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A Dynamic Network Model of Societal Complexity and Resilience Inspired by Tainter's Theory of Collapse

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Abstract: In recent years, several global events have severely disrupted economies and social structures, undermining confidence in the resilience of modern societies. Examples include the COVID-19 pandemic, which brought unprecedented health challenges and economic disruptions, and the emergence of geopolitical tensions and conflicts that have further strained international relations and economic stability. While empirical evidence on the dynamics and drivers of past societal collapse is mounting, a process-based understanding of these dynamics is still in its infancy. Here, we aim to identify and illustrate the underlying drivers of such societal instability or even collapse. The inspiration for this work is Joseph Tainter's theory of the "collapse of complex societies", which postulates that the complexity of societies increases as they solve problems, leading to diminishing returns on complexity investments and ultimately to collapse. In this work, we abstract this theory into a low-dimensional and stylized model of two classes of networked agents, hereafter referred to as "laborers" and "administrators". We numerically model the dynamics of societal complexity, measured as the fraction of "administrators", which was assumed to affect the productivity of connected energy-producing "laborers". We show that collapse becomes increasingly likely as the complexity of the model society continuously increases in response to external stresses that emulate Tainter's abstract notion of problems that societies must solve. We also provide an analytical approximation of the system's dominant dynamics, which matches well with the numerical experiments, and use it to study the influence on network link density, social mobility and productivity. Our work advances the understanding of social-ecological collapse and illustrates its potentially direct link to an ever-increasing societal complexity in response to external shocks or stresses via a self-reinforcing feedback.

Keywords: societal complexity; social-ecological collapse; resilience; network model; agent-based model

1. Introduction

Human societies have always faced a wide variety of challenges that have tested their resilience and resulted in a median longevity of premodern societies of around 200 years [1]. These challenges include external stresses such as invasions and environmental catastrophes as in the case of Mesopotamia [2] or internal pressures such as corruption, rebellion and mismanagement, as in ancient Egypt [3]. Historically, both cases led to societal collapse. Other classic and widely studied examples include the fall of the Western Roman Empire in the 5th century CE, the collapse of the Maya in the 9th century CE or the fall of the Minoan civilization of Crete in the 14th century BCE [4,5]. Many more historical civilizations and

their trajectories have also been studied extensively in order to identify common factors of collapse [6].

Modern societies face similarly severe challenges, such as global climate change, pandemics, or financial instability. The estimated costs of climate change [7] alone will put additional strain on already overindebted nation states [8]. While the 2008 housing crisis was still reverberating in several countries, the COVID-19 pandemic has caused major impacts on health systems and may have long-term consequences for economies, political institutions and social structures [9].

The above examples illustrate the long-standing question of whether it is possible to identify underlying principles that determine a society's ability to cope with such large-scale challenges and thus increase its resilience to collapse. As reviewed by Cumming and Peterson [10], there are several explanatory models for collapse. Explanations focusing on resource limitations such as the classical Malthusian trap [11] and limits to exponential growth and overuse of resources have been widely discussed in the past [12–15]. But climate change, extreme events, inequality, overpopulation, social tension, revolutions and wars also point to collapse, e.g., [1,16,17]. These theories can be broadly divided into external and internal drivers of collapse [1].

Tainter's *theory of the collapse of complex societies* specifically emphasizes the notion of societal complexity and its tendency to self-amplify in response to stress as the primary cause of collapse [4,18]. Within this framework, complexity emerges continuously through problem solving and is manifested in the differentiation and specialization of social roles, hierarchies and control of behavior, growing population, technical abilities and increased information flows. These investments in complexity have associated costs, such as calories, natural resources, time or money, all of which can be reduced to an abstract form of *energy*. Although increases in complexity can be beneficial to societies and contribute to well-being, it is hypothesized that the surpluses generated by increases in complexity will diminish as the "low-hanging fruits" become fewer. Thus, diminishing marginal returns to investments in complexity (or simply returns on complexity *ROC*) ultimately drive a society into collapse due to a loss of resilience to external perturbations or internal crises. In addition, the benefits of complexity may be short-lived and quickly consumed by population growth or increased living standards. In contrast, increased energy demand tends to be persistent [19], leading to new problems once previously earned energy surpluses are used up. Within the framework proposed in Tainter [4], this process is described as the *energy–complexity spiral* [18].

Contemporary evidence for complexity gains in specialization and differentiation of social roles as well as technical abilities are the decline of labor in the agricultural sector and increase of the financial and communications sectors in the 20th century, while productivity in primary sectors has increased [20]. Further evidence of complexity increase is the growing burden of bureaucracy. In the U.S., the number of administrative regulations increased linearly from 1950 to 2000 by approximately 2500 pages per year [21]. For example, in cancer research, complexity is introduced by formalities or regulations for opening trials. Such complexity increments diminish the returns on investment (ROI) of research, resulting in reduced innovativeness in the field [22]. Hall et al. [23] estimates the energy return on investment (EROI) of a sustainable society to be well above 3:1, which can be interpreted as a proxy for complexity. Despite these examples, overall empirical evidence on societal complexity and its change over time is scarce, even though it seems ubiquitous in our lives.

Mathematical models and simulations are important tools for exploring the implications of different theories of collapse and resilience and for testing competing hypotheses about the processes underlying the dynamics of societal complexity and collapse. In the archaeological domain, agent-based models are employed to understand the interaction of water and land-use change [24] or to compare trajectories of biocultural evolution [25]. Dynamical systems models are frequently employed to model small-scale social-ecological systems [26–28]. Examples include a resource economics model of the collapse of the Rapa Nui population [29] or an exploration of vulnerabilities in forager–resource systems [30].

