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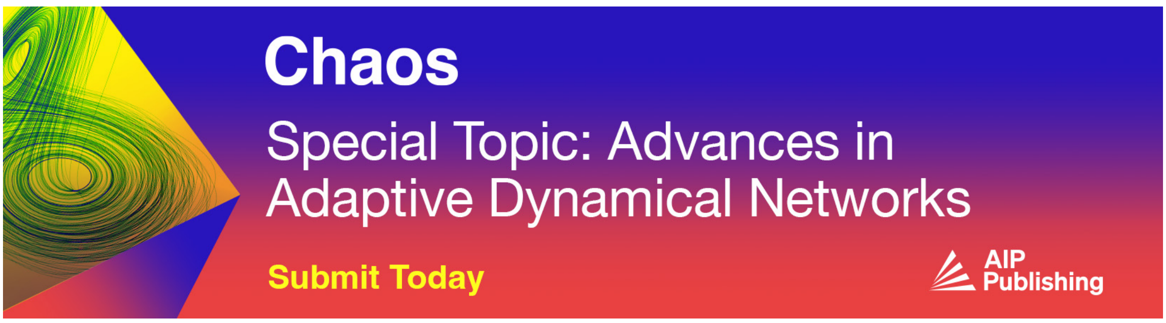

Mean exit times as global measure of resilience of tropical forest systems under climatic disturbances—Analytical and numerical results

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ABSTRACT

Both remotely sensed distribution of tree cover and models suggest three alternative stable vegetation states in the tropics: forest, savanna, and treeless states. Environmental fluctuation could cause critical transitions from the forest to the savanna state and quantifying the resilience of a given vegetation state is, therefore, crucial. While previous work has focused mostly on local stability concepts, we investigate here the mean exit time from a given basin of attraction, with partially absorbing and reflecting boundaries, as a global resilience measure. We provide detailed investigations using an established model for tropical tree cover with multistable precipitation regimes. We find that higher precipitation or weaker noise increases the mean exit time of the forest state and, thus, its resilience. Upon investigating the transition times from the forest state to other tree cover states, we find that in the bistable precipitation regime, the size of environmental fluctuations has a greater impact on the transition probabilities from the forest state compared to precipitation.

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We investigate how stochastic environmental noise and mean annual precipitation interact to induce transitions in tropical vegetation systems. The mean exit time for absorbing and reflecting boundaries is proposed as a global resilience measure and used to quantify the directional transition from forest to savanna states. We investigate the dependence of the mean exit time for mixed boundaries on mean annual precipitation and noise strength. We examine the effect of precipitation and noise strength on mean exit times and find that higher precipitation or weaker noise increases the mean exit time. In the tristable precipitation region, environmental fluctuations have a greater impact on transition probabilities compared to precipitation. The unstable state between forest and savanna is the state where the rate of change in transition time is the greatest.

I. INTRODUCTION

The Amazon rainforest is at risk of large-scale decline due to human deforestation and global climate change. A dieback of the Amazon would have severe consequences on regional climate but also globally in terms of the release of large amounts of carbon dioxide, which would further accelerate global warming.[1–4] Analysis of tropical tree cover fractions from MODIS satellite data suggests that there are three alternative equilibrium states, which correspond to forest, savanna (tree–grass mosaics), and a treeless (barren or grassy) state.[5,6] Fire and mean annual precipitation (mean annual precipitation) can be regarded as a self-stabilizing mechanism for creating an alternative stable state of tropical rainforest and savanna.[6–8] On the one hand, the grass and low tree cover in savanna enhances natural or man-made fires.[9] After a number of fires, a savanna ecosystem

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may be established. On the other hand, higher rainfall creates a moist understory microclimate that suppresses fires, thereby stabilizing the forest state itself.[10,11] In addition to precipitation, temperature fluctuations,[12] different soil types,[13] and biodiversity[14] are key variables determining the stability of different tropical vegetation states.

