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Research Paper

Green transition and macroeconomic stabilization

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A R T I C L E I N F O

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A B S T R A C T

This paper looks into the crucial macroeconomic feedback mechanisms emerging from the interplay among the goods market, the labor market, the financial sector, and monetary policy, particularly in the context of transitioning towards a climate-neutral economy. The investment decisions of firms, pivotal in this interaction, can trigger feedback loops with potentially destabilizing effects, underscoring the critical role of investment within the complex interplay of market and sector dynamics in the macroeconomy. Governmental intervention is highlighted as a key factor in steering the green transition while preserving economic stability. A carbon tax on fossil fuel consumption is proposed as a primary tool for facilitating this green transition. Our investigation employs a disequilibrium model of monetary growth, a la Keynes-Metzler-Goodwin (KMG), incorporating a portfolio perspective across three asset markets - money, bonds, and stocks. This framework allows for an in-depth analysis of how a carbon tax influences real production, inflation, and inequality during the transition. Our findings indicate that imposing a carbon tax on production does not markedly disrupt economic stability, as long as the carbon pricing and its growth rate remain within low bounds.

1. Introduction

Under the broader theme of the green transition, the focus of academic research and policy discourse has predominantly revolved around questions concerning the optimal policies and tools (i.e., carbon price) for climate change mitigation, as well as their implications on social welfare and households' well-being. However, these policies and tools may give rise to macrodynamic feedback effects, which are often overlooked or not adequately represented in standard macroeconomic models. These feedback effects can be either of stabilizing or destabilizing nature. In addition to the household and firm sectors, the implementation of abatement strategies will impact other sectors and markets within the economy. The objective of this paper is to comprehensively examine and analyze the dynamics and feedback effects that arise from the interaction between markets and sectors in our model economy, all within the context of a transition towards a green economy.

While it may have experienced a decline in popularity throughout the years, the study of macroeconomic feedback loops is a long-standing tradition in the fields of macroeconomics. Feedback mechanisms can cause instability in the goods market, the financial market and in pricing dynamics. It is therefore important to take into account those feedback mechanisms when implementing policies to incentivize a transition to a low-carbon economy, such as the carbon tax. Given the paper's scope, we employ a macro-based methodology to describe our economy. We adopt and expand upon a model of monetary growth that is part of the disequilibrium macroeconomic dynamic modeling tradition, specifically referred to as the *KMGT* model. The integrated macrodynamics approach

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known as the Keynes-Metzler-Goodwin-Tobin (*KMGT*) approach forms the fundamental framework of the model formulated by Flaschel et al. (1997), Chiarella et al. (2000), Chiarella and Flaschel (2000), and Chiarella et al. (2005).

The label (*KMG*) underscores the significant macroeconomic mechanisms established by the cited economists; Keynes (1937) alludes to the causal connection between financial and real markets, Metzler (1941) to inventory dynamics, and Goodwin (1982) to the dynamics of distributive shares. On the other hand, *T* refers to Tobin's (1969) General Equilibrium Approach to Monetary Theory. Overall, these are the fundamental mechanisms that we believe should lie at the core of descriptive macroeconomic models. By combining a comprehensive disequilibrium approach on the real side of the economy with a general equilibrium approach on the financial side, a number of interesting considerations regarding the dynamics that ultimately drives the economy are generated.

Introducing a climate policy in the form of a carbon tax into our economy will yield additional feedback effects that may exhibit stabilizing or destabilizing characteristics. It is plausible that, in the context of a rapid transition towards a carbon-neutral economy, the economy will not necessarily establish balanced growth patterns; with no contractions in labor markets, no global recessions, no risk of debt deflation and financial meltdown, and no significant fluctuations in economic activity. One of the primary objectives of this paper is to present a framework for analyzing all such tendencies when implementing a carbon tax. On a macroeconomic level, the behavior of firms and their investment decisions, the actions of financial markets and their portfolio choices (driven by returns), and the potential impact of policymakers are of key significance.

