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ABSTRACT

Dynamical stability of the synchronous regime remains a challenging problem for secure functioning of power grids. Based on the symmetric *circular model* [Hellmann *et al.*, Nat. Commun. **11**, 592 (2020)], we demonstrate that the grid stability can be destroyed by elementary violations (motifs) of the network architecture, such as cutting a connection between any two nodes or removing a generator or a consumer. We describe the mechanism for the cascading failure in each of the damaging case and show that the desynchronization starts with the frequency deviation of the neighboring grid elements followed by the cascading splitting of the others, distant elements, and ending eventually in the bi-modal or a partially desynchronized state. Our findings reveal that symmetric topology underlines stability of the power grids, while local damaging can cause a fatal blackout.

A particular complexity of the power grid stability is caused by the fact that the desired synchronous state is only locally stable, not globally. In the system phase state, it repeatedly co-exists with many other desynchronized states. In such a case, the desired grid synchrony can be secured only against small perturbations but not against large impacts, even applied to a single grid element or to a single connection. If so, the system's dynamics can switch to another, desynchronized attractor as soon as a large perturbation is applied. The essential difficulties of the power grid studies are also induced by intricate, highly asymmetric architectures of the realistic grids, often caused by geographical and historical reasons. What is the role of asymmetry for the stability? Which grids with symmetric or asymmetric topology are more reliable? We attack this problem by examining a symmetric *circular* power grid model and compare its stability with the situation when the symmetry is broken by elementary violations of the network structure.

I. INTRODUCTION

Despite many studies, both theoretical and engineering,[1–16] stability of power grids remains a challenging problem. The solution to this riddle is noteworthy as it concerns the safety of our everyday life. As was found in Ref. 11, the multistability grid problem is a consequence of the presence in the system phase space of the so-called *solitary states*,[17–24] in which one or a few network oscillators (generators or consumers) deviate from the collective synchrony and start to rotate with a different frequency. The problem becomes even more puzzled due to the fact that stability regions of the solitary states are practically coinciding (at the lower level of connectivity) with the regime for the operating synchronous state. Such unavoidable grid multistability can perhaps explain the never-ending chains of the blackouts happening in many countries of the world, even in these with the highest level of grid security. The U.S. Department of Energy (DOE) estimates that power outages cost businesses annually $150 billion.[25,26]

A well known example is the European blackout, which originated in Northern Germany on 4 November 2006 (see Refs. 27 and 28). The blackout was caused by the planned disconnection of a high-voltage line, which was switched off to allow a ship to pass underneath the overhead cables. After the implementation of the procedure, the lines were overloaded, and as a consequence, one of the zones was out of power. Millions of customers from France, Germany, Belgium, Netherlands, Italy, and Spain were deprived of electricity that evening. It is interesting to note that the same disconnection was performed before more than ten times.

A question arises whether the multistability problem is indeed unavoidable,[29–31] and, more generally, how to design future "smart grids"[32–34] with minimum risk and the maximum protection against blackouts.

Nonlinear dynamics of complex networks, including stability of the synchronous state, is predominantly defined by the network architecture.[35–40] Due to the historical and geographical reasons, most of the existing power grids have a high non-regular topological structure with an individual, visually "random" coupling structure far from symmetric. This fact makes the study of the power grid dynamics more involved. How is then the stability affected by the level of the asymmetry? Which grids are more reliable, symmetric, or asymmetric? In Ref. 41, this problem was analyzed for inhomogeneous grids with different mass generators, and it was found that this kind of asymmetry can improve the stability of the grid.

II. SYMMETRIC CIRCULAR GRID MODEL

In this Letter, we analyze the grid asymmetry problem from a different perspective based on the symmetric model proposed in Ref. 11. In the model, all generators and consumers are considered identical and coupled alternately on a ring [Fig. 1(a)]. Note that despite maximum symmetry, this model shows an adequate correspondence with the realistic Scandinavian power grid.[11] Our task here is to analyze the impact of asymmetry for the model by cutting a connection between any of two chosen elements or removing one of them, generator or consumer. We explore how does the imposed asymmetry affects the grid stability. Can it improve or disprove it?

In this study, we have considered five elementary motifs of the asymmetric grid damaging and found that in four of them, the stability can be destroyed, and only in one of them, it is not affected. The asymmetry impairs the grid stability causing, under certain conditions, cascading desynchronization as illustrated in Figs. 1(b) and 1(c). We conclude that at the contrary to the mass generator inhomogeneity,[41] topological asymmetry underlines instability of power grids.