The perspective of the Amazon rainforest as a multistable dynamical system suggests that transitions between the alternative stable states, and, in particular, from the forest to the savanna state can be triggered by noise, representing fluctuations in environmental and climatic conditions, including droughts and fire activity. Such noise-induced transitions are fundamentally unpredictable yet become more likely as the rainforest's resilience decreases. Recent work based on satellite data suggests that the resilience of the Amazon rainforest has indeed declined in recent decades;[15] however, these results are based on the theory of critical slowing down and are, thus, based on local stability concepts. To explore the effect of noise on the transition behavior in the vegetation system, we focus here on the mean exit time (MET) as a global stability measure to quantify forest resilience. The mean exit time measures the expected time it takes for the system to cross a threshold such as the border of a given basin of attraction.[16,17] Instead of the time evolution of single stochastic paths, the mean exit time generalizes the macroscopic behavior of diffusing particles. Estimating the mean exit time is arguably a more useful way to characterize forest resilience perturbed by stochastic environmental variables than purely local measures such as those related to critical slowing down.[18] The expected lifetime of tropical forests is an intuitive resilience measure.[19] The mean exit time also provides a feasible method and technique for systems such as financial markets,[20] genetic transcriptional,[21] and climate.[22]

Mathematically, a diffusive system is a deterministic system perturbed by Gaussian noise. Following from large deviation theory in the small noise limit, the classic Arrhenius formula gives an approximate expression for the mean exit time, with a potential difference decay exponent.[16] For finite noise intensity, an elliptic partial differential equation with extra boundary has been proposed to describe the evolution of the mean exit time at different initial points.[23,24] Theoretical and numerical studies have investigated the mean exit time when a diffusive process is stopped after the first encounter with a boundary or a target domain.[17,25] Such a boundary is called absorbing condition or Dirichlet boundary condition, i.e., once a particle arrives at the boundary, it is immediately transferred, reacted, or relaxed. Instead, we consider here a more general case of partially absorbing and reflecting boundaries, corresponding to Dirichlet–Neumann boundaries.[26] This boundary condition is the natural setting to describe a rainforest, with reflecting boundary condition for very high precipitation (and tree cover) states and an absorbing boundary condition at the unstable fixed point that separates the rainforest from the savanna state. Once the vegetation system approaches such a boundary, it may either shift from the forest to the savanna state or be reflected back toward the forest states to remain for a longer time.

Our objective is to assess how stochastic environmental noise and mean annual precipitation may interact to induce regime shifts from forest to savanna. In Sec. II, we will propose the mean exit time satisfying the elliptic partial differential equation with two kinds of boundary conditions. In Sec. III, a simple differential equation model for tropical tree cover under stochastic fluctuations is introduced based on earlier works, in order to capture feedback between the climate and vegetation. Numerical experiments are conducted to investigate the impact of Gaussian noise and mean annual precipitation on the mean exit time with a mixed boundary for the forest state in Sec. IV.

II. MEAN EXIT TIME WITH BOUNDARY CONDITION

We consider a stochastic differential equation with Gaussian noise,

$$dX(t) = f(X(t))dt + g(X(t))dB(t), \quad X(0) = X_0. \tag{1}$$

The function $f(X(t))$ and $g(X(t))$ are referred to as drift and diffusion, respectively. The stochastic differential equation Eq. (1) has a unique solution if $f(X(t))$ and $g(X(t))$ satisfy the Lipschitz and linear growth conditions.[27]

Mean exit time. The first exit time is defined as the first time when X_t hits the boundaries of a bounded domain $D = (a, b)$ in $\mathbb{R}^1$,[24]

$$\tau(\omega, x) = \inf\{t \geq 0, X_t(\omega, x) \notin D\},$$

and the mean exit time is denoted as the mean value of the random variable $\tau(\omega, x)$ at the initial point x,

$$u(x) \triangleq \mathbb{E}\tau(\omega, x) \geq 0.$$

By the Dynkin formula for Markov processes, the mean exit time satisfies the elliptic partial differential equation for proper boundaries condition[16,17,24]

- a and b **both absorbing**

$$\begin{cases} f(x)u'(x) + \dfrac{g(x)^2}{2}u''(x) = -1, & x \in D = (a, b), \\ u(a) = 0, \quad u(b) = 0. \end{cases} \tag{2}$$

  The mean exit time of a particle initially at $x \in D$ shifts immediately when it hits the boundary, i.e., end points of $D = (a, b)$ at a or b in $\mathbb{R}^1$.

- a **absorbing and** b **reflecting**

$$\begin{cases} f(x)u'(x) + \dfrac{g(x)^2}{2}u''(x) = -1, & x \in D = (a, b), \\ u(a) = 0, \quad \dfrac{du}{dx}(b) = 0. \end{cases} \tag{3}$$

  The derivative at point b indicates the particle will be reflected back at this boundary. The transition occurs at the boundary $x = a$.