More recently, Motesharrei et al. [31] developed a dynamical systems model to study trajectories to collapse in a coupled hierarchical consumer–resource system. Similarly, stylized models of coupled carbon, population and capital stocks have been used on a planetary scale to identify preconditions for societal collapse and sustainability and their implications for the world–Earth system resilience [32,33].

In the last decade, a new generation of models has been conceptualized turning to larger and up to global scales and embracing social-ecological complexity by integrating concepts from dynamic land-use models and agent-based models [34,35]. Building on this framework, Brown et al. [36] identified societal breakdown as an emergent property of large-scale behavioral models of land-use change under different climate-economic scenarios. To strengthen the human side of the equation(s), much attention has been paid to increase realism in modeling human decision-making in socio-economic and social-ecological models [37–40] as well as modeling collective behavior and social tipping dynamics [41]. Recently, these efforts have culminated in a comprehensive modeling framework for so-called World–Earth models [42]. The work of Tainter has been previously approached from a system dynamics angle [43], focusing on complexity increase by the growth of a nonbeneficial bureaucracy class and the overuse of a limited resource, which lead to diminishing marginal returns and collapse.

In this work, we take an alternative approach and explore the theory of *collapse of complex societies* in terms of the mechanisms that lead to increasing societal complexity, diminishing resilience and ultimately collapse. To this end, we employ an agent-based network model with worker and administrator nodes connected by edges. We introduce a complexity-generating mechanism wherein added complexity through converting workers to administrators yields positive feedback, enhancing the productivity of connected worker nodes. This implementation successfully recaptures the theory's postulated behavior of collapse due to diminishing marginal returns to complexity. We then study its macroscopic dynamics in detail and identify two escape mechanisms from collapse: increasing the productivity of labor or allowing social mobility by stochastic transitions of nodes between productive and administrative states. In order to approach the theory of societal collapse from a conceptual angle, we propose here to reduce it to three key elements:

1. The basic currency of any society is energy, since labor and material goods can be viewed as driven by or derived from energy.
2. Problems need to be solved when energy availability is deficient as a consequence of stochastic events or shocks. According to the self-reinforcing process denoted as *energy–complexity spiral* (see above), this process always increases societal complexity.
3. Increases in complexity can be modeled as increases in administrative capacity because they encapsulate increases in specialization of social roles, hierarchies and control, and information flow. Moreover, the size of an administrative body relative to the overall size of the system is a very tangible example of complexity.

The remainder of this paper is organized as follows. We begin with a thorough description of our proposed network model in the next section. We first present the basic model and then describe a model variant that includes a mechanism to counteract collapse. In the results section, we first present the stochastic model by discussing exemplary trajectories of the system followed by the derivation of an analytical approximation. Based on this approximation we present a comprehensive ensemble analysis of all crucial model parameters. The paper concludes with a discussion of all relevant results and an outlook on future work.

2. Model Description

2.1. Tainter Inspired Network Model of a Hierarchical Society Steering into Diminishing Marginal Returns and Collapse

In the following, we introduce the model that is used in this work to exemplify the dynamics of collapse proposed in Tainter [4]. Its goal is to illustrate the increase in societal complexity as a response to the solving of problems, here the sustaining of a certain level

of energy supply *E*. We define complexity as an increase in the diversification of the society (the creation of a new role) and the corresponding increase in control over the behavior of others (an increasing administrative body). We show that by only including these two processes, one already observes an increase in an appropriately defined measure of complexity and hence, we do not need to specifically account for additional phenomena, such as specialization or an increase in hierarchies.

In particular, we represent an artificial society by means of a complex network [44,45] as complex networks have been successfully used in the past to model social systems with respect to collapse [46], sustainability [47] and their ability to adapt to new conditions [48]. They are usually comprised of entities of two basic types, i.e., nodes and links. For our case the nodes indicate (representative) small but long-lived entities that supply labor, such as family lines, in a society. Links between the nodes or entities represent some comparatively stable social tie, e.g., a professional relationship, between them. *N* counts the total number of nodes in the network and remains fixed throughout the entire simulation (Figure 1).
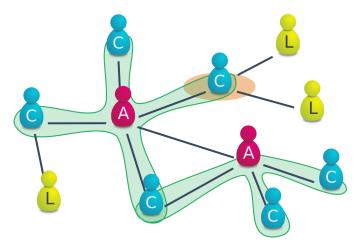

Figure 1. Sketch of the model setup. Nodes/agents are either attributed to *administrators A* or *laborers L*. laborers that are connected to at least one administrator are denoted as *coordinated laborers C* that produce higher energy outputs due to an increased productivity. The influence of each administrator is marked with green shading. In case the energy production falls below a critical value, the coordinated laborer that is connected with most other nodes (here marked in orange) becomes an administrator as well.

The sole goal of the society is to produce energy *E* at a certain level in order to sustain its function. The corresponding per "capita" (actually, per node) energy requirement of the society is denoted as ϵ. In order to produce this energy, certain nodes (denoted as laborers *L*) harvest an individual energy source R_{max}. R_{max} is subject to external shocks that represent an abstract form of a problem that needs to be solved. In order to increase the energy that is produced from the resource, some laborers *L* can also be appointed to become administrators *A*. The purpose of the administration is to increase the productivity of the laborers *L* that are under its direct management.

