airfoil systems. These unwanted behaviors will seriously threaten the flight safety of an aircraft and usually lead to structural damage and even catastrophic failure of an aircraft due to accumulation of fatigue. As a consequence, identifying the onset and predicting the occurrence of undesired and dangerous flutter behaviors and achieving their early warning before flutter occurs is thus extremely crucial and necessary in the design and health monitoring of the wing structure. In the past few years, the problem of early warning has been a particularly interesting topic[140–142] and has been considered in the aeroelastic fields.[134–139]

Usually, complex dynamical behaviors appear in the unstable region of the subcritical Hopf bifurcation, as indicated in Fig. 1(b), in particular, the interesting intermittency behavior, which is dangerous and can destroy the wing structure. Hence, the unstable region is also called a high-risk region to the airfoil model and must be precursed before it occurs. In recent years, Venkatramani *et al.*[134] explored the dynamical responses of an NACA 0012 airfoil through a wind tunnel experiment. They observed intermittent bursts of periodic oscillations in the pitch and plunge degrees of freedom from the experimental data. At the same time, a powerful tool called a recurrence plot was exploited to characterize the dynamical behaviors of an airfoil system in the phase space, which is obtained by[134,135,143]

$$\mathbf{R}_{ij} = \Theta\left(\bar{\varepsilon} - \|\mathbf{x}_i - \mathbf{x}_j\|\right), \quad i, j = 1, 2, \ldots, N, \tag{11}$$

where N is the number of measured points $\mathbf{x}_i$, Θ is the Heaviside function, $\bar{\varepsilon}$ is a predefined threshold, and $\|\mathbf{x}_i - \mathbf{x}_j\|$ is the Euclidean distance between the state points $\mathbf{x}_i$ and $\mathbf{x}_j$. The recurrence matrix constructed by Eq. (11) is a symmetric matrix comprising the elements of zeros and ones. They found that statistics from recurrence plots can be employed to develop model-free precursors for achieving early warning of the unwanted aeroelastic flutter behaviors.[134] Moreover, they attempted to understand the physical mechanisms that cause the intermittency bebavior in the airfoil system with a random fluctuation flow[135] and found that the time scales of the input flow fluctuations can cause two different types of intermittency, i.e., "on–off" and "burst" type intermittency. Furthermore, they developed a new indicator, i.e., the Hurst exponent, and explored its potential to warn against the impending flutter behavior.[136] At the same time, Venkatramani *et al.*[137] further presented a set of model-independent indicators that are obtained directly from the time history, including entropy measures, the Lempel–Ziv complexity, Hurst exponents, and measures from recurrence plots to achieve early warning of the impending aeroelastic flutter.[137] Recently, effects of random fluctuation flow on the stability of a two degrees of freedom airfoil model with cubic structural nonlinearity were examined.[138] A Shannon entropy measure was presented to quantify the stochastic P-bifurcation regime, which can also be regarded as a precursor to warn the dangerous flutter behavior.

More recently, Ma *et al.*[144] explored the prediction of noise-induced tipping from low-amplitude to high-amplitude oscillations on a two degrees of freedom airfoil system with the narrow-band random fluctuation (7). More importantly, a new concept, i.e., the high-risk region defined as the escape probability $P_E \geq 1/2$,