III. STOCHASTIC TREE COVER MODEL

A simple dynamic model has been proposed to capture feedback between climate and vegetation to describe tropical vegetation.[28,29] The model describes the dynamics of tree cover T (fraction) as a function of precipitation P in mm yr^{-1}. The model consists of a logistic growth function of tree cover with the expansion rate $r_m(\text{yr}^{-1})$ and two nonlinear loss terms. The first loss term is called the Allee effect, known in ecology to describe increased

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mortality at very low tree cover densities. The second loss term represents fire mortality. Staal *et al.* have proposed that the fire-induced tree cover mortality contains a variable landscape continuity $C(T)$ and a soil moisture index $SMI(P)$.[29] $C(T)$ is a Hill function representing the negative relation between tree cover and fire-induced mortality. Meanwhile, the flammability of fuel depends on the soil moisture $SMI(P)$, which is expressed as a S-shaped Hill function based on mean annual precipitation P,

$$\frac{dT}{dt} = \frac{P}{h_p + P} r_m T\left(1 - \frac{T}{K}\right) - m_A T \frac{h_A}{T + h_A} - T\frac{1}{FRI}\frac{C(T)^r SMI(P)^r}{h_I^r + C(T)^r SMI(P)^r}, \tag{4}$$

where the landscape continuity $C(T)$ is

$$C(T) = \frac{h_c^\beta}{h_c^\beta + T^\beta}$$

and the soil moisture index $SMI(P)$ is

$$SMI(P) = \frac{h_{SMI}^\alpha}{h_{SMI}^\alpha + P^\alpha}.$$

Additionally, we impose here some environmental noise representing, e.g., fluctuations in precipitation, local evapotranspiration, in the patterns of monsoon circulation,[30,31] or in strong El Niño events.[5] This is modeled as the Gaussian Brownian motion with amplitude ε. We, thus, obtain the following stochastic tree cover system:

$$dT = \left(\frac{P}{h_p + P} r_m T\left(1 - \frac{T}{K}\right) - m_A T \frac{h_A}{T + h_A} - T\frac{1}{FRI}\frac{C(T)^r SMI(P)^r}{h_I^r + C(T)^r SMI(P)^r}\right) dt + \varepsilon\, dB_t. \tag{5}$$

TABLE I. Description of parameters and model variables.[5,28,29]

Parameter	Description	Value
α	Power in the soil moisture index function	4
β	Power in the continuity function	6
γ	Power in the fire-induced mortality term	6
FRI	Fire return interval	7 yr
h_A	Half saturation of the Allee effect	10%
h_C	Half saturation of grass (non-forest) cover continuity	57%
h_I	Half saturation of the fire-induced mortality term	0.15
h_p	Half saturation of the growth term	80 mm yr^{-1}
h_{SMI}	Half saturation of the soil moisture index	1800 mm yr^{-1}
K	Maximum tree cover	90%
m_A	Mortality due to the Allee effect	0.15 yr^{-1}
r_m	Maximum tree-cover growth rate	0.3 yr^{-1}

All parameters are given in Table I, for which parameter values in the model are provided through fitting tree cover data. The parameters in logistic growth function are from Van Nes *et al.*,[28] and the parameters value for the fire term are from Hirota *et al.*[5] and Staal *et al.*[29]