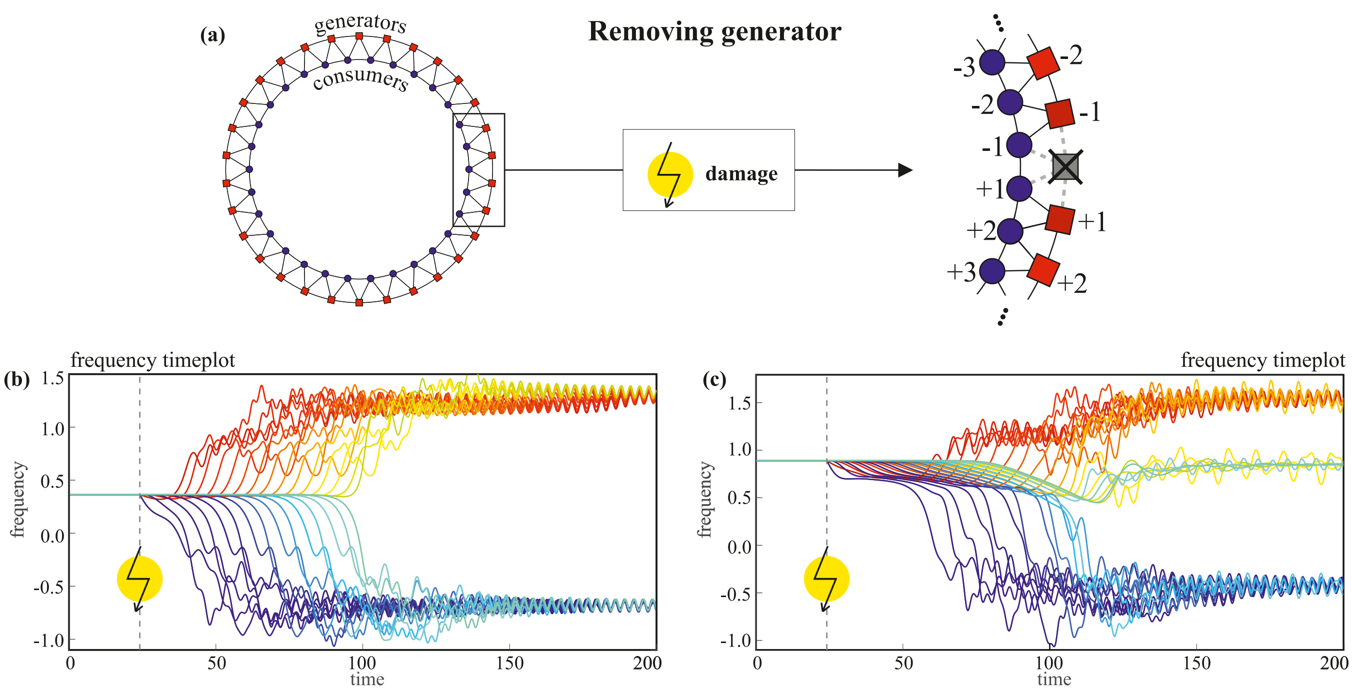

FIG. 1. Grid desynchronization in model (1) obtained by removing a generator. (a) Schematic representation of the circular grid model closed in a ring (red squares denote generators, whereas blue circles denote consumers). The damage is illustrated in the zoom to the right, where the numbers next to consumers and generators indicate neighboring deviation from the damage point: (−1, −2, etc.) to the left and (+1, +2, etc.) to the right. Frequency timeplots of cascading grid failure resulting eventually in (b) the complete desynchronization (parameters: $\alpha = 0.21$, $\mu = 0.208$) or (c) in partial desynchronization (parameters: $\alpha = 0.3$, $\mu = 0.28$). Frequencies of the generators are shown in warm colors from red to yellowish-green, while the frequencies of consumers are shown by cold colors from dark blue to turquoise. Other parameters $m = 1.0$, $\varepsilon = 0.1$, $\omega_{1,2} = \pm 1$; network size $2N = 60$ (30 generators and 30 consumers).