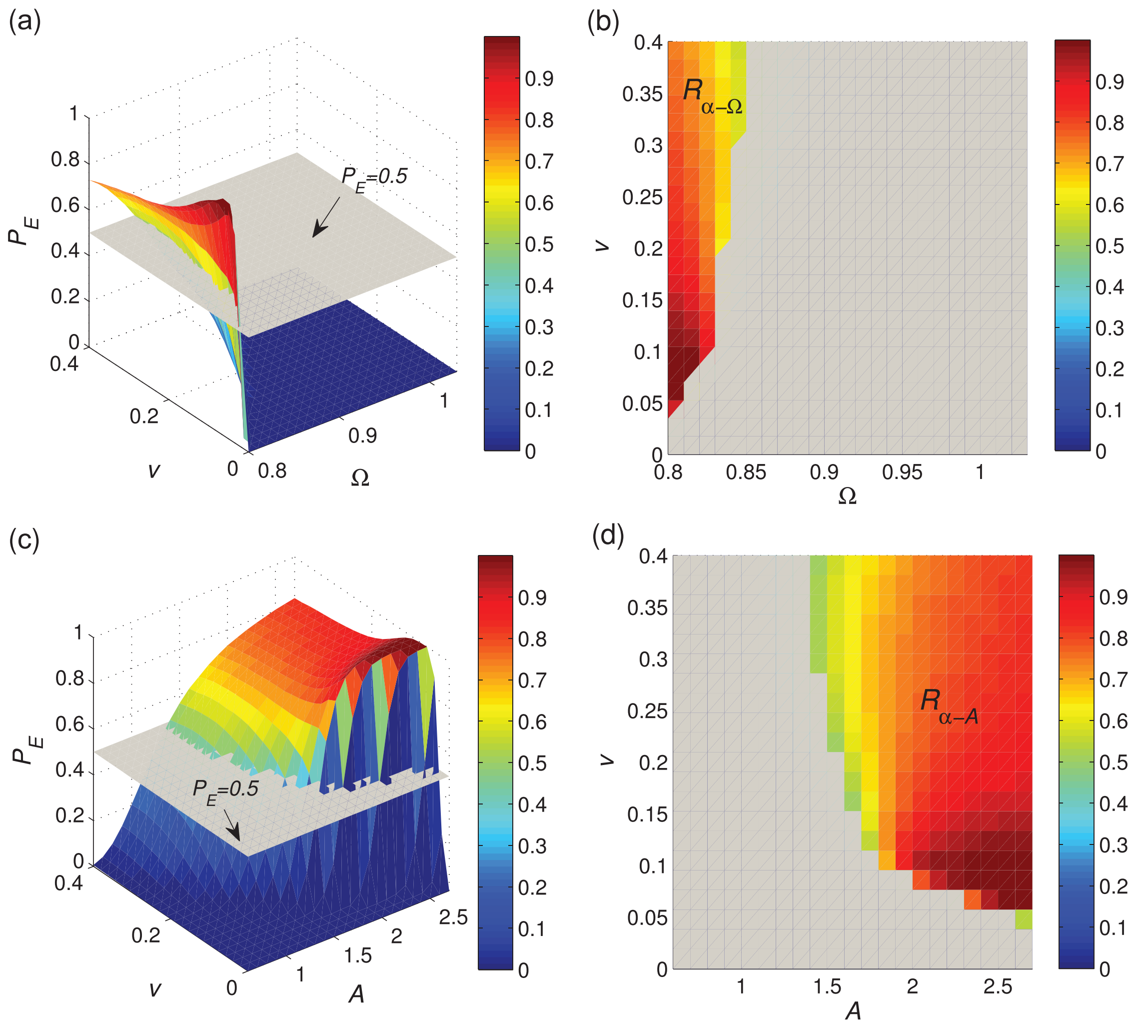

FIG. 6. High-risk regions of the pitch degree of freedom of the airfoil system (1) with the narrow-band random fluctuation (7): (a) and (b) $R_{\alpha-\Omega}$ with respect to the noise intensity ν and the frequency Ω. (c) and (d) $R_{\alpha-A}$ with respect to the noise intensity ν and the amplitude *A*. Reproduced with permission from Ma *et al.*, Chaos **32**, 033119 (2022). Copyright 2022 AIP Publishing LLC.

as an efficient early warning indicator, was proposed to approximately quantify the high-risk regions where the noise-induced catastrophic high-amplitude oscillations may occur in advance.[144] Figure 6 demonstrated the high-risk regions $R_{\alpha-\Omega}$ and $R_{\alpha-A}$ of the pitch motion with respect to the noise intensity ν, frequency Ω, or amplitude A of the random excitation, respectively. It can be seen that both the high-risk regions $R_{\alpha-\Omega}$ and $R_{\alpha-A}$ increase with the increase of the noise intensity ν. The quantified high-risk regions are the important geometric structures that can be exploited to predict in advance and early warn the noise-induced catastrophic high-amplitude oscillations in the aeroelastic airfoil systems before such unwanted behaviors occur.[144]

We have found that over the past few decades, there have been several studies potentially providing a series of early warning approaches for impending aeroelastic flutter or catastrophic tipping behaviors in conceptual aeroelastic airfoil systems. These findings suggest that critical behavior in airfoil systems could have promising applications even for real-time structural health monitoring and prediction.

C. Suppression of random vibration

Sustained random vibration is particularly dangerous and unexpected to the engineering design, which generally leads to damage or structural fatigue to the aircraft wing. During the past decade, several control approaches, such as optimal control, feedback control, and others, have been developed to suppress the undesired random vibration of the aeroelastic airfoil system (1) with different random fluctuations.[95,98,123,145–153] For instance, Liu *et al.*[95] exploited an SMC technique to address vibration suppression of a two degrees