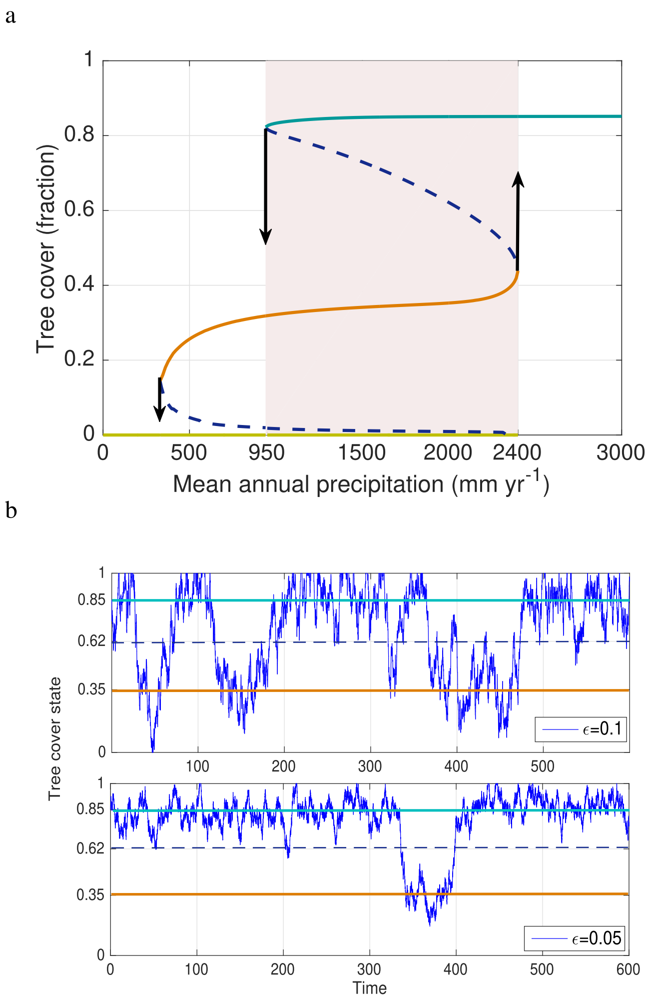

FIG. 1. (a) Bifurcation diagram of the tree cover model introduced in Sec. III, giving the stable tree cover states as functions of mean annual precipitation. There are three stable states: forest (turquoise line), savanna (orange line), and treeless state ($T = 0$ yellow line). Stable states are represented by solid curves and unstable states are dashed. Fold bifurcations show classical tipping points for transition between them. Within the bistable regime, roughly $950 < P < 2400$ mm yr^{-1} for the employed model parameters (pink), both forest and savanna can be maintained. (b) Example trajectories for $P = 2000$ mm yr^{-1}, showing transitions between the stable forest state at $T_F = 0.851$ and the savanna state at $T_S = 0.351$, triggered by Gaussian noise with strength $\varepsilon = 0.1$ and $\varepsilon = 0.05$, respectively. The green solid line, orange solid line, and dashed line present the forest stable state T_F, savanna stable state F_S and unstable state T_u, respectively.

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IV. RESULTS

We illustrate the dynamical behavior of tree cover by examining the three equilibrium states of the model in Eq. (4), namely, forest (green line), savanna (origin line), and treeless state ($T = 0$) (yellow line), cf. Fig. 1(a). Distinct vegetation states are dominant for particular mean annual precipitation ranges. For precipitation levels above 2400 mm yr^{-1}, the natural vegetation is almost exclusively tropical forest, whereas savanna with sparse tree cover dominates regimes with precipitation values below 950 mm yr^{-1}. For mean annual precipitation values in between $950 < P < 2400$ (pink area), forest, savanna, and treeless can all be maintained. The bifurcation points mark the critical mean annual precipitation values for which branches of equilibria meet and vanish. A system gradually approaching a bifurcation point presents a critical transition from one state to another. In the tri-stable precipitation range, transitions between forest, savanna, and treeless states can be triggered by stochastic fluctuations. Equation (5) satisfies the Lipschitz and linear growth conditions, thus, we can use the Euler–Maruyama scheme with a time step of 0.01. For $P = 2000$ mm yr^{-1}, $\varepsilon = 0.1, 0.05$, we obtain stochastic sample paths showing noise-induced transitions from the forest state [Eq. (5)] $T_F = 0.851$ across the unstable state $T_u = 0.621$ to the savanna state $T_S = 0.351$. The probability of transitions increases with the noise intensity and, hence, the strength of the fluctuations increases [Fig. 1(b)].