The circular grid model is written in the general form

$$m\ddot{\theta}_i + \varepsilon\dot{\theta}_i = \omega_i + \frac{\mu}{2P}\sum_{j=i-P, j\neq i}^{i+P}\sin(\theta_j - \theta_i + \alpha), \tag{1}$$

where $\theta_i(t)$ describes the phase of the ith oscillator, $i = 1, \ldots, 2N$. The coefficients m, ε, μ, and α denote mass, damping, coupling strength, and phase lag, respectively. The oscillators' natural frequencies $\omega_i/\varepsilon m$ are positive for generators and negative for consumers, and we put, therefore, $\omega_i = (-1)^i\omega$. In our study, we fix $m = 1.0$, $\varepsilon = 0.1$, $\omega = 0.1$ and explore stability dynamics of the synchronous state of (1) when varying $\mu \geq 0$ and $0 \leq \alpha \leq \pi/2$ as control parameters. The coupling radius P will be fixed to 2, which corresponds to the node degree 4. The number of all oscillators in the grid ring will be $2N = 60$, including equal number $N = 30$ of generators and consumers placed alternately: consumers at odd places $\theta_{i=2k-1}$ and generators at even $\theta_{i=2k}$, $k = 1, \ldots, N$ [see Fig. 1(a) for a schematic representation of the grid].

III. IMPOSING THE GRID ASYMMETRY

The results of the direct numerical simulations of the model (1) in the two-parameter plane of the phase lag α and coupling strength μ are summarized in Fig. 2. This figure exhibits and allows one to compare between stability regions of the synchronous grid state before and after the elementary motif damages. The regions are bounded from below by the bifurcation curves denoted as S_j, $j = 0, \ldots, 5$. This figure reveals that the symmetric model (i.e., this not damaged) has the widest stability region, which is bounded from below by the curve S_0 (the lowest curve in the figure). Only one of the other curves, S_1, coincides with S_0, and four remaining S_{2-5} lie above. The curves bound stability regions from below for the following basic asymmetric motifs:

(1) S_1: cutting link between two generators;
(2) S_2: cutting link between two consumers;
(3) S_3: cutting link between a generator and a consumer;
(4) S_4: removing a consumer; and
(5) S_5: removing a generator.

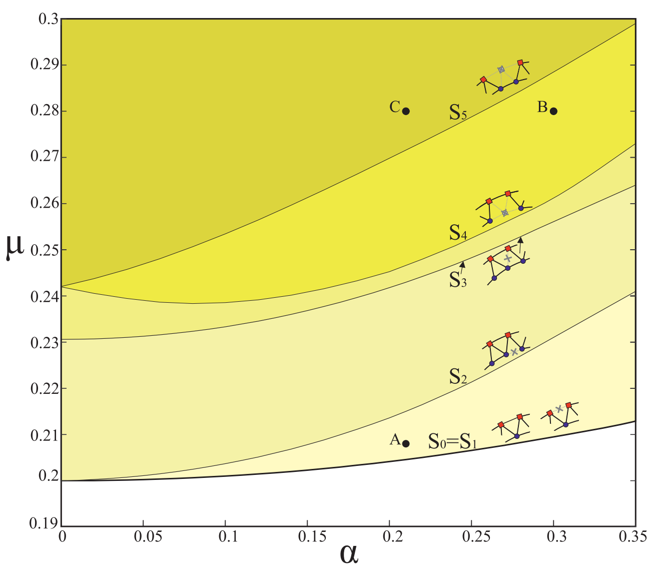

FIG. 2. Regions of stability of a synchronous state for different node and edge removal (cases S_1–S_5) and for system without damage S_0. Coordinates (α, μ) of chosen parameter points: $A = (0.21, 0.208)$, $B = (0.3, 0.28)$, and $C = (0.21, 0.28)$.

We conclude, therefore, that the asymmetric grid modifications can destroy the grid stability. This occur as soon as the grid operating point lies in the gap between the curves S_0 and S_{2-5}. The only conditionally secure modification is cutting a link between two generators (the curve S_1 coincides with S_0). Two examples of the critical desynchronization transitions, when removing a generator, are illustrated in Figs. 1(b) and 1(c). The synchrony is violated by a cascading grid failure, leading eventually to complete [Fig. 1(b)] or partial [Fig. 1(c)] desynchronization.