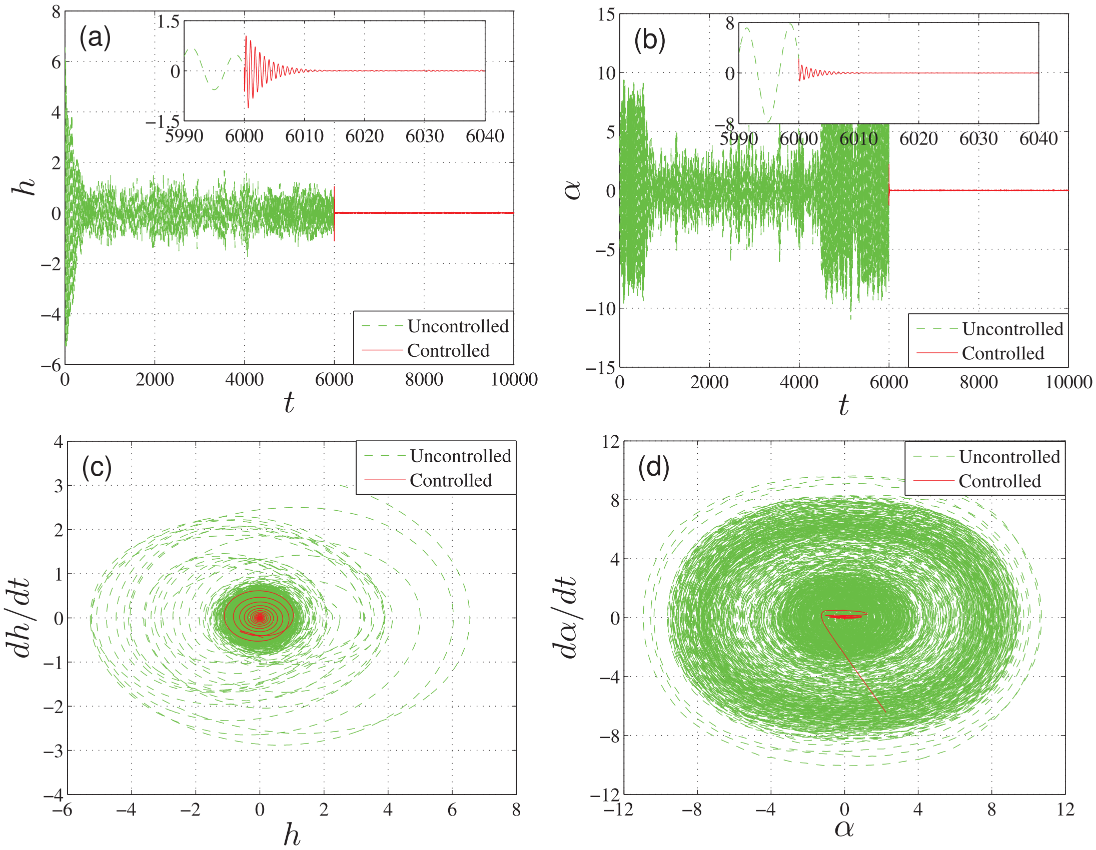

FIG. 7. Time history and phase diagrams of the airfoil system (1) with the colored Gaussian noise (6) and the SMC law: (a) and (c) plunge motion. (b) and (d) Pitch motion. Reproduced with permission from Liu *et al.*, Appl. Math. Modell. **64**, 249–264 (2018). Copyright 2018 Elsevier.

of freedom airfoil model excited by a combination of harmonic force and colored Gaussian noise. Therefore, an integral sliding mode vector function was selected to construct the SMC law.[95] More importantly, the concepts of ultimate reachability with an arbitrary small bound and practical stability were introduced to analyze the reachability and stability of the controlled airfoil system.[95] Figure 7 shows the time history and the phase diagrams of the stochastic airfoil system with the designed SMC control, in which the insets in Figs. 7(a) and 7(b) display the corresponding partially enlarged view. A stochastic jump can be observed when the SMC controller is not activated in the airfoil system, whereas the system states will converge to zero under the SMC controller. Numerical simulation results indicated that the presented control scheme is very effective and feasible to suppress such a stochastic jump due to the effect of random fluctuations on the aeroelastic airfoil system. Moreover, Huang and Tao[98] proposed a feedback control strategy to examine vibration control of a two degrees of freedom binary airfoil model subject to non-Gaussian colored noise. At the same time, the stability of the controlled airfoil system was also analyzed via the largest Lyapunov exponent. Zhang *et al.*[123] proposed an adaptive controller with an estimation update law to stabilize the unexpected vibration of a three degrees of freedom airfoil model with stiffness and damping uncertainties. Besides, the excitation of atmospheric gust was also considered to model the effects of the external flight environment on the wing structure.

However, most of them may not work effectively if actuator faults include in the systems.[154–157] The existing works rarely consider the effect of actuator faults in the airfoil models, but it is more practical and significant to take the fault-tolerant control problem. In recent years, Gao *et al.*[154–156] addressed flutter suppression of two-dimensional airfoil models by means of the RBF neural network and the fault-tolerant control technique. Therefore, both actuator faults and uncertain disturbances were considered in the airfoil system, and a RBF neural network was utilized to construct the control law.[154–156] Simulation results demonstrated that the proposed fault-tolerant control strategy is reliable and robust to the aeroelastic airfoil system with actuator faults.

V. DATA-DRIVEN PROBLEMS OF THE AIRFOIL MODEL

There are still several problems that need to be solved in the existing research studies on the dynamics and vibration suppression of the conceptual aeroelastic airfoil models. In terms of modeling, most of the current studies are mainly based on accurate mathematical models, but it is difficult and is challenging to obtain an accurate mathematical representation of the governing equation (1) for complex wing structures. In terms of methodology, traditional theories and methods rely on the mathematical model (1) with all parameters known and show low computational efficiency in dealing with complex situations, especially for high-dimensional or stochastic cases. Although numerical simulations can provide insights into dynamical responses of the aeroelastic airfoil systems, it is impractical because the high computational expense is inevitable in practice. On the other hand, with the development of sensor technology, people can collect a large amount of structural response monitoring data by placing a large number of sensor devices on the wings of an aircraft to monitor the vibration state of the airfoil structure from a wind tunnel experiment or a flight test. An important emerging problem is to leverage data to improve modeling and prediction of complex nonlinear dynamical systems. Consequently, developing fast and efficient methods for processing these available data are highly desirable. Considering the increasing complexity of real wing structures and the increasing amount of available data, data-driven problems of the aeroelastic airfoil models have been an increasingly popular research topic. At the same time, machine learning has recently received considerable attention for its powerful modeling and characterization capabilities, which offers the possibility of solving practical nonlinear problems from a data-driven perspective.[81–84,158–161] As an illustration, Xu *et al.* have leveraged the advantages of deep learning for high accuracy solving of integer- and fractional-order Fokker–Planck–Kolmogorov equations,[158–160] meanwhile explored global dynamical behaviors and response forecasting of a nonlinear dynamical system by combining the generalized cell mapping method and deep learning.[161] Thus, inspired by the development of data science and machine learning, new theories and methods for the data-driven studies of complex dynamics and vibration control of aeroelastic airfoil models should be presented. In the past decade, data-driven techniques, including statistical and machine learning, have become a critical complement to the nonlinear aeroelastic systems.