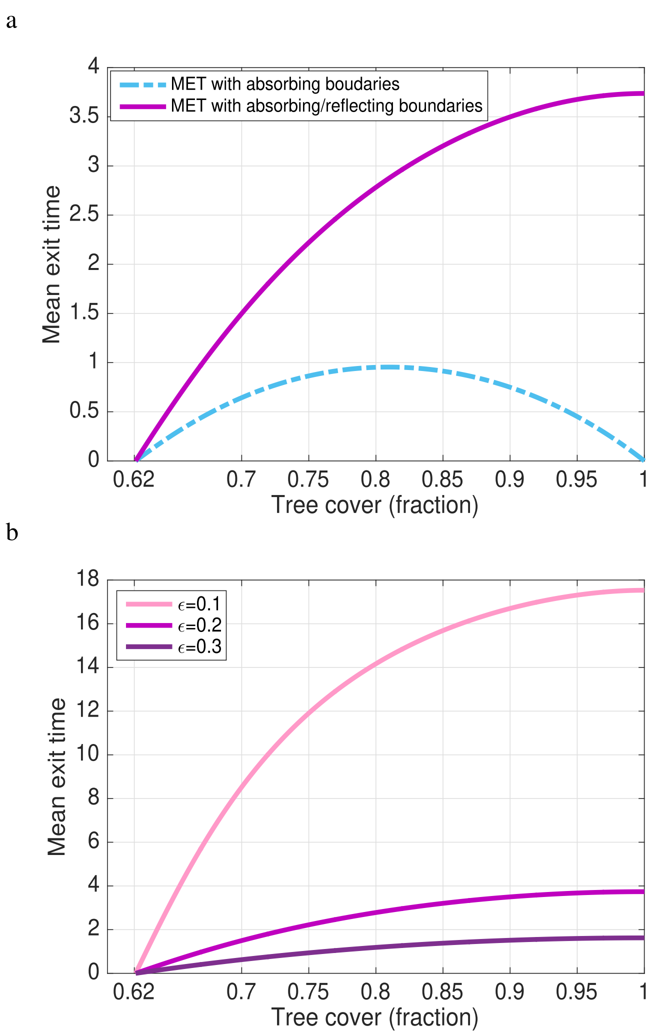

FIG. 2. (a) Mean exit times for absorbing and reflecting boundaries defined by Eq. (3) (magenta) and both absorbing ones defined by Eq. (2) (cyan) in the forest basin of attraction $D = (0.621, 1)$ with $P = 2000$ mm yr^{-1} and $\varepsilon = 0.2$. (b) Mean exit time for absorbing/reflecting boundaries in the forest basin $D = (0.621, 1)$ for $P = 2000$ mm yr^{-1} and $\varepsilon = 0.1, 0.2, 0.3$.

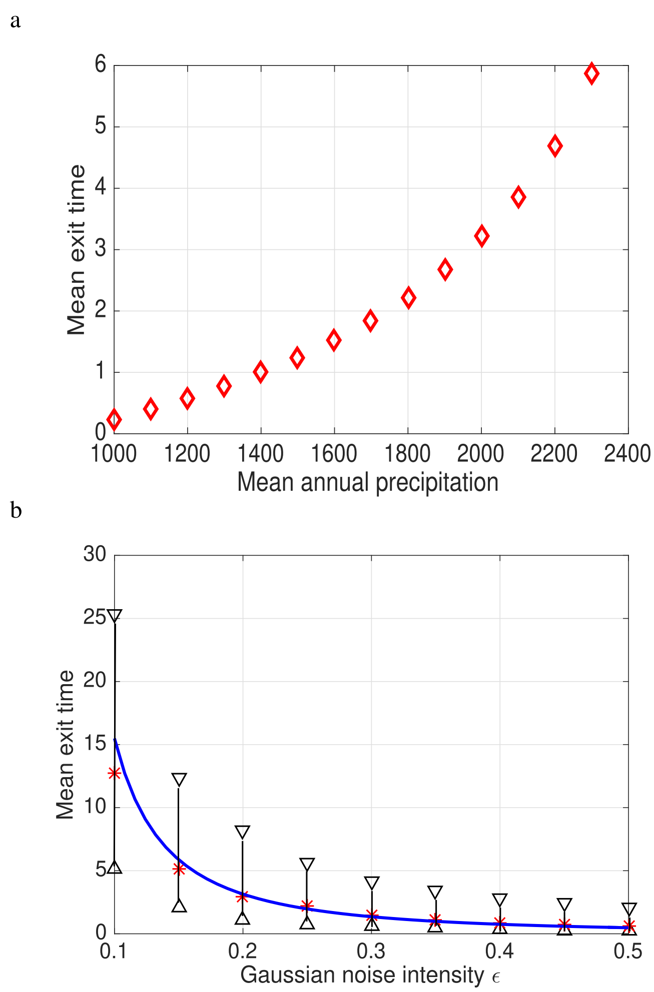

FIG. 3. Dependence of the mean exit time on mean annual precipitation and noise strength ε, for mixed boundaries defined by Eq. (3) starting at the forest stable state T_F in the forest basin $D = (T_u, 1)$. (a) The mean annual precipitation changes in the tristable region for $\varepsilon = 0.2$. (b) Gaussian noise strength ε varies from 0.1 to 0.5 for $P = 2000$ mm yr^{-1}. The blue line shows the analytical solution. The red stars denote the medians of the simulated exit times, the triangles are the 25th and 75th percentiles.