To better understand the additional effects of carbon tax on the stability of the economy, we employ in our dynamical analysis a systematic step-by-step approach; whereby we focus first on a lower dimensional system, namely the real side of the economy and study the impact of carbon tax on the labor market. Subsequently, we augment our model and extend the stability analysis by enabling the interaction of the real economy with the assets market. In a final step, we complete the analysis by incorporating the monetary and the fiscal policy. Our incremental approach to the analysis of the dynamical system allows us to gain a better comprehension of the feedback effects between different sectors. Furthermore, by working with a higher-dimensional model, we attempt to construct a more realistic setting with more relevant policy implications. Policy-oriented models typically operate with low-dimensional macrodynamics, and it remains unclear to what extent those policy effects (for instance, stabilizing policies, formulated for low-dimensional dynamics) may remain valid in higher-dimensional macrodynamics.

Based on our analysis, the introduction of a carbon tax does affect the stability of the real-side of the economy when the adjustment parameters associated with carbon policy are high and when the growth rate of the carbon tax per period is also high. The effect of a carbon tax extends beyond the firm's output, as it also influences the firm's expected rate of profit and consequently affects the asset portfolio of capitalist households. In a higher dimensional setting that considers the interactions between the real and financial markets, the implementation of a carbon tax exacerbates this instability. So far, our focus has been on simple climate policies, such as the imposition of a carbon tax on fossil-fuel energy use. However, the *KMG* framework provides us with additional opportunities to employ alternative climate policy instruments and examine their impact on macroeconomic stability. For example, it would be interesting to divide the firms' sector into two distinct representative firms (one in the dirty sector and one in the green sector) and investigate the investment behavior of these firms in light of the carbon tax or other policy tools, such as subsidies for green capital. In the standard *KMG* model, we only considered conventional bonds, but we can assume, for instance, that the government issues green bonds as a means to contribute to the transition towards a low-carbon economy.

To summarize, we analyze in this paper the impact of carbon policy, aimed at accelerating the green transition, on the stability of the overall economy. To that aim, we build upon the work of Chiarella and Flaschel and derive a comprehensive framework outlining the functioning of a closed market economy. The structure has a descriptive value and focuses on the analysis of economic interdependence, macroeconomic feedback loops, and the stability problems that emerge when the core markets interact with the macroeconomy. Within this model, an adaptive expectation learning mechanism is employed to capture the dynamics of anticipated variables in terms of expectations.[1] The high-dimensional integrated method to macrodynamic analysis is a very important part of knowing how real and financial markets interact in the real world.

The rest of the paper is as follows: section 2 introduces the *KMGT* model and provides a detailed description of the model's expanded form. Section 3 derives the model's intensive form, including all essential laws of motion. The fourth section examines numerical simulations of particular situations of the general system, namely the influence of the carbon tax and the role of the financial markets and monetary policy in that context. Finally, section 5 comes to a close.

2. The model

In this section, we present the structural form of a growth model employing a portfolio approach and building on the behavior of heterogeneous agents in the assets market. The economy in our model is comprised of a heterogeneous household sector, a productive sector, the government sector, and a financial market. There are two distinct classes within the household sector: the workers who receive labor income, and the capitalists who receive capital income. A complete set of stock-flow interactions is specified, in addition to the well-defined budget constraints of all the model's sectors. Expectations are model-consistent, but heterogeneous expectation formations are also allowed. For further references, the choice of symbols and letters to represent the variables and other parameters in the model below is close to the ones used in Chiarella et al. (2014).

[1] Richters (2021) employs a similar approach of adaptive expectation to his General Constrained Dynamics model.

2.1. Households (workers and asset-holders)

The two types of households in our model, workers and pure asset-holders, are differentiated by their source of income and their saving habits. We model the consumption and the saving decisions of worker households in a fairly simple manner, while the modeling of the capitalist households' income, consumption and wealth is slightly more evolved, in part due to the inclusion of an active asset market. Capitalist households do not supply any work, and they consume and save entirely out of interest income.

$$\omega = w/p, \tag{1}$$

$$Y_w^{Dn} = (1-\tau_w) w L^d, \quad Y_c^{Dn} = (1-\tau_c)[\mathrm{i}\ B_c^s + B^l + \rho^e (1-b_f^s) pK], \tag{2}$$

$$pC_w = Y_w^{Dn}, \quad pC_c = C_{c_1} Y_c^{Dn} + C_{c_2} W_c^n, \tag{3}$$

$$S_w = 0, \quad S_c^n = Y_c^{Dn} - pC_c = \dot{M}2 + p_b \dot{B}^l + p_e \dot{E} \tag{4}$$

$$W_c = (M2 + p_b B^l + p_e E)/p \quad \text{with } M2 = M_c + B_c^s, \text{ and } W_c^n = pW_c \tag{5}$$