A. Data-driven identification and reconstruction

As we all know, accurate mathematical models allow for robust aeroelastic flutter analysis, response prediction, and control design, which are very critical for aircraft safety, gust-load alleviation, and others. However, the governing equation (1) of the conceptual aeroelastic airfoil models is often unknown, but only limited measured data or time series are available. It is imperative to identify or reconstruct the complex airfoil system from the available but limited data.[162–192] Usually, the nonlinear aeroelastic behaviors of the typical nonlinear airfoil models are strongly determined by the system parameters, and therefore, it is of great importance and necessary to accurately identify the structural parameters of the aeroelastic airfoil systems from the measurement response data before carrying out relevant nonlinear analysis. Liu *et al.*[180,181] employed an enhanced response sensitivity approach to address parameter identification of two-dimensional aeroelastic airfoil systems with cubic structural nonlinearity and without or with time-delay consideration cases, respectively. They transferred the parameter identification into a nonlinear least-squares optimization problem. Furthermore, Ding *et al.*[182] proposed an enhanced Jaya algorithm to achieve parameter identification for a two-dimensional aeroelastic airfoil model with an external store. They constructed the following objective function under the discrepancy between the measured and calculated responses:[182]

$$\mathbf{P}^* = \arg\min_{\mathbf{P}} \left[\sum_{ss=1}^{N_{\text{point}}} \sum_{tt=1}^{T_{\text{time}}} \left(q_{ss,tt}^{\text{measured}} - q_{ss,tt}^{\text{calculated}} \right)^2 \right], \tag{12}$$

in which the model vector $\mathbf{P} = \left[p_1, p_2, \ldots, p_n\right]^T$ consists of the identified systemic parameters, N_{point} is the total number of measurement points used for the purpose of identification, T_{time} represents the total response time of each measurement point, $q_{ss,tt}^{\text{measured}}$ is the measured response at time tt of the ss-th point, and $q_{ss,tt}^{\text{calculated}}$ means the corresponding calculated response. Then, an enhanced Jaya algorithm was employed to optimize this objective function (12). The results showed that the enhanced Jaya algorithm can acquire more accurate results than other algorithms. In recent years, Balatti *et al.*[183] performed an investigation on gust-load identification from the measured flight data. Three approaches, including Tikhonov regularization (TIKH), truncated singular value decomposition (TSVD), and damped singular value decomposition (DSVD), were employed to deduce the identified gust load. All the obtained numerical results suggested that the TIKH and TSVD regularization can achieve a more significant reduction than the DSVD one on the reconstruction error.[183]

Recently, Liu *et al.*[184] explored fixed-interval smoothing of a conceptual two-dimensional airfoil model with structural nonlinearities in incompressible flow via a state-of-the-art technique. Under the system (2) and by means of the classical Runge–Kutta algorithm, for any pair of estimated system states $\left(\hat{\mathbf{x}}_j, \hat{\mathbf{x}}_{j+1}\right)$ and intermediate values $\{\hat{\mathbf{x}}_j^{(i)}\}_{i=1}^{s}$, denote $\hat{\mathbf{X}} = \{\hat{\mathbf{x}}_j\}_{j=1}^{m}, \tilde{\mathbf{X}} = \{\hat{\mathbf{x}}_j^{(1)}, \ldots, \hat{\mathbf{x}}_j^{(s)}\}_{j=1}^{m}$, the system states and even the unknown parameters can be estimated simultaneously via minimizing the following loss function:[184]

$$\mathcal{L}\left(\hat{\mathbf{X}}, \tilde{\mathbf{X}}\right) = \sum_{j=1}^{m-1} \left[\kappa_1^{(j)} \mathcal{L}_1^{(j)} \left(\hat{\mathbf{X}}, \tilde{\mathbf{X}}\right) + \kappa_2^{(j)} \sum_{i=1}^{s} \mathcal{L}_2^{(j,i)} \left(\hat{\mathbf{X}}, \tilde{\mathbf{X}}\right) \right] + \bar{\lambda} \|\mathbf{Y} - \hat{\mathbf{X}}\|_p^p, \tag{13}$$

where

$$\mathcal{L}_1^{(j)} \left(\hat{\mathbf{X}}, \tilde{\mathbf{X}}\right) = \left\| \hat{\mathbf{x}}_{j+1} - \left[\hat{\mathbf{x}}_j + \Delta t_j \sum_{i=1}^{s} b_i \mathbf{f}\left(\hat{\mathbf{x}}_j^{(i)}\right) \right] \right\|_2^2,$$

$$\mathcal{L}_2^{(j,i)} \left(\hat{\mathbf{X}}, \tilde{\mathbf{X}}\right) = \left\| \hat{\mathbf{x}}_j^{(i)} - \left[\hat{\mathbf{x}}_j + \Delta t_j \sum_{l=1}^{s} a_{il} \mathbf{f}\left(\hat{\mathbf{x}}_j^{(l)}\right) \right] \right\|_2^2,$$