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In Fig. 2(a), we compare the mean exit time for mixed boundaries defined by Eq. (3) and absorbing ones defined by Eq. (2) in the forest basin $D = (0.621, 1)$. Here, $T_u = 0.621$ is an unstable state between the forest and savanna basins of attraction for $P = 2000\,\mathrm{mm\,yr^{-1}}$. We assume that $T_u = 0.621$ is an absorbing boundary, i.e., the system will shift immediately from the forest state to the savanna state at this boundary. At the tree-cover boundary $T = 1$, the exit time is 0 at the border for both absorbing boundaries. By contrast, the derivative of mean exit time is 0 for the reflecting boundary. The comparison indicates that the value of $u(1)$ is different due to the selection of the boundary at $T = 1$. The reflection condition keeps the system in the forest basin, thus it takes longer in this case for the system to escape from the forest basin. Considering the units of the parameters in Table I, dimensional analysis of Eq. (4) reveals that the time unit used here is in years. We refer to the mean exit time as the mean time for all random trajectories starting from initial values x to exit the region. Thus, it only depends on the initial position. This is well demonstrated in Fig. 2(a), where we can calculate the mean time for each vegetation cover within the forest basin to shift to the savanna under the influence of environmental noise.

Furthermore, Fig. 2(b) illustrates the mean exit time for mixed boundaries in the forest basin $D = (0.621, 1)$ for $P = 2000\,\mathrm{mm\,yr^{-1}}$ with varying noise strengths $\varepsilon = 0.1, 0.2, 0.3$. For a fixed starting state in the forest basin D, the mean exit time increases as the noise intensity decreases, keeping other factors the same.