The bifurcation curve S_0 for the symmetric circular model (1) can be obtained analytically. Indeed, in the synchronous regime, all generators obtain equal synchronized phase θ_g, as well as all consumers—phase θ_c. Then, the respective system in the difference $\eta = \theta_g - \theta_c$ takes the form

$$m\ddot{\eta} + \varepsilon\dot{\eta} = 2\omega + \frac{\mu}{2}\sin(-\eta + \alpha) - \frac{\mu}{2}\sin(\eta + \alpha). \tag{2}$$

In the synchronized state, all nodes have the same constant frequency; therefore, $\ddot{\eta} = \dot{\eta} = 0$. Then, the solution of (2) exists if and only if $\mu \geqslant \frac{2\omega}{\cos\alpha}$. The equality in this formula just describes the curve S_0, above which the synchronous state is (locally) stable, herewith all network oscillators, generators, and consumers, rotate with constant frequency $\overline{\omega} = \frac{\sin(\alpha)}{\varepsilon}\left(\sqrt{\frac{\mu^2}{4} - \frac{\omega^2}{\cos(\alpha)^2}} + \frac{3}{4}\mu\right)$ and with a constant phase difference $\eta = \arcsin\left(\frac{2\omega}{\mu\cos\alpha}\right)$.

For the asymmetric motif modifications of the model (1), analytical formulas for the bifurcation curve S_1–S_5 cannot be obtained due to the high-dimensionality of the nonlinear dynamics in the cases. Our accurate numerical simulations (with using two different codes) allow us nevertheless to make the following conclusions about the changes in the system behavior caused by the asymmetry, described in Sec. IV.

IV. CASCADING FAILURE

Typical scenarios for the desynchronization transition in each of the four critical damaging motif S_2–S_5 (listed above) are illustrated in Fig. 3, where we have fixed the control parameters to the point $A = (\alpha, \mu) = (0.21, 0.208)$ in the gap above S_0 and below S_{2-5}. In each case, first, the symmetric model (1) has been simulated, starting from random initial conditions, up to time moment $t_c = 25$, and then the damage has been applied. As A lies above S_0, the solution behavior synchronizes (quite fast) at the beginning time interval to equal phases θ_g and θ_c for generators and consumers, respectively, all rotating with equal frequency $\overline{\omega}$.

Figures 3(a)–3(d) reveal that each of the four elementary grid damages destroy the synchronization via cascading splitting of the oscillators, bringing eventually the network solution to a bi-modal