To describe energy production in the present model we draw upon a commonly used utility function in economic models, Equation (2). The equation considers the *productivity* of labor, scaling linearly with the number of workers, and the *output elasticity to scale*, which is a measure of the marginal change in energy production resulting from each additional coordinated worker. Output elasticity to scale is typically used to describe scaling in production process in economy and scales exponentially with increases in production factors, e.g., the number of laborers. We assume each laborer *L* who is directly connected with at least one administrator *A* becomes part of a more productive body of the labor force which has a larger total factor productivity, expressed by a productivity factor $c > 1$ and an output elasticity to scale of labor of *b*, where the rest of the labor force has an output elasticity to scale of labor of $a \leq b$. In line with common economic models, we assume

overall decreasing returns to scale, i.e., $a \leq b < 1$. We denote this class of more productive laborers the *coordinated laborers C* (Figure 1). While they increase the productivity of their immediate surroundings, the administrators A themselves do not produce energy any longer (thus indicating a certain cost of maintaining an administrative body). Note that being additionally connected with more than one administrator A does not further increase the productivity c of the coordinated laborers C. Along the lines of Tainter [4], we measure the resulting societal complexity S in terms of the size of the administration $N_A = |A|$ as it represents the level of behavioral control in our model society.

We model the temporal dynamics within our artificial society according to the following rules. At each time step t, the maximally available resource per node, R_{max}, is reduced by randomly drawn disturbances from a beta distribution with parameters $\alpha = 1$ and $\beta = 15$.

$$R = R_{max}(1 - B), \text{ with } B \sim \text{Beta}(\alpha, \beta) \tag{1}$$

The stochasticity of R represents the occurrence of shocks or problems to be solved by the model society. Larger values of B represent larger shocks or problems. We have chosen the parameters for the beta distribution such that drawing shocks mostly results in R close to R_{max} (i.e., no or small problems) with a rare chance of $R \ll R_{max}$ (i.e., large problems). Depending on the number of laborers $N_L(t)$ (uncoordinated) and $N_C(t)$ (coordinated), the total produced energy $E(t)$ is computed as

$$E = R(N_L^a + cN_C^b), \tag{2}$$

where a and b determine the output elasticity to scale of the nodes L and C, respectively, and c is the productivity factor of coordinated workers C. R_{max} is scaled by the output elasticity of uncoordinated workers $R_{max} = N/N^a$ to initialize the unperturbed per capita energy production at $E_{cap} = 1$. If the energy per capita falls below a minimum value representing the society's vital needs, i.e., if $E(t)/N < \epsilon$ for some parameter ϵ, we assume the society tries to solve this problem by appointing exactly one (additional) administrator such that the productivity of some laborers is increased. The new administrator is selected as follows:

(a) If currently, no administrators exist in the society (i.e., if $N_A = 0$), the laborer L with the most network connections becomes the sole administrator.
(b) If at least one administrator currently exists (i.e., if $N_A > 0$), the coordinated laborer who has the most network connections becomes an additional administrator.

The model then proceeds to the next time step and the availability of R as well as the produced energy $E(t)$ are again computed according to Equations (1) and (2).

Following [18], we define the potential onset of societal collapse, i.e., the vicinity to a critical state, once the system approaches diminishing returns on complexity ROC. We operationalize ROC into our model by computing the differences in energy E and complexity S after the recruitment of one additional administrator A at time t,

$$ROC(t) = \frac{E(t) - E(t-1)}{S(t) - S(t-1)}. \tag{3}$$

Note that $ROC(t)$ depends on the specific structure of the underlying social network, in particular, on whom the newly appointed administrator A is connected to and how many laborers turn from L to C as a consequence. As outlined above, we measure societal complexity S simply in terms of the size of the administration N_A, i.e., $S(t) = N_A(t)$. Since the size of the administration is increased by one due to the recruitment of a single additional administrator at time t, we obtain $N_A(t) = N_A(t-1) + 1$ such that $S(t) - S(t-1) = 1$. Therefore, the return on complexity ROC reduces to $ROC(t) = E(t) - E(t-1)$.

Once $ROC(t)$ becomes negative, our artificial society is expected to begin its decline into collapse since the energy production E becomes smaller and approaches zero with

increasing time t and a correspondingly ever-increasing size of the administration N_A. In the following section, we therefore directly display results for the energy production $E(t)$ over time and interpret a positive slope as increasing returns to complexity and a negative slope as decreasing returns to complexity. In the scope of this model, we define collapse as the state where $E = 0$. While this is unrealistic in a real-world scenario, it serves as a reasonable endpoint for the abstract model proposed here, since qualitative differences are not expected when using collapse signals between $ROC \leq 0$ and $E = 0$.

For the initial model setup, we considered an Erdős–Rényi random network [49]. It consisted of N ($N = 400$ for our case) nodes that were all initially assumed to be laborers ($N_L(0) = N$, $N_A(0) = N_C(0) = 0$). Additionally, we placed a link between each pair of nodes with a fixed probability ρ (which we varied between $\rho = 0$ and $\rho = 0.1$ for our analyses). Thus, ρ gives the expected density of links in the resulting network, while $N(N-1)/2$ is the maximum number of possible links. Starting from this setup, we simulated the system for a maximum of $t_{max} = 10{,}000$ time steps following the model logic that was given above. In the case when all nodes becomes administrators (i.e., if $N_A(t) = N$), we stopped the simulation even though t_{max} may not have been reached. In that case, no more energy is produced ($E = 0$ due to the lack of laborers L), and we considered the society to be entirely collapsed.

The code of the entire model and the described analysis is publicly available at https://github.com/flo-schu/tainter (accessed on 12 December 2023) including instructions for downloading pre-simulated datasets. Table 1 gives the parameters used in the simulation unless otherwise given.

Table 1. Model parameters, their function in the model and their values used in the analysis unless otherwise mentioned. The values in squared parentheses express the range scanned for the parameter analysis shown in Figure 4.