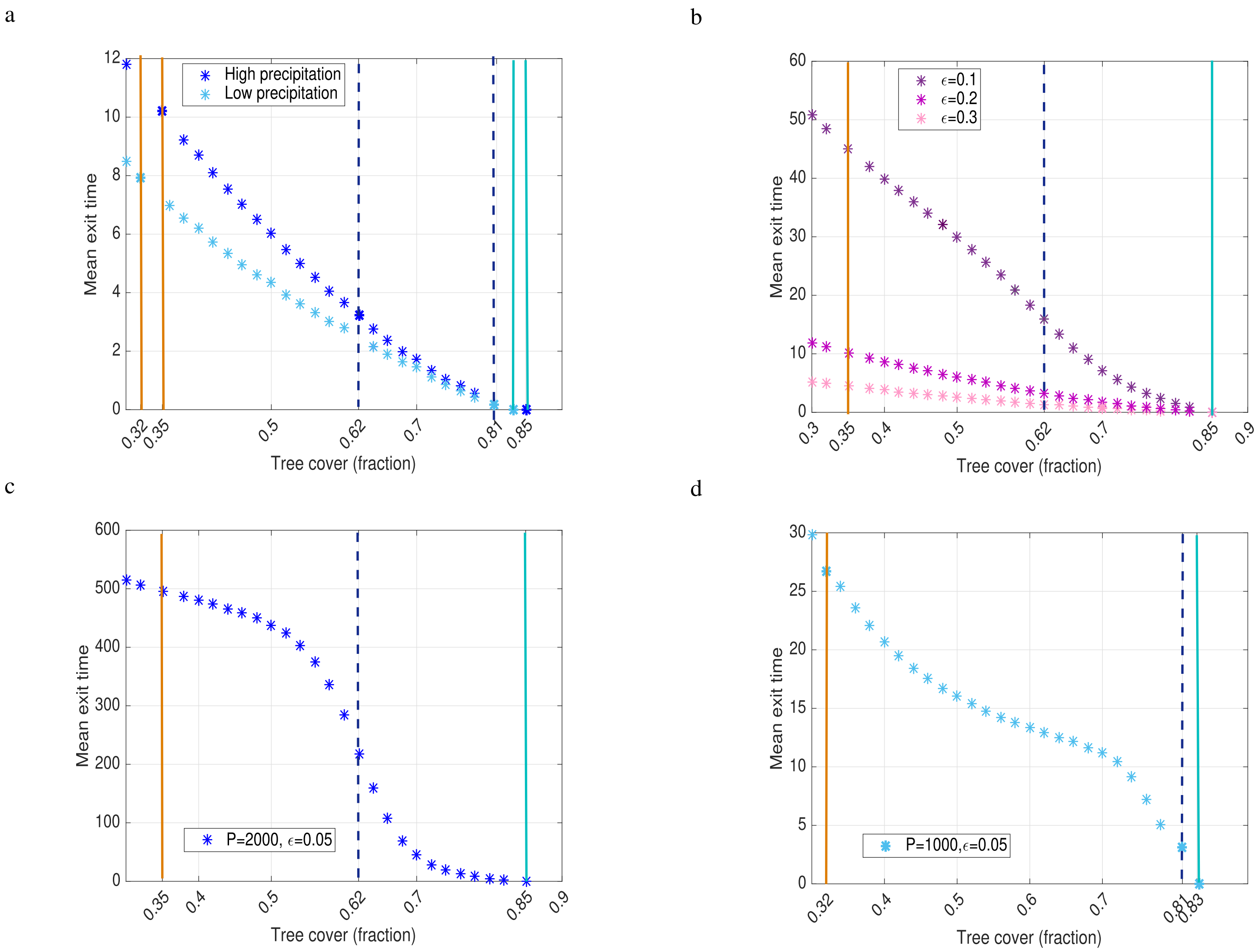

FIG. 4. Effect of mean annual precipitation and noise strength ε on mean exit time shapes for mixed boundaries staring from the forest state T_F to a target state $u(T_F \to T)$ defined by Eq. (3), indicated by the values on the x-axis. (a) For high ($2000\,\mathrm{mm\,yr^{-1}}$) and low precipitation ($1000\,\mathrm{mm\,yr^{-1}}$) and $\varepsilon = 0.2$. (b) For $P = 2000\,\mathrm{mm\,yr^{-1}}$, $\varepsilon = 0.1, 0.2, 0.3$. (c) For $P = 2000\,\mathrm{mm\,yr^{-1}}$, $\varepsilon = 0.05$. (d) The same as (c) except $P = 1000\,\mathrm{mm\,yr^{-1}}$. The green solid line, orange solid line, and dashed line present the target are forest stable state T_F, savanna stable state F_S, and unstable state T_u, respectively.

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Therefore, we mainly focus on the changes in the mean exit time for mixed boundaries, starting at the forest stable state T_F, in the following.

To illustrate the dependence of the mean exit time for mixed boundaries on the mean annual precipitation P and the noise intensity ε, we examine the mean exit time starting at the forest stable state T_F and exiting from the forest basin $D = (T_u, 1)$ through the boundary T_u. Here, the absorbing boundary T_u is the unstable state between the forest and savanna basins.

The modeled tree cover distributions for different precipitation ranges show that, although there is an overall increase of average tree cover with precipitation, the distinct character of the three states remains for P changing in the tri-stable region from 1000 to 2300 mm yr^{-1}. Figure 3(a) demonstrates that the mean exit times are longer for larger values of P with fixed Gaussian noise $\varepsilon = 0.2$. The behavior is in agreement with the corresponding result in the relationship between ecosystem pattern and precipitation.[5] It implies that higher annual rainfall sums increases forest resilience under the same strength of random environmental perturbations. With the precipitation reductions projected by the IPCC,[32] transitions from forest to savanna will thus become more likely in the future.

Additionally, we investigate the effect of local climate or environmental noise fluctuation strength ε on the mean exit time for mixed boundaries starting at T_F. Keeping the mean annual precipitation fixed at $P = 2000$ mm yr^{-1}, we obtain a stable forest state at $T_F = 0.851$, the unstable state at $T_u = 0.621$, which we identify with the absorbing boundary. Clearly, as the strength of Gaussian noise ε increases, the mean exit time decreases as shown in Fig. 3(b). We also simulated 1000 stochastic paths starting from the forest stable state $T_F = 0.851$ using the Monte Carlo method. The first time when it reaches the absorbing boundary $T_u = 0.621$ is statistically reported in terms of the median (red star), and the 25th and 75th percentiles (triangles). The distribution characteristics of the numerically obtained values agree well with our analytical results for the mean exit time. It implies that the exit time distribution characteristic of forest state shift to savanna can be measured by the mean exit time.