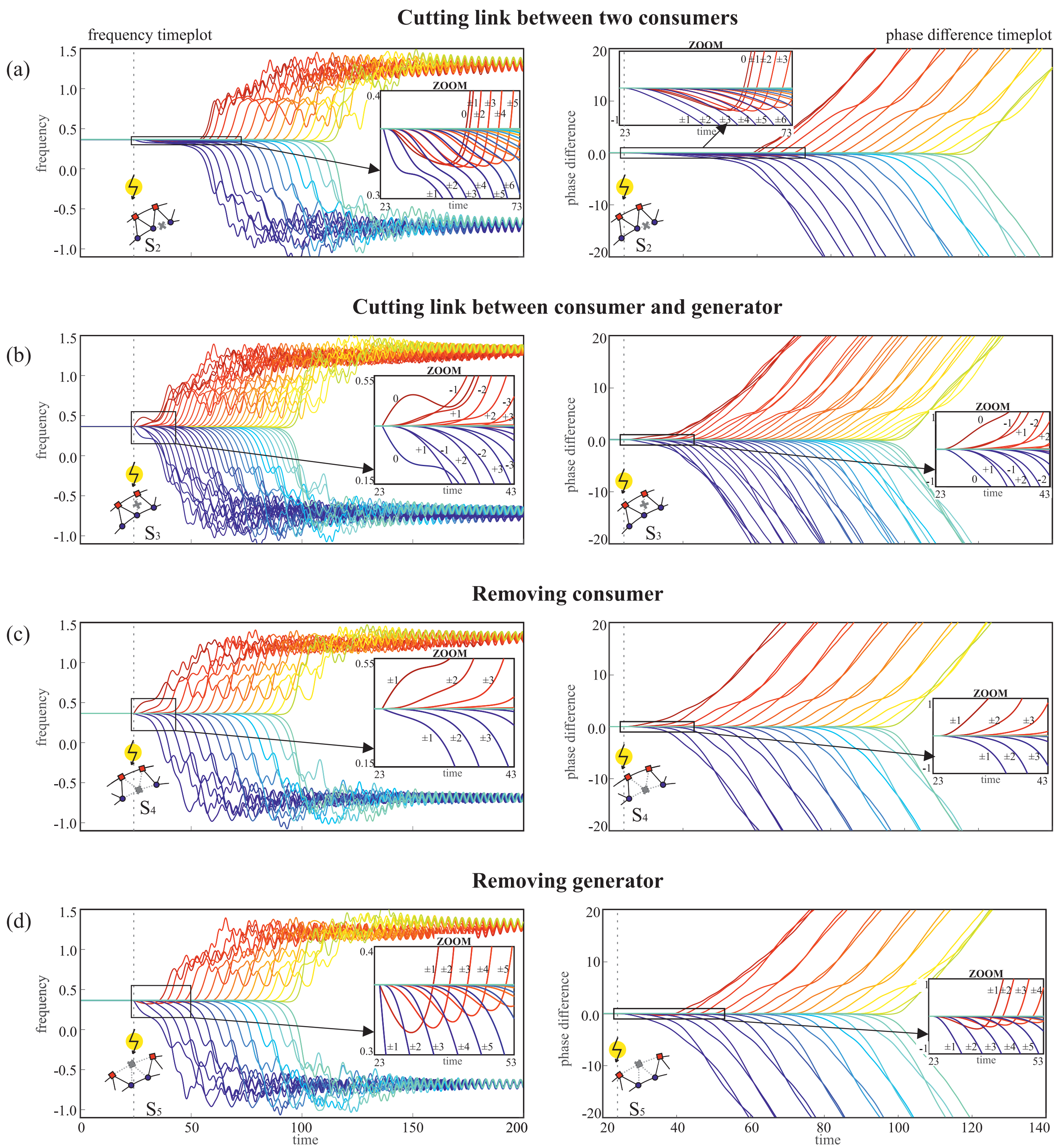

FIG. 3. Cascading failure in the model (1) as a result of elementary damaging of the grid: (a) cutting consumer–consumer connection, (b) cutting generator–consumer connection, (c) removing consumer, (d) removing generator. Left column—frequency timeplots, right column—phase difference timeplot (difference between the actual phase in the simulations and the theoretical phase if the system would be still left synchronized). Parameter $(\alpha, \mu) \equiv A$. Point A is shown in Fig. 2. Other parameters as in Fig. 1. Multimedia views: https://doi.org/10.1063/5.0131931.1; https://doi.org/10.1063/5.0131931.2; https://doi.org/10.1063/5.0131931.3; https://doi.org/10.1063/5.0131931.4

state with two different frequencies ω_g and ω_c for all generators and all consumers, respectively. The transition starts with a sharp voltage picking of the nearest-neighboring oscillators, i.e., linked directly to the removed connection or node, followed by the secondary desynchronizations of the distant nodes, pair by pair. Let us focus on the transitional behavior in each case by analyzing frequency and phase timeplots shown in the left and in the right panels, respectively.

Figure 3(a) (Multimedia view) corresponds to cutting the coupling link between two consumers (S_2 case). The first elements to desynchronize are two disconnected consumers (denoted by ± 1 in the figure) and nearly simultaneously with them—the generator located opposite to the cut link (denoted by 0 in the inset). After some time, the next distant consumers ± 2 and generators ± 1 desynchronize in turn. This process continues successively for other nodes until the full desynchronization of all oscillators. Frequency of the customers decreases monotonously: first, very slowly, then faster, approaching eventually ω_c. To quantify the change of pace of escape, we have calculated escape acceleration for consumer ± 1: average acceleration in first 25 time units after damage is equal to -0.0034, then in the next 10 time units (before first oscillations occurs) is equal to -0.048, i.e., around 14 times larger in the module. Interestingly, the frequency of each desynchronized generator first decreases (escape acceleration in the first 23 time units for generator ± 1 is equal to -0.0015) and a bit after starts to increase (escape acceleration is equal to 0.0117 in the next 10 time units) reaching eventually the ω_g value. As a result, the described cascading frequency distribution in the grid becomes bi-modal.