Parameter	Function	Value
N	Network size (number of nodes)	400
t_{max}	Maximal runtime of the simulation (time steps)	10,000
α	Parameter of shock regulating the Beta distribution	1
β	Parameter of shock regulating the Beta distribution	15
ϵ	Energy threshold for coordinating a new A	1.0
a	Output elasticity to scale of uncoordinated workers L	0.75
b	Output elasticity to scale of coordinated workers C	0.75
c	Productivity of coordinated workers C	1.05 $[1.0, \ldots, 3.0]$
ρ	Link probability between nodes	0.02 $[0.0, \ldots, 0.1]$
p_e	Exploration probability between node states	0.00 $[0.0, \ldots, 0.2]$

2.2. Social Mobility as a Possible Countermeasure to Collapse

The basic model that we introduced in the previous section represents closely the assumptions put forward in Tainter [4]. Here, a society may get caught in the energy–complexity spiral as the increased or even flat demand for energy increases its complexity (measured as the size of the administrative body $N_A(t)$) when being faced with ever new problems.

One obvious downside of the above model setup is that once selected, an administrator A does not change its role back to become a laborer L (or C) again (ratchet effect). This decision is motivated by findings that social mobility is predominantly stable or upward rather than downward within [50] and across generations, e.g., [51]. Additionally, it is only possible to become an administrator A if one is selected from the set of coordinated laborers C, meaning that a spontaneous jump in hierarchy is not permitted within the logic of the model.

In order to increase social mobility, we deviated from Tainter and aimed to investigate a version of our model that allowed for the nodes to randomly change their state with a fixed small probability. Such dynamics can be interpreted as the loss or gain in socio-

economic status within or across generations. Within the logic of our model, such an additional process was implemented as follows. At the beginning of each time step t, every node changes its state with probability p_e either from A to L or C (depending on whether it is connected to another administrator), or from L or C to A. We expected that this would allow us to regulate the complexity of the society and possibly avoid its unlimited growth (as for this setting, the complexity may also decrease as the problems/shocks become smaller again). Hence, such process should cause a sustainable ratio between the size of the administration N_A and the body of laborers $N_L + N_C$. Note that if we set $p_e = 0$, we again obtain the dynamics of the model setup outlined in the model description.

3. Results

3.1. Example Trajectories

Figure 2A shows the implementation of the original approach to Tainter's theory of societal collapse with dynamics described above in the model description. Initially, the model society benefits from its response to external shocks. The reason is that initially, each new administrator A has a much higher number of connections to laborers L, which are thus turned into coordinated laborers C producing an energy output at increased productivity (here, $a = 1.05$). As the number of laborers L decreases, the marginal returns on complexity (*ROC*) of each new administrator A decreases until it becomes negative. Figure 2B displays the energy output as a function of the administration, which resembles well the assumed underlying principle of diminishing marginal returns on complexity in Tainter [18]. In our first approximation to the theory of collapse, the model society can only react to shocks by choosing the best possible administrator each time the energy requirements are not met. We interpret the formation of an administrative body as an increase in societal complexity. After a period of increased energy output, marginal returns decrease while correspondingly, the share of administration rises, caught in an energy–complexity spiral until the society collapses (here, $E = 0$) (Figure 2A).

Next, we studied whether a small adaptation mechanism of the model society enabled the model to overcome the fast collapse shown in Figure 2A. For this purpose, we allowed for a random exploration of node states enabling the model agents to change their state ($A \longrightarrow L/C$ or vice versa) with a low probability ($p_e = 0.00275$). For a network of $N = 400$ individuals, this amounted to an average of 1.1 nodes changing their status at random per time step.

As Figure 2C shows, an exploration rate of this low magnitude can already be sufficient to delay the collapse that follows from the event-triggered selection of administrators indicated by a higher survival time of the society; compare Figure 2A–C. However, the energy output E was still very low at $p_e = 0.00275$, due to the high degree of administration in the network over sustained periods of time.

We also studied the case of larger rates of exploration, i.e, $p_e = 0.02$ (Figure 2D). We found an initially sharp increase in administration, facilitated by the high exploration probability. The trajectory then converged to a stable administrator share slightly above $N_A/N = 0.5$, slightly increased by an additional effect-based promotion of nodes, because the energy production per capita was always below the threshold ($\epsilon = 1.0$). After reaching this (meta)stable state, the model society seemed to survive for a long time (at least longer than $t = 5000$) while faced with a decreased mean energy production at around $E \approx 0.5$.

It is noteworthy that in each instance of the simulation (Figure 2A,C,D), the society reached an energy production above the energy requirements, which was then however quickly surpassed until the energy level either tended towards zero for zero or low exploration rates in Figure 2A,C or converged to a stable regime for large exploration rates in Figure 2D. Also note that in the standard "fast-collapse" scenario, the society initially fared better with respect to energy production than under exploration scenarios ($p_e \geq 0$).
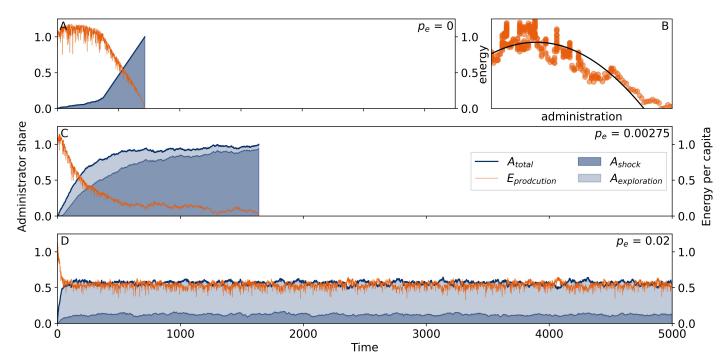
Figure 2. Exemplary network simulations of a Tainter-like model society ($N = 400$) according to the model description. The panels show only the first 5000 time steps to focus on the initial dynamic. Blue curves show the share of the administration in the network (light blue: administration as a result of decreased resource availability; dark blue: administration resulting from exploration with rate p_e). Orange curves show the average energy produced per node. (**A**) shows the typical development of a network reacting to shocks by changing one node from coordinated laborer C to administrator A. (**B**) displays a moving average of the energy measured in (**A**) against the size of the administration. The black curve indicates stylized parabola-shaped diminishing marginal returns as in [18]. (**C**) displays the network development at an intermediate exploration probability ($p_e = 0.00275$). (**D**) shows the development of a network with a high probability of exploration ($p_e = 0.02$).