The first equation defines the real wage ω before taxation, where w denotes the nominal wage and p the actual price level. Equation (2) exhibits the current disposable income of workers and of asset holders respectively. The latter consists of interest on short-term bonds B^s (with fixed price, set equal to 1, and a variable interest rate i), interest on long-term bonds B^l (which we can consider as perpetuities with implied interest rate $1/p_b$), and dividend payments of firms (based on their expected sales and obtained by deducting depreciation, gross wages, loan interest payments of firms and other expenses including the carbon tax). Workers consume all their income, while the consumption function of pure asset holders is a linear combination of their nominal income Y_c^{Dn} and nominal wealth W_c^n, as shown in (3). Since workers do not save, their wealth is zero at every point in time. On the other hand, nominal savings S_c^n of asset holders are used to acquire new short- and long-term bonds and equities as issued by the government and firms.[2]

2.2. Asset markets—portfolio adjustments

We consider an active asset market that feeds into the real sector. To reflect a realistic framework, we model the asset market in an exhaustive manner as in Chiarella et al. (2014). The following are the asset demand functions and market clearing conditions for money, short-term bonds, long-term bonds, and stocks respectively:

$$\tilde{W}_c^n = B_c^s + p_b B^l + p_e E = B_c^{sd} + p_b B^{ld} + p_e E^d, \quad \text{with} \quad W_c^n = \tilde{W}_c^n + M_c \tag{6}$$

$$M_c^d = \alpha_{m_c^d} B_c^s, \quad \text{and} \quad \dot{M}_c = \beta_{mc}\left(M_c^d - M_c\right) + (\phi + \bar{\pi}) M_c. \tag{7}$$

$$B_c^s = B_c^{sd} := f_s(\mathrm{i}, \rho_{b^l}^e, \rho_e^e)\tilde{W}_c^n; \quad B_c^l = B_c^{ld} := f_l(\mathrm{i}, \rho_{b^l}^e, \rho_e^e)\tilde{W}_c^n / p_b; \tag{8}$$

$$E = E^d := f_e(\mathrm{i}, \rho_{b^l}^e, \rho_e^e)\tilde{W}_c^n / p_e;$$

Equation (6) illustrates the *Walras' Law of Stocks* which states that, in nominal terms, total asset demand must be always equal to asset supply. M_c^d in (7) denotes money demand of capitalists, and it changes solely by way of a dynamic inventory approach[3]; hence, money holdings is simply adjusted to match a chosen fraction of short-term bond holdings or saving deposits. We only have to consider then the three asset market-equilibrium conditions (8), which reduces to only two independent equilibrium conditions given the employed *Walras' Law of Stocks*. Overall, equilibrium in the assets market is established through the market clearing conditions of the long-term bonds and equity prices, p_{b^l} and p_e. Equations (11) and (12), defined later in section 2.3, determine the equity and bond prices on the basis of their expected rates of return. The determination of the short-term interest rate, denoted as "i", is not contingent upon the LM curve. Instead, it is viewed as a policy variable, influenced by the monetary authority's conduct, represented by a Taylor rule.

In this portfolio approach to asset market equilibrium, asset holders seek to maximize their portfolio returns by optimally re-allocating their holdings of stocks, short-term and long-term bonds. Maintaining equilibrium in the asset markets is achieved by trading financial instruments among asset owners, which results in price fluctuations in either an upward or downward direction. The functions introduced above in (7) and (8) must satisfy the following conditions:

$$f_b(\mathrm{i}, \rho_{b^l}^e, \rho_e^e) + f_{b^l}(\mathrm{i}, \rho_{b^l}^e, \rho_e^e) + f_e(\mathrm{i}, \rho_{b^l}^e, \rho_e^e) = 1, \tag{9}$$

$$\frac{\partial f_b(\mathrm{i}, \rho_{b^l}^e, \rho_e^e)}{\partial x_i} + \frac{\partial f_{b^l}^e(\mathrm{i}, \rho_{b^l}^e, \rho_e^e)}{\partial x_i} + \frac{\partial f_e^e(\mathrm{i}, \rho_{b^l}^e, \rho_e^e)}{\partial x_i} = 0 \quad \forall x_i \in \{\mathrm{i}, \rho_{b^l}^e, \rho_e^e\} \tag{10}$$

Financial assets and capital gains are assumed to be imperfect substitutes. Gross substitution indicates that as the price of one asset increases, the demand for all other assets rises as well.