$\mathbf{Y}$ represents the noisy measurement data, and $\bar{\lambda} \in (0, +\infty)$ denotes a penalty/regularization parameter. For the reason of simplicity, we have omitted the time dependence in the vector fields $\mathbf{f}$. The norm in the second term of Eq. (13) can be selected as the l_2-norm $\|\cdot\|_2^2$ for the Gaussian noise or the l_1-norm $\|\cdot\|_1^1$ for the heavy-tailed noise. Other symbols and more details for this algorithm can be found in Ref. 184. The accuracy and effectiveness of the introduced algorithm on the aeroelastic airfoil system with cubic or freeplay structural nonlinearities are performed under several numerical experiments. The simulated measurement data are generated according to the following equation:[180,184]

$$\mathbf{Y} = \mathbf{X} + \text{noise\_level} \cdot \text{std}(\mathbf{X}) \cdot \epsilon_{\text{noise}}, \tag{14}$$

in which $\mathbf{X} = \{\mathbf{x}_j\}_{j=1}^N$ is the sampling data from the simulation trajectory as the true state, ϵ_{noise} is the measurement noise, noise_level represents the level of measurement noise in percent, and std $(\mathbf{X})$ is a diagonal matrix and its diagonal elements consist of the standard deviation of each state.[184] Both typical Gaussian or heavy-tailed measurement noises were considered in Eq. (14). As an illustrated example, the results of the pitch motion of the airfoil system with cubic nonlinearity are performed in the chaotic regime corrupted by 40% zero-mean Gaussian white measurement noise, as shown in Fig. 8, in which partial system parameters, including nonlinear stiffness coefficients and flow velocity, are assumed to be unknown. The obtained results suggested that the introduced algorithm can achieve excellent estimation for the system state and the unknown system parameters simultaneously.[184]

Additionally, Mannarino and Mantegazza[185,186] proposed two techniques for nonlinear aeroelastic reduced-order modeling by exploiting continuous- or discrete-time recursive neural networks from input–output data, respectively, and the feasibleness of the proposed approaches was verified via a two degrees of freedom typical aeroelastic airfoil section. Bai *et al.*[187] and Ghorbani and Khameneifar,[188] respectively, investigate reconstruction of airfoil data and airfoil profile from unorganized noisy point cloud data via a compressive sensing or a recursive weighted local least-squares scheme. Dawson and Brunton[189] and Sun *et al.*[190] employed sparse identification of nonlinear dynamics to improve approximations of the Wagner function and identify reduced-order models for the unsteady lift of an oscillating airfoil. Li *et al.*[191] employed the LSTM neural network to fit the relationship between the output and input for varying Mach number and mean angle of attack. At the same time, they also investigated the transonic flutter boundaries of a NACA 64A010 airfoil to illustrate the performance of the proposed technique. Beregi *et al.*[192] carried out a study that focuses on discovering universal differential equation (UDE) models for a nonlinear aeroelastic airfoil section. Both neural networks and Gaussian processes were considered as universal approximators against the mechanistic models from the first principles. Numerical and experimental results indicated that the proposed algorithm has superior performance on an aeroelastic airfoil system, and the trained UDE model can fit well with the measurement data.

B. Data-driven prediction and control

To ensure the safe operation of aeroelastic airfoil systems, it is very necessary to achieve a fast and accurate prediction of complex dynamical behaviors. Catastrophic consequences can be avoided and controlled if we can know about the possibility that the aeroelastic airfoil system undergoes the undesired vibration behaviors before they occur. As a consequence, there is of particular interest developing effective techniques to predict long-term behaviors of the aeroelastic systems from a short segment of transient data. During the past decade, data-driven prediction and control problems of the aeroelastic airfoil models have been popular due to the development of data science and machine learning.[165,191,193–215]

For example, in the aspect of response prediction, Sudha *et al.*[193] employed an autoregressive model to investigate flutter prediction of a typical three degrees of freedom airfoil model based on simulation data. To improve the accuracy and efficiency of the flutter prediction procedure, Gu and Zhou[196] focus on flutter onset prediction utilizing the well-known autoregressive moving-average (ARMA) parametric model. Both numerical and experimental results were performed to test the effectiveness of the ARMA parametric model for aeroelastic flutter prediction of the airfoil model. In these works,[200–205] a novel method based on a critical slowing down was proposed for forecasting the bifurcation behaviors only from the much fewer observation data of the pre-bifurcation regime. Simulation results suggest that the proposed method can predict the post-bifurcation regime accurately. Recently, Wang *et al.*[206] employed the generative deep learning to develop an important method for extracting and predicting the flow fields around supercritical airfoils. Additionally, Wang and Wang[209] analyzed the nonlinear aeroelastic behaviors using deep learning and compared the differences between the deep neural network (DNN) and the long short-term memory (LSTM) applied to the prediction of flutter speed. Figure 9 shows, respectively, the prediction results of the flutter speed obtained from the DNN model and the LSTM model.[209] In Fig. 9, the red dots indicate the results of the true flutter speed vs the predicted flutter speed, while the dashed line represents the prediction result of the DNN or LSTM model. They found that accuracy of 95.6% is achieved for the case of DNN, while the accuracy of 96.8% is achieved for the case of LSTM, which indicates that the LSTM model can obtain higher accuracy for predicting the flutter speed than the DNN one.[209] In addition, Li *et al.*[211] proposed a new technique based on deep reinforcement learning for reducing the aerodynamic drag of supercritical airfoils. Results show that the learned policy is effective and can be applied repeatedly to achieve greater drag reduction.