3.2. Deterministic Macroscopic Approximation of the Stochastic Micromodel

We now derive a macroscopic approximation of the above stochastic micromodel in terms of an ordinary differential equation describing the average time evolution of an aggregate quantity (here, the total number N_A of administrators). This time evolution is governed by average transition rates between the three groups. The rates are based on approximations of the probabilities with which individual agents switch between the groups and the assumption that N is large so that the law of large numbers applies.

There are two processes which make the number of administrators change, exploration and targeted recruiting in response to energy demands. Due to exploration, from the N_A administrators, on average, $p_e N_A$ many leave the administration per unit time, and from the $N - N_A$ nonadministrators, on average, $p_e(N - N_A)$ are hired as administrators per unit time, making a balance of $p_e(N - 2N_A)$ additional administrators on average per unit time.

For targeted recruiting, we make the simplifying assumption that the density of links inside and between the three groups remains approximately constant and can thus be estimated by the overall link density ρ of the Erdös–Renyi network. Then, if N_A is the number of administrators, the probability that a nonadministrator is not linked to any of the N_A administrators (and is thus uncoordinated) is approximately $(1 - \rho)^{N_A}$; hence, the numbers of uncoordinated and coordinated laborers are approximately $N_L \approx (N - N_A)(1 - \rho)^{N_A}$ and $N_C \approx (N - N_A)(1 - (1 - \rho)^{N_A})$; hence, we have $E = Re$ with $e = N_L^a + cN_C^b \approx [(N - N_A)(1 - \rho)^{N_A}]^a + c[(N - N_A)(1 - (1 - \rho)^{N_A})]^b$.

An additional administrator is recruited iff $E/N < \epsilon$, i.e., iff $R < N\epsilon/e$. We have Equation (1) ($R = R_{max}(1-B)$ with $B \sim \text{Beta}(\alpha, \beta)$); hence, the condition $R < N\epsilon/e$ is equivalent to $1 - B < N\epsilon/eR_{max} = N^a\epsilon/e$, as $1 - B \sim \text{Beta}(\beta, \alpha)$. Hence, the probability of $E/N < \epsilon$ is given by the cumulative probability function of the Beta distribution, $P = F(N^a\epsilon/e, \beta, \alpha)$.

The expected number of administrators hired additionally due to shocks per time unit is then also equal to this probability P.

In all, we obtain the approximation

$$\frac{dN_A}{dt} \approx p_e(N - 2N_A) + F\left(\frac{N^a\epsilon}{[(N-N_A)(1-\rho)^{N_A}]^a + c[(N-N_A)(1-(1-\rho)^{N_A})]^b}, \beta, \alpha\right), \tag{4}$$

$$N_C \approx (N - N_A)(1 - (1-\rho)^{N_A}), \tag{5}$$

$$N_L \approx (N - N_A)(1-\rho)^{N_A} \tag{6}$$

as long as $N_A < N$.

Simulation results were well predicted by the approximation, as shown in Figure 3A,B. Particularly, at higher values of p_e, the approximation yielded excellent results both for the energy production and the administration share. Only at an exploration probability of $p_e = 0$ (dashed line and light colors in Figure 3) did the macroscopic approximation slightly overestimate the velocity of collapse. This can be explained by one simplifying assumption of the macroscopic approximation, namely, the network degree or local link density is homogeneous, i.e., approach ρ, across the entire network. In contrast, due to the links being put at random when setting up the network, this distribution in a node's number of connections becomes heterogeneous with some nodes showing an above-average number of connections. Because those nodes are preferred over those with few connections in the process of administration selection, initially selected administrators will result in an above-average energy return. This produces the prolonged slow-growth phase of the administration in the numerical simulation. In contrast, the macroscopic approximation assumes that at a given point in time any new administrator would have the same average effect.