Similar behavior is observed for respective phase differences, Fig. 3(a) (right panel). Phase differences are here understood as the difference between the actual phase in simulations and the theoretical phase if the system will still be synchronized. Looking at the right panel, however, no sharp picking can be seen. All phase differences grow gradually in a power law with power > 1. Another peculiarity of the desynchronization is that after the deviating, the first escaping consumers ± 1 wait for the second pair ± 2 and further they rotate together. In the case of generators, the scenario is analogous but first 0 and ± 1 waits for ± 2. After some time, the second group is separating, in which each element makes the same number of phase flips. Eventually, the phase difference grows, and all oscillators obtain approximately the same phase, with multiples of the 2π difference between the groups. In the end, all oscillators group by fours in the group (regarding the number of full flip rotations) except the first and the last groups with five and only one.

Note that when removing the link between two consumers [S_2 case, Fig. 3(a)], the grid is separated into two parts that are a mirror reflection—that is the reason of complete overlapping of the $\pm$ curves. The symmetry is broken in the next S_3 case (Fig. 3), where the link between the consumer and generator is cut. Looking at the frequency timeplot (left panel), one can see that the first escape is very rapid and it concerns both the generator and consumer with a broken connection, i.e., the elements are denoted as 0. Soon after, cascading splitting appears, but this time (due to topological asymmetry), graphs for consumer to the left and to the right are not coinciding; the same is for generators. Nevertheless, the closer node is to the damage point, the earlier the desynchronization occurs. Finally, as in the previous case, the system reaches the bi-modal desynchronized state.

The phase difference presented in Fig. 3(b) (Multimedia view) shows that also in this case, we observe phase grouping by four in the group (except five in the first and last groups), but only for generators. For consumers, the groups are not equal and vary from two to five elements in the group. As we assume, such grouping of consumers can be caused by not equal phases, which create a smooth wave profile, while for generators, the phases are approximately identical.

Figure 3(c) (Multimedia view) illustrates the cascading desynchronization caused by removing a consumer (S_4 case). The topology of the damage is symmetric; so, as expected, the frequencies of the escaping elements are also symmetrically grouped (in the $\pm$ denoting sense). Generators ± 1 and consumers ± 1 are the first to desynchronize, but the generators escape earlier than the consumers. The next groups escape in the similar way, and the phases are grouped by four (similar to the S_2 case), whereas the last group of generators consists of six elements and the last group of consumers of five.

Desynchronization transition in the last damaging case (S_5: removing a generator) is illustrated in Fig. 3(d) (Multimedia view). Here, the frequency cascading is similar to the case S_2 [Fig. 3(a)]: first, both generator's and consumer's frequencies decrease approaching eventually the bi-modal distribution. Again, the phases group by fours, and only the first and last groups break up from the rule, containing two, four, six, or seven elements.

In all damaging motifs demonstrated in Fig. 3, the escaping oscillators' behavior mostly obeys the rule of "grouping-by-four" (regarding the number of full flip rotations in the desynchronization procedure). Due to the fact that $N = 30$ is not a multiple of 4, the first and last groups have different numbers of elements, but the inner grouping number is always four for the S_2, S_4, and S_5 motifs. Interestingly, this rule does not work for consumers in the S_3 motif, when the phases create a rotating wave. In order to explain the phenomenon of grouping in fours, we have performed numerous simulations with grids with different node degrees. It seems that any rule for grouping is only visible for "strongly" symmetrical coupling, i.e., for example, when each consumer is connected to four other consumers, each generator to four other generators, and each generator to four consumers. We do not have any explanation for this kind of "quantization" of the network behavior and leave this puzzle for future study.

To finalize our consideration, in Fig. 4, we present three peculiar examples of possible different system behavior after the grid damaging. The first example shown in Fig. 4(a) (Multimedia view) shows the system behavior for motif S_1: cutting the link between two generators, which does not affect stability of the synchronous state (bifurcation curve S_1 coincides with S_0; see Fig. 2). Frequency timeplot and six phase snapshots for this damage motif are presented for parameter point A. It can be seen than although the system behavior returns eventually to the complete synchronization (on a slightly lower frequency level), before this, it undergoes rather large perturbations as a result of the damage: The nearest-neighboring generators obtain a strong perturbation (losing some amount of frequency) followed by the milder cascading perturbations of the subsequent couples. Analogous behavior is observed for the consumers. This example shows that the perturbations arising at damaging can also represent a way back to the synchrony.