Besides, in the aspect of vibration control, Zhang and Söffker[212] proposed a novel data-driven criterion and design a controller for stabilization of unknown nonlinear discrete-time systems based on online black-box system identification. Importantly, the proposed stability criterion directly uses the time series instead of the dynamical models of the aeroelastic airfoil system. Jia *et al.*[215] proposed a novel optimal controller, i.e., a data-driven adaptive dynamic programming control strategy, for an aeroelastic airfoil system, which can effectively avoid the effects of uncertainty. Numerical results indicated that the proposed control method can eliminate the LCO phenomenon of the aeroelastic airfoil system within several seconds and possess superiority and feasibility.

VI. FUTURE DIRECTIONS FROM PAST INSPIRATION

During the past decade, we have already witnessed that there has been some significant progress in complex nonlinear dynamics and vibration suppression of the conceptual two-dimensional airfoil models, in particular, the aspects of stochastic dynamics and data-driven studies. Despite all this, there are still several open problems that are worth further exploring in-depth in the aspects of

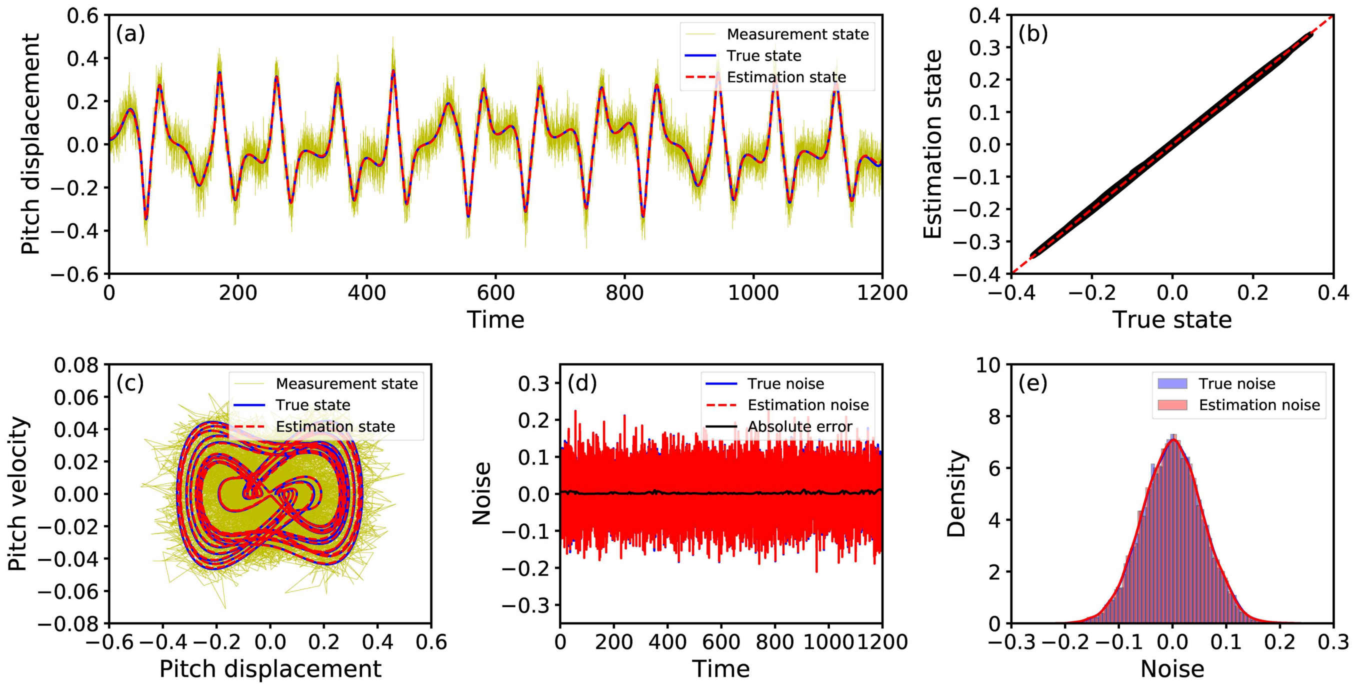

FIG. 8. Estimates of the pitch motion for the airfoil system (2) with cubic nonlinearity in the chaotic regime corrupted by 40% zero-mean Gaussian white measurement noise: (a) time history, (b) relationship between the true and estimation state, (c) phase diagram, (d) measurement noise, and (e) the density histogram of the measurement noise. Reproduced with permission from Liu *et al.*, Acta Mech. Sin. **37**, 1168–1182 (2021). Copyright 2021 Springer Nature.

accurate modeling, response solving, and dynamics control of conceptual airfoil models.