To illustrate the probability of collapse under different exploration assumptions, histograms of survival times from the stochastic model were computed (Figure 3C). At high rates of social mobility p_e, the model converged to a stable state where the society persisted for infinite times due to its ability to absorb shocks. For lower $p_e \approx 0$, we also find isolated cases of infinite survival even though the society mostly collapsed comparatively early (Figure 3C). Such observations can be explained by a fragmentation of the network into smaller worker networks that are mutually disconnected. This behavior cannot be captured by the macroscopic approximation and here, the results of the approximation can differ from the numerical simulation. The scattered survival times along the time axis for intermediate exploration rates illustrate the stochastic survival of the model society due to a random sequence of shocks.
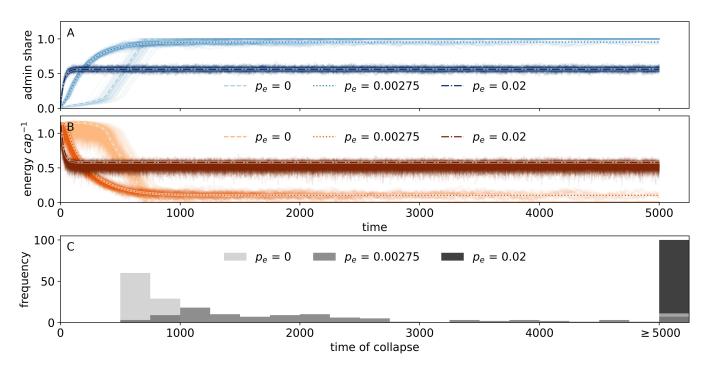
Figure 3. Analytic approximation of the model dynamics with the share of administration N_A/N (**A**) and corresponding energy production per capita (**B**). Transparent solid lines show the results of an ensemble of 100 simulations of the microscopic network model for different values of p_e. Dashed lines display the respective analytical approximation. (**C**) Histograms of collapse frequencies of the model, i.e., frequency of times at which energy production approaches zeros, $E = 0$. Bars located at time $t \geq 5000$ show right-censored cases of the simulation.

3.3. Influence of Model Parameters on Survival Time

In the third step of our analysis, we used the approximation proposed in the previous section to estimate model outcomes for a broad range of parameter values ρ, c and p_e. Note that the additional parameters of resource availability (α, β) and threshold (ϵ) had a major influence on the outcome of the model as well. Specifically, a high probability of low resource availability led to much shorter survival times and vice versa, while a low threshold ϵ to appoint further administrators considerably increased survival times. However, since these effects did not reveal any additionally remarkable results (not shown), we focused our attention on the major drivers of survival time and energy output, i.e, the exploration probability p_e, link density ρ and productivity of labor c.

Figure 4(B$_1$–B$_4$) show the survival time as a function of link density ρ and labor productivity c. Figure 4(B$_1$) displays a parameter analysis for the case of no exploration, $p_e = 0$, and can hence be interpreted as showing the minimum requirements of a successful administration, the most important requirement being a sufficiently large connectivity of the social network (indicated by the increasingly gray area with increasing ρ). Also, the extreme case of $\rho = 0.0$, i.e, a network with no connections, must be noted since in this case, nodes cannot be converted at all to administrators and thus the model remains in its initial state. As p_e approaches $\frac{1}{N}$ (Figure 4(B$_2$,B$_3$)), the relevance of ρ diminishes and the survival time becomes mainly a function of the productivity c.
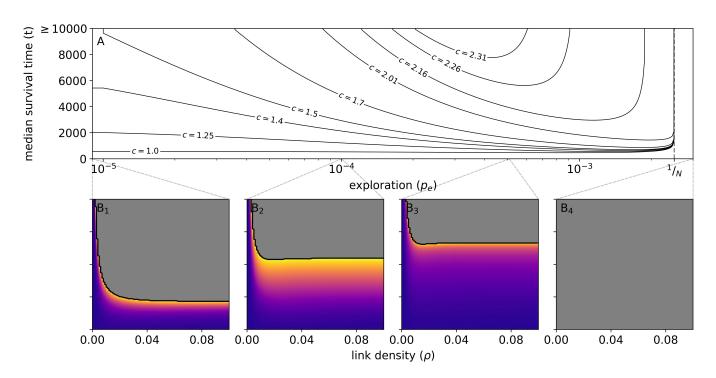
Figure 4. Survival-time analysis of the parameters exploration probability (p_e), link density (ρ) and productivity (c). (**A**) The effect of p_e between zero and $\frac{1}{N}$ for some select values of c on median survival time over all computed values of ρ. The vertical dashed line defines the exploration threshold of $\frac{1}{N}$. Lower panels (**B₁**–**B₄**) display the relation between c and ρ for different exploration probabilities. Gray areas in the lower panels indicate a survival time $\geq 10{,}000$, potentially going to infinity.

Figure 4A demonstrates the effect of exploration for selected values of *a*. For exploration probabilities $p_e < 1/N$, i.e., when less than one individual per time unit switches their type, collapse is likely, which is indicated by reduced median survival times, Figure 4A. Note that the per capita energy production of the macroscopic approximation is dependent on N in so far as the link density and thus the number of connected nodes per A does not scale linearly with N. Only if ρ is rescaled with N, i.e., we replace ρ with $\rho_N = 1 - \sigma^{1/N}$ for some fixed $\sigma \in (0, 1)$, does the per capita energy production become effectively independent of N. In fact, our macroscopic approximation shows that $N_A = N$ is a stable fixed point of the approximate dynamics for $p_e < 1/N$ since the RHS of Equation (4) remains strictly positive for $N_A \to N$. This means that the labor force will vanish in finite time, thereby also resulting in a finite survival time. The optimal number of administrators derived from Equation (4) is given in Appendix A. Additionally, in this "low exploration regime", a larger coordinated output productivity c tends to increase the median survival time. Indeed, the term $F(\cdot)$ in Equation (4) is a decreasing function of c. This effect is the more pronounced the larger N_A is, because for a large N_A, the energy production of coordinated laborers C is dominant. Hence, a larger c slows down the collapse more and more as we approach $N_A = N$. The effect of a p_e on survival time is more difficult to understand because it is twofold, as can again be seen in Equation (4). Initially, when $N_A < N/2$, a larger p_e increases dN_A/dt and speeds up the process of recruiting administrators, as seen on the left of Figure 3A. As soon as $N_A > N/2$, the reverse happens and a larger p_e slows down that process. For a small p_e below some turning-point value $p_e^0 < 1/N$, the first effect dominates, so that survival time decreases. For p_e between p_e^0 and $1/N$, however, the second effect dominates, so that survival time increases again. For some values of c, the two effects are clearly distinguishable in different phases of the evolution, as can be seen in the realization depicted in Figure 5, where the system stays a long time close to $N_A = N/2$ before reaching $N_A = A$ and thereby collapsing. As p_e approaches

the critical value of $1/N$, this effect becomes ever stronger, and expected survival times approach infinity.
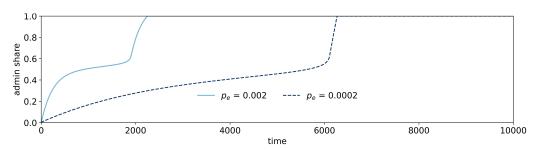