Given a starting at the metastable forest state T_F, it is of interest to know the transition time from T_F to other tree cover states $u(T_F \to T)$ defined by Eq. (3), where T is a target state in the vicinity of the unstable state. The transition time $u(T_F) = 0$ indicates starting from T_F.

We investigate this for a high precipitation ($P = 2000$ mm yr^{-1}) and low precipitation regime ($P = 1000$ mm yr^{-1}). Figure 4(a) shows that the transition time gradually increases as the tree cover state approaches the savanna basin. Meanwhile, transition times increase with larger mean annual precipitation, as in Fig. 3(a). For $P = 2000$ mm yr^{-1}, we analyze the effect of the noise intensity on the transition time from the forest state $T_F = 0.851$ to other tree cover states $u(0.851 \to T)$. In Fig. 4(b), weak noise will make the transition time longer compared with strong noise. Compared to the impact of mean annual precipitation on the mean exit time mentioned above, it is worth noting that the change in transition time is considerably larger in response to the noise. Therefore, in the tristable precipitation range, our results show that random noise perturbations play a major role in triggering transitions from the forest to savanna, compared with the change of precipitation.

Finally, we examine the transition time $u(T_F \to T)$ for smaller noise intensity $\varepsilon = 0.05$. For $P = 2000$ and $P = 1000$ mm yr^{-1}, the transition time, considered as a function of the transition target state T, changes most strongly at the unstable state T_u (blue dashed line) as shown in both Figs. 4(c) and 4(d). For larger precipitation, the transition time changes relatively slowly in the forest basin and the savanna basin. These results mean that the rate of change of transition time is the greatest in the vicinity of the unstable state that serves as the boundary between forest and savanna basins.

V. CONCLUSIONS

We considered the mean exit time with both absorbing and reflecting boundary conditions as a global measure of the resilience of tropical forests and exemplified it using a differential equation model of tropical tree cover, exhibiting three alternative stable states. We added white Gaussian noise to this model to mimic the effect of environmental and climatic perturbations. The mean exit time is a deterministic quantity used to capture the stochastic behavior of the system, with a focus on the global stability properties of alternative basins of attraction. It corresponds to the expected time it takes for the system to cross a threshold between the different basins. We examine the deterministic and stochastic components of the underlying tree cover dynamical system. The stochastic environmental fluctuations are modeled by Brownian Gaussian noise with varying noise intensity.

Compared to the case where both sides of a given domain are absorbing boundaries, the reflecting boundary condition keeps the system in the forest basin and it thus takes longer time for the system to escape from the forest state. We investigated the effect of mean annual precipitation and noise intensity on the mean exit time of the forest state. On the one hand, under fixed environmental fluctuations, it takes longer for the forest state to shift to the savanna state under higher precipitation. On the other hand, keeping MAP fixed, we find that the transition from forest to savanna is more likely to occur for stronger environmental fluctuations. Furthermore, we investigated the transition times from the forest state to other tree cover states $u(T_F \to T)$. In the bistable precipitation range, we find that environmental fluctuations play a major role in the transition from the forest to savanna compared with the change of precipitation.

Our results show that the mean exit time should indeed be suitable as a global measure of tropical forest resilience. The approach thus provides a valuable complement to local notions of resilience based on the framework of critical slowing down, such as the lag-one autocorrelation, see Boulton *et al.*[15] and Smith *et al.*[33] We would like to emphasize that the research results described in our paper, obtained from an established model of tropical rainforest dynamics, are consistent with the results of empirical data analysis and ecological understanding. Our theoretical framework is suitable and can be readily applied to observational data, once the potential problems with the empirical data are solved. We are actively working on an application of our method to real-world data. Given the complexity of the problem, we believe that this is beyond the scope of the present paper and outline this as a highly interesting topic for future research.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Yayun Zheng: Formal analysis (equal); Methodology (equal); Writing – original draft (equal). **Niklas Boers:** Conceptualization (equal); Investigation (equal); Writing – review & editing (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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