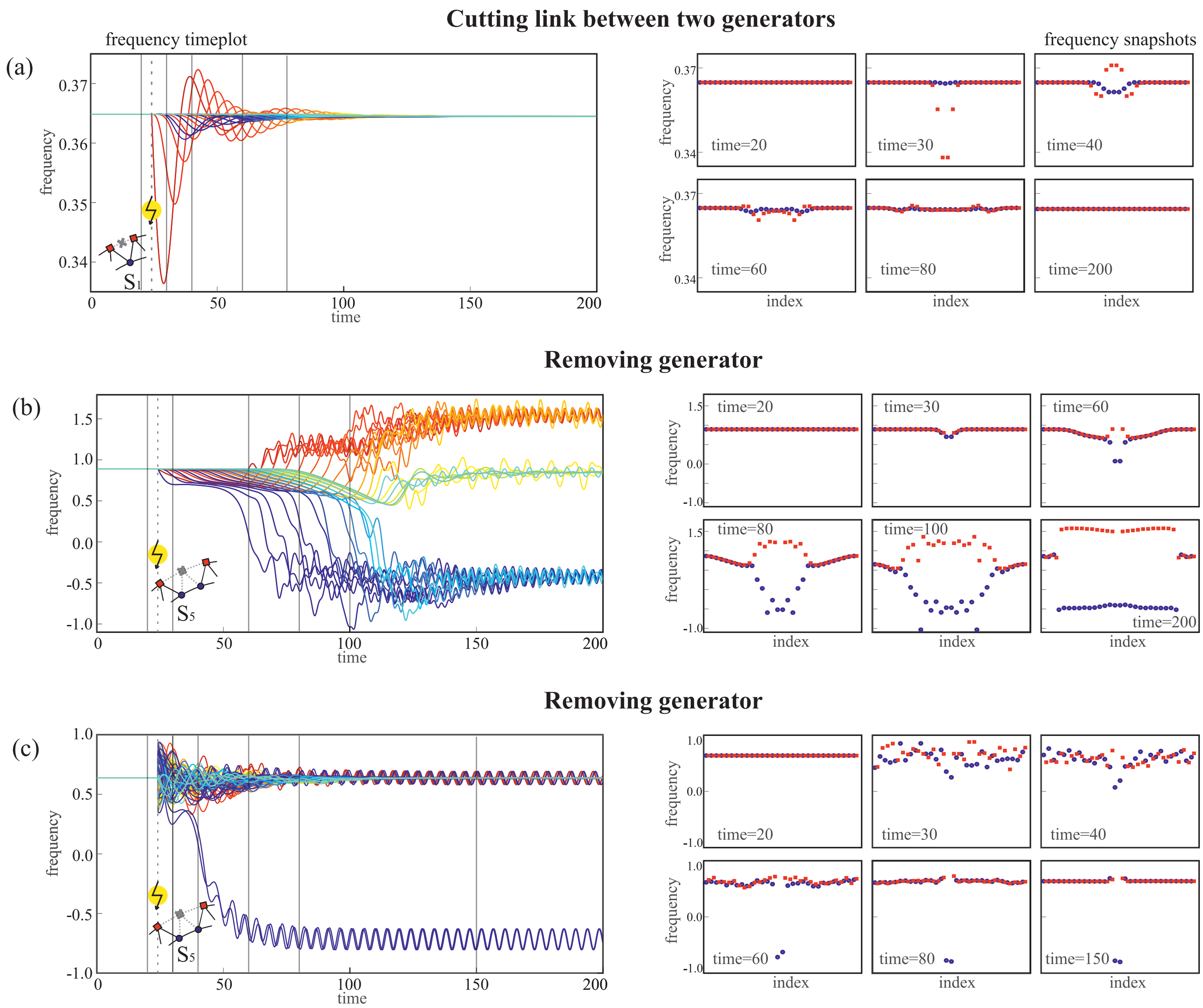

FIG. 4. Diversity in the desynchronization transition after grid damaging. (a) Removing generator–generator line ending up in synchronization with slightly different frequency, $(\alpha, \mu) \equiv A$. (b) Cascading failure to partial synchronization obtained after removing a generator, $(\alpha, \mu) \equiv B$. (c) Solitary state reached after removing generator and adding perturbations in the moment of damage, $(\alpha, \mu) \equiv C$. Left column—frequency timeplot, right column—frequency snapshots. Points A, B, and C are shown in Fig. 2. Other parameters as in Fig. 1. Multimedia views: https://doi.org/10.1063/5.0131931.5; https://doi.org/10.1063/5.0131931.6; https://doi.org/10.1063/5.0131931.7