(1) *Stochasticity quantification of complex flight conditions*. The stochasticity in the existing studies is mostly characterized as simple cases, such as Gaussian or uniform distributions. However, in a complex flight environment, the excitations of an aircraft subjected to them are very complicated. On the one hand, an aircraft inevitably encounters various kinds of severe flight conditions, such as extreme high or low temperatures, strong winds, thunderstorms, etc., which will have a significant threat on the flight safety of an aircraft. On the other hand, in a turbulent environment, there may be random fluctuations with a long-correlation feature. It is of considerable importance to accurately understand the nonlinear dynamics of the conceptual airfoil models under extreme flight conditions and long-correlation noise to ensure the flight safety of an aircraft. Consequently, we need to further explore the excitation models acting on the aeroelastic airfoil models that can characterize the complex flight conditions of an aircraft, such as Poisson or Lévy noises,[216–218] with a large jump describing the extreme flight conditions, fractional Gaussian noise[219] with a long-correlation property, and even other complex cases.

(2) *High-dimensional nonlinear airfoil models*. For simplicity, most of the existing works only consider the simplified two-dimensional aeroelastic airfoil models. However, more complex wing models, such as beam or plate structural models, folding wings, high-aspect-ratio wings, etc., can better reflect the situation in practical engineering, but it also brings some chalenges, especially in the aspect of theoretical analysis of the airfoil systems. For such complex situations, the governing equations of the aeroelastic airfoil models are usually described as high-dimensional nonlinear ordinary or partial differential equations, which may lead to the course of dimension. It will severely limit the application of the classical solving methods for the dynamical response analysis of complex wing structures. Effective decoupling and reducing for the high-dimensional nonlinear airfoil systems becomes a central challenge in the theoretical and numerical analysis and the applications of data-driven techniques. The key is to ensure fidelity between the solution of the original system and the solution of the reduced-order system. Fortunately, the averaging principle can guarantee this property.[220–222] Thus, it is necessary to further discuss more complex airfoil models, in particular, the cases with random fluctuations. Meanwhile, the development of high-fidelity reduced-order theory and efficient numerical algorithms is extremely urgent for nonlinear aeroelastic analysis of complex airfoil models.

(3) *Dynamics control of conceptual airfoil models*. Complex dynamical phenomena, including unwanted subcritical Hopf bifurcations, discontinuous or catastrophic saddle-node bifurcations, chaotic motion, stochastic switching, and even extreme events, can be induced due to the mutual coupling of nonlinearity and stochasticity in the conceptual airfoil models. These behaviors

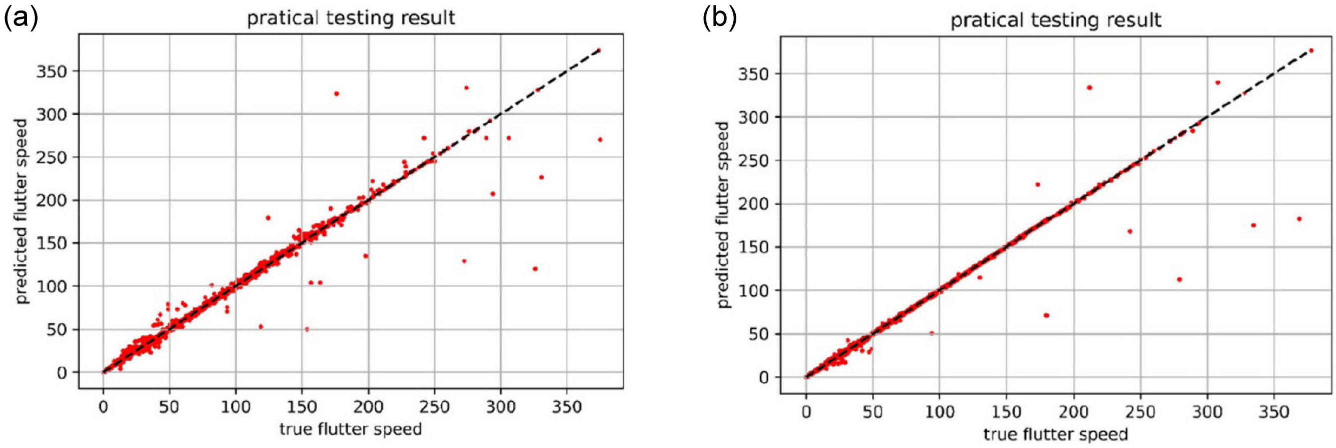

FIG. 9. Prediction results of flutter speed of the airfoil model obtained from different methods: (a) DNN model. (b) LSTM model. Reproduced from Y. R. Wang and Y. J. Wang, Adv. Mech. Eng. **13**, 16878140211062275 (2021). Copyright 2021 Author(s), licensed under a Creative Commons Attribution (CC BY) License.

would cause an aircraft to become unstable. For example, abrupt jump and hysteresis phenomena are caused in the airfoil systems by the saddle-node bifurcations, in which two stable attractors exist with an unstable one in between at a certain interval of the control parameter. Several control strategies, such as bifurcation control methods,[223–225] have been studied in the last several decades to achieve better dynamics control of a variety of nonlinear dynamical systems. However, as the wing structure and service environment become more complex, the airfoil model exhibits strong nonlinearities, strong coupling, model uncertainties, external disturbances, and others, resulting in more complex dynamical behavior, which makes the design of control methods extremely difficult. In particular, it will become more complicated when random or non-smooth factors are present. As a result, future research is needed to propose some effective controller design methods for solving the stability of complex wing systems, such as active or passive control strategies, particularly data-driven techniques combined with machine learning, in order to better realize the complex dynamics control of typical airfoil models. Meanwhile, it is also imperative to consider the practical application of control methods in the control of complex dynamics of conceptual airfoil systems.