Figure 5. Trajectories of administration share for $c = 2.1$ and $\rho = 0.05$ for two choices of p_e below the critical value $p_e = \frac{1}{N}$ above which a collapse of the system is mitigated. The trajectories show how the administration share is stabilized by exploration. This mechanism effectively slows the growth of the administration until it reaches a critical size. At this point, the administration growth enters the event-driven phase.

Of course, since the microscopic network model is of stochastic nature, individual survival times still vary between realizations for the same set of parameters. As survival time also depends on other parameters, we display only the central tendency of survival time in Figure 3A. Since we terminated our simulations after $t = 10{,}000$ time steps, we cannot distinguish higher survival times, which means we cannot use the arithmetic mean of survival times as the measure of central tendency. This is why we display the median instead, which has the additional advantage of being a more robust statistic than the mean.

For larger values of c, we see that some realizations survive for very long, even though $p_e < 1/N$. This is because for a large enough c, the approximate dynamics in Equation (4) have a second stable fixed point despite $N_A = N$. Indeed, for $c \to \infty$, the $F(\cdot)$ term vanishes, and $dN_A/dt = 0$ for some value close to $N_A = N/2$. In that case, realizations of the stochastic model starting with a small N_A are likely to stay close to $N_A = N/2$ for very long (which is thus a "metastable" state) before eventually escaping into the basin of attraction of the "collapse" fixed point $N_A = N$ due to a large enough shock.

When p_e crosses the critical value $1/N$, the collapse fixed point $N_A = N$ of the approximate dynamics becomes unstable, so that collapse can only occur due to a sequence of large shocks, which becomes increasingly unlikely. The median survival time is then very large. For a large p_e, one can also see from Equation (4) which levels of N_A occur most likely over the course of the simulation. Since the $F(\dots)$ term is between zero and one, $dN_A/dt = 0$ implies $N/2 \leq N_A \leq N/2 + 1/2p$. In other words, when either p_e or c are large enough, one can expect that after some transient phase, there will only be slightly more administrators than laborers on average for a long time, before a large shock eventually causes the system collapse after all.

4. Discussion

We constructed an agent-based network model to illustrate the emergent dynamics of the *theory of societal collapse* as postulated in Tainter [4]. In our model, problem solving increased complexity and ultimately led to system collapse by diminishing marginal returns to investment. Problems were represented abstractly by random shocks to energy production, which were countered by adding complexity to the networked social system in the form of an increasing administration. We were able to derive a well-performing macroscopic approximation of the proposed model, which provided a simple mathematical description of the theory in the form of an ordinary differential equation. Using this approximation, we showed that increased social mobility through random status exploration ($A \longrightarrow L/C$, $A \longleftarrow L/C$) of at least $p_e = 1/N$ was sufficient to mitigate a collapse at the expense of skipping a phase of high energy returns on labor. A minimum network link density $\rho \approx 0.02$ was essential for long survival times but had no further effect beyond

a threshold. Finally, the positive effect of complexity, expressed as the productivity c of workers connected to administrators, was found to be the dominant factor increasing survival time and also increasing energy returns.

Similar to Motesharrei et al. [31], we found that an unequal society in which the elite consumed more (in their case) or produced less (in our case) than the commoners was likely to run into collapse. In addition, we found that the absence of an elite (i.e., egalitarian society), which is comparable to the random status exploration mechanism in our model, produced more pathways to avoid collapse than a hierarchical (complex) society. In comparison, previously reported simulations of competing premodern societies showed that collapse becomes less likely as societies grow more complex and develop intensive agriculture at sufficiently large scale [52]. This finding contrasts with our result for two specific reasons: First, in Guzmán et al. [52], complexity is a binary state and cannot continue to increase once a society is denoted as being *complex*. Second, and more importantly, Guzmán et al. [52] modeled a warrior class comparable to administrators in our model, which was directly coupled to available food (energy). Therefore, complexity in terms of hierarchies could not exceed the level of energy available and thereby grew to an unsustainable size. A more detailed investigation into the interactions of complexity and economic development in future work would certainly help in understanding the importance of complexity in collapse. Butzer [3] points out that collapse itself should be modeled by sophisticated social-ecological models that do not rely on simplifying assumptions. Nevertheless, Butzer [3] concludes that the preconditions for breakdown are typically economic or fiscal decline (i.e., diminishing marginal returns). We agree and emphasize that our model of the theory of *collapse of complex societies* offers a potential path leading to exactly such preconditions. The model implemented in this work bears some resemblance to the Peter Principle [53], which assumes that promoting workers based on their merits leads to losses in productivity as workers are promoted away from the jobs they are most qualified for. As in the present work, it was previously shown that performing random status exploration can be an effective mechanism to evade the productivity-reducing effects of the Peter principle [54]; however, in the present work, the mechanism mainly acts by removing administrators and reassigning them to productive labor.