Another example of peculiar behavior in model (1) after the damaging is presented in Fig. 4(b) (Multimedia view). It is for motif S_5 (removal of a generator) in the parameter point B close to the bifurcation curve S_5. As it can be seen from the figure, after the damage, the solution approaches eventually not the bi-modal but only a *partially synchronized state* with three frequencies. The generators and the consumers close to the damage deviate, one after another, from the synchronous frequency. The cascading process does not lead, however, to complete separation into two groups: it stops in such a way that seven distant generators and six consumers remain synchronized creating a *partially synchronized* state.

Our last example in Fig. 4(c) (Multimedia view) shows the arising of a partially synchronized state for motif S_5 (removing a generator) in point C above the bifurcation curve S_5 (see Fig. 2), i.e., inside the stability region of the synchronous state. This synchronous state, as it has been noted previously, is only locally not globally stable as there are many other co-existing stable partially synchronized states. Then, if the perturbations in the grid are not small, the damage can bring the system phase point beyond the basin boundary of the synchronous state and the solution will approach one of the co-existing partially synchronized states. Figure 4(c) shows the example where this state contains only two desynchronized oscillators (the closest to the damage), and therefore, it is the so-called *solitary state*.[11]

Each video representing Figs. 3(a)–3(d) (Multimedia views) and Figs. 4(a)–4(c) (Multimedia views) presents phase evolution for each grid element after the introduction of the damage.

V. DISCUSSION AND CONCLUSION

In our study, we have analyzed how elementary violations of the symmetric power grid topology can affect its stability based on the circular model (1) proposed in Ref. 11. We have found out that in most cases, the imposition of asymmetry can cause the cascading desynchronization in the grid, leading eventually to complete splitting of the generator and consumer frequencies. The generators and consumers escape, one by one, from the operating synchronous state until the grid is fully disintegrated. The closer to the damage point, the earlier node is desynchronized. In most cases, we observe an interesting phenomenon of "grouping-by-four" for the phase in the process of the desynchronization. The desynchronized networks stabilize on the bi-modal (mostly) or partially synchronized (less common) state.

We have examined five different asymmetric motifs for the grid damaging and found parameter regions for the grid stability. The most dangerous damage seems to be the removal of a generator: in this case, the region shrinks most drastically. Cutting a connection between two generators, on the other hand, is rather secure transformation. It does not affect the stability region; however, there arise voltage surges, which can also cause disturbance to the functioning of the grid.

An interesting open question concerns the asymptotic behavior of the grid solutions after the desynchronization. Our study demonstrates a perfect cascade of the successive desynchronizations of the nodes, one after another, as a wave originated in the damage point to the periphery. In the majority of the considered cases, the wave goes to the end causing complete grid uncoupling. The bi-modal state is created with two different frequencies for generators and consumers, respectively. We show also that the cascading grid disintegration can stop after some number of escapes giving rise to a partially synchronized state. Moreover, it can happen that only two neighboring nodes escape, which corresponds to a solitary state. Nevertheless, we believe that deeper characterization of cascades using, for example, line flow and overload conditions is needed.

Our results demonstrate that topological symmetry encourages stability of power grids. Elementary interruptions of the symmetric architecture can destroy the stability causing full or partial grid disintegration. We believe that this integral property indicates a common, probably universal phenomenon in the power grids of different typologies, which can have some consequences for designing new power grids.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Patrycja Jaros: Conceptualization (equal); Formal analysis (equal); Investigation (lead); Software (equal); Validation (equal); Visualization (lead); Writing – original draft (equal); Writing – review & editing (equal). **Roman Levchenko:** Conceptualization (equal); Investigation (supporting); Software (equal); Validation (equal). **Tomasz Kapitaniak:** Conceptualization (equal); Funding acquisition (lead); Methodology (supporting); Supervision (supporting); Writing – review & editing (equal). **Jürgen Kurths:** Conceptualization (equal); Methodology (supporting); Supervision (supporting); Writing – review & editing (equal). **Yuri Maistrenko:** Conceptualization (equal); Formal analysis (equal); Methodology (lead); Supervision (lead); Validation (equal); Visualization (supporting); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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