(4) *Hypersonic aeroelastic airfoil models with uncertain disturbances*. The existing studies on the airfoil models with uncertain disturbances have focused on the subsonic, transonic, and supersonic cases, but little research studies have been worked on the stochastic dynamics of airfoil models in hypersonic flow, especially in the theoretical analysis. Hypersonic vehicles have become a development trend and requirement in the aerospace field. In hypersonic flow, the flight environment is more severe, and the fluid-interaction coupling problem is also more complicated where both structural and aerodynamic nonlinearities have to be considered, resulting in the wing structure being more prone to aeroelastic instability. Meanwhile, the coupling problems of multiple physical fields, including aerodynamic, structural, control, and thermal, have to be also considered in the hypersonic scenario. These will cause difficulties in accurate modeling, response solving, and vibration suppression of the hypersonic airfoil models. For this reason, new theories and methods are highly anticipated to be developed for fluid–structure interaction analysis and vibration suppression of the hypersonic aeroelastic airfoil models, which can help ensure good flight characteristics of hypersonic vehicles and avoid catastrophic consequences due to the unwanted aeroelastic behaviors of the wing structures.

(5) *Data-driven study of conceptual airfoil models with complex uncertain disturbances*. In recent years, several data-driven techniques have been exploited to carry out preliminary explorations of related data-driven problems of typical aeroelastic airfoil models, related to data-driven modeling, flutter prediction and control, and others. However, it should be emphasized that many of the existing studies mainly consider deterministic airfoil models but neglect the effects of uncertain disturbances on the aeroelastic airfoil systems. The presence of random perturbations will limit the application of existing data-driven methods and present some challenges in the interpretability and generalization of the machine learning methods. As a result, we should develop new data-driven approaches and theories applicable to the study of data-driven problems in conceptual aeroelastic airfoil models with uncertainty disturbances. At the same time, interpretation and generalization of machine learning algorithms on the aeroelastic airfoil models are also highly recommended.

(6) *Data-driven problem of conceptual airfoil models with a small amount of data.* Many of the state-of-art data-driven techniques, in particular, deep neural network models, still require a large amount of training data, which are often unavailable, impractical, or interpretable to their engineering application with small datasets. In fact, for many practical engineering problems, the available data are generally quite limited and have a

characteristic of small data, such as life testing data, on-line transient data, etc. Particularly, the amount of data has a considerable effect on the interpretation of machine learning techniques. It is extremely crucial and of great importance for engineering practical applications to obtain accurate understanding of the aeroelastic airfoil systems with a small dataset. More importantly, we prefer to predict the long-term dynamic behaviors of an aircraft using a small amount of transient data, in order to meet the real-time requirements during the actual flight. Accordingly, we should be devoted to developing several effective machine learning techniques that is applicable to the small data problems by combining the mechanistic models and the available data, such as physics-informed neural network, transfer learning, data fusion, and others.

VII. CONCLUSIONS

In this paper, we review several significant theoretical, numerical as well as experimental developments on the complex dynamic response and vibration suppression of conceptual two-dimensional aeroelastic airfoil models over the past decade, in particular, the deterministic and stochastic dynamical responses and the relevant data-driven problems. To begin with, the general form of the governing equation for the conceptual two-dimensional aeroelastic airfoil models and a brief description of representative structural nonlinearities arising from the wing structure, including cubic, freeplay, and hysteresis nonlinearities, are presented. Subsequently, the results related to the nonlinear dynamics and vibration suppression of the deterministic airfoil models are summarized, including analytical or semi-analytical, numerical, and experimental methodological studies of flutter analysis, and active and passive vibration control methods of unexpected flutter responses. At the same time, the recent advances on the aeroelastic airfoil models with different types of random fluctuations are summarized, including stochasticity quantification, noise-induced dynamics, early warning of unwanted aeroelastic behaviors, and other interesting topics. Furthermore, several main results in recent years in data-driven problems of the aeroelastic airfoil models are summarized, especially for data-driven response prediction and data-driven inverse problems. Finally, further fascinating research directions that are worth attention are also recommended, in particular, accurate modeling, response solving, control of complex dynamics, and even the problems of a small amount of data in conceptual aeroelastic airfoil models. This mini-review aims at helping scholars in related fields to be able to quickly understand the latest research results and provides valuable insight and inspiration for them to carry out related research.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

NOMENCLATURE

ARMA	Autoregressive moving-average
DNN	Deep neural network
DSVD	Damped singular value decomposition
LCO	Limit cycle oscillation
LSTM	Long short-term memory
NES	Nonlinear energy sink
RBF	Radial basis function
SMC	Sliding mode control
TIKH	Tikhonov regularization
TSVD	Truncated singular value decomposition
UDE	Universal differential equation

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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