As in Watson and Lovelock [55] or Granovetter [56], the objective of our research was not to derive operational guidelines for building resilient societies but to illustrate the mechanism of collapse due to diminishing marginal returns of an increasingly complex society. We acknowledge that our approach to modeling collapse is highly conceptual; nevertheless, our results underline the relevance of understanding the emergence and consequences of complexity in the resilience discourse.

One major critique of the model is the built-in ratchet effect, where administrators, once recruited, cannot convert back into the productive labor force for $p_e = 0$. This results in a model where collapse is hardwired into the system. While this assumption is questionable for real-world scenarios, it serves to exemplify the following statement, which we believe encapsulates J. Tainter's theory: *If the energy available to a system is limited, if a system is confronted with problems that occasionally require it to increase its complexity, and if these increases in complexity have associated costs and the system's complexity cannot be reduced, then given enough time, the system will inevitably collapse.* The general validity of assuming a persistent complexity is supported by Tainter [4], who argues that the endogenous reduction in complexity as a problem-solving strategy is not observed in historical cases, at least not as a dominant process. Furthermore, the assumed persistence of complexity can be motivated by persistent power structures [19], which come into existence much more easily than they are removed. Finally, the administration depicted in this model can be compared to expanding elite bureaucracies according to Parkinson and Lancaster [57] and Weber [58]. It would be interesting to consider more complex dynamics of intragenerational and intergenerational mobility of social status and wealth and their respective changes over time.

Unlike more economically spirited models, our simple model does not include any form of competition, economic externality, population growth or resource depletion, which may be alternative or additional drivers of collapse in the real world.

Finally, we acknowledge that our proposed model represents only one feature of complexity, i.e., the emergence of one additional layer of hierarchy, administration. Of course, modeling efforts exist that consider much more intricate mechanisms, such as human behavior, shared-resource systems, interacting societies, etc., [31,32,34,35,37,38]. But keeping Occam's razor in mind, we believe that this very simple model (without population growth, additional hierarchical layers, dynamic networks, etc.) is sufficient to make a case for the direct influence of complexity with respect to social-ecological collapse.

With the availability of databases of collapse like Seashat [59] and the recently published empirical survival analysis of states [1], the data are available to ground models on real data. The complexity-generating mechanism proposed in this work can provide the mechanistic basis to simulate the expected slowdown of recovery from perturbations [1], as the marginal returns on complexity decrease with an increasing complexity of the society. The logical next step is to use a survival analysis to combine empirical data with mechanistic models by modeling hazards through advanced mechanistic models in order to test specific hypotheses of societal collapse. For this, the presented model needs to overcome its deficiencies and include resource acquisition and population dynamics not as abstract quantities like energy E but physically based quantities. Therefore, our model should be extended in future studies to analyze how complexity relates to collapse in potentially more realistic scenarios, which could be represented by further social stratification or more complex and adaptive network topologies. In this context, the complexity costs of social connectivity should be considered. In addition, the impacts of innovations and energy substitutes (coal, oil, etc.) on collapse could be studied to identify additional strategies to slow down or avoid collapse [60]. Furthermore, the interrelation between energy and complexity could also be studied in more elaborate World–Earth and Human–Earth system models to study under which circumstances the results obtained in this work could be transferred to less abstract applications. For this purpose, appropriate measures of energy, complexity and resilience need to be defined depending on the specific model and research question at hand [61].

With regard to the open research topics posed by Cumming and Peterson [10], such as moving the debate from "whether" to "why" collapse occurred and possible strategies to "avoid, slow or hasten collapse", we wish to contribute our findings to the discourse on the drivers of social-ecological resilience and collapse. Growing complexity has recently been identified as a source of increased societal risk, e.g., due to high levels of socio(-ecological) interconnectedness that can lead to cascading failures [62,63]. In this regard, complexity is mainly considered as a facilitator of collapse by shaping systems to become more vulnerable. In our research, we show that complexity may be even more directly linked to collapse, as societies may reach a point where the associated costs outweigh the benefits, thereby directly causing their breakdown. This dimension is, to our understanding, an underrepresented view of social-ecological collapse that should be strengthened by future research efforts.

While increasing complexity has brought vast benefits to modern industrialized societies, our findings also raise the question of how far contemporary societies can increase complexity without creating large-scale risks of collapse due to ever-increasing energy demands. Along those lines, it is of great interest to further investigate whether increasing complexity is always associated with increased risk, or whether certain forms of complexity exist that do not necessarily increase the risk of societal collapse. Such an assessment would allow the development of design principles for resilient infrastructures and social structures that do not facilitate complexity in the understanding of Tainter [4].

Abbreviations

The following abbreviations are used in this manuscript:

ROC   Returns on complexity
EROI   Energy return on investment

Appendix A. Optimal Number of Administrators

From the analytical approximation, we know that the expected energy input E is proportional to

$$E_0 := [(N - N_A)(1 - \rho)^{N_A}]^a + c[(N - N_A)(1 - (1 - \rho)^{N_A})]^b, \tag{A1}$$

which has a maximum in N_A where

$$0 = \partial_{N_A} E_0 = -a[(N - N_A)(1 - \rho)^{N_A}]^{a-1}[1 - (N - N_A)\ln(1 - \rho)](1 - \rho)^{N_A} \tag{A2}$$

$$+ bc[(N - N_A)(1 - (1 - \rho)^{N_A})]^{b-1}[(1 - \rho)^{N_A}[1 - (N - N_A)\ln(1 - \rho)] - 1]. \tag{A3}$$

E.g., for $N = 400$, $\rho = 0.02$, $a = b = 0.75$ and $c = 1.05$, the optimal number of administrators is $N_A \approx 22$ or $\approx 6\%$, leading to $E \approx 461$ or $E/N \approx 1.15$.

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