[2] There is always consistency between the inflow of new bonds (short- and long-term) and equities and the amounts that are actually purchased by asset-holders and the central bank.

[3] Pure cash holdings are assumed to only serve simple transaction purposes.

The nominal demand for asset holders' $M2$ balances is determined by a function that depends on the short-term asset interest rate i, the expected rate of return on long-term bonds, and the expected rate of return on stocks. The function $f_{x_i}(\cdot)$ describes the fraction of nominal wealth that is allocated to each asset in the market. We assume, as it is common in portfolio approaches, that investors desire assets in a quantity equal to their nominal wealth. Asset holders adjust their wealth distribution based on new information about their assets' return rates, thus managing their wealth constraint.

2.3. Asset markets—rates of return

Expected returns on equities E and long-term bonds B^l are defined in a straightforward manner; it is the sum of dividends (or interest rate of return for bonds) and expected capital gains.

$$\rho_e^e = \rho^e \frac{(1-b_f)(pK+pN+M_f)}{p_e E} + \pi_e = \rho^e \frac{1}{q} + \pi_e, \tag{11}$$

$$\rho_b^e = \frac{1}{p_b} + \pi_b = r_l + \pi_b. \tag{12}$$

$$\pi_e = \alpha_e \pi_{ef} + (1-\alpha_e)\pi_{ec}, \quad \pi_b = \alpha_b \pi_{bf} + (1-\alpha_b)\pi_{bc}. \tag{13}$$

As in Chiarella et al. (2014), we consider two categories of traders (i.e., the fundamentalists and the technical traders) who, within the group of asset holders, form distinct expectations on capital gains with regards to long-term bonds and stocks. Fundamentalists anticipate capital gains approaching the rate of growth of fundamental stock prices (prices that equal Tobin's q). Expectations of technical traders (also called chartists) on the other hand adjust to delayed evolution of real share prices. This divergence in expectations leads to the formation of financial bubbles. The average expected rates of return for traded assets are determined by the average expectations of these heterogeneous groups of traders, as shown in (13).[4]

2.4. The entrepreneurial sector

The entrepreneurial sector is a key component of our model economy. We analyze firm behavior through two submodules: one outlining the production structure and investment in fixed capital by firms, and the other detailing the Metzlerian perspective on how inventories fluctuate with anticipated sales, actual sales, and firm's output.

2.4.1. Firms' production and investment decisions

The firm produces final good Y using a production technology that transforms fixed capital K, fossil fuels F (as source of energy) and labor L. The fossil fuel intensity z is endogenized and associated with a clean capital stock K_c that maps one-to-one with the technology z. Total private fixed capital K is composed of both standard capital K_s and clean capital K_c.

$$Y = \min\{y^p K, xL_1^d, zF\}, \tag{14}$$

$$Y^p = y^p K,\ u = Y/Y^p,\ y^p = \text{const.}, \tag{15}$$

$$L_1^d = Y/x,\ e_1 = L_1^d/L,\ \hat{x} = \text{const.}, \tag{16}$$

$$L_2^d = z/\phi_2,\ e_2 = L_2^d/L,\ \hat{z} = \phi_1 \hat{K}_c. \tag{17}$$

The parameter y^p, representing the output-capital ratio, determines the potential output Y^p of firms, whereas $y = Y/K$ reflects the actual ratio of output to capital. The utilization rate of private capital is represented by u. The labor demand of firms hinges on their desired output Y and a constant labor productivity x. The decision-making process of the firm is structured as follows: it assesses the demand for final goods, which then guides its use of labor and energy for production, taking into account the current capital and fossil fuel efficiency technology z that are fixed in the short term. In the intervals between production periods, the firm decides on investments in two types of capital - clean capital K_c, influencing technology z, and standard capital K_s. The growth speed parameter ϕ_1 for fossil fuel efficiency z is critical in determining the rate at which energy usage - and by extension, carbon emissions - decreases. Energy intensity, the measure of energy required for a given output or activity level, inversely correlates with efficiency level z; hence, an enhancement in z leads to a reduction in the energy intensity of products or services (implying lower energy consumption for production or delivery).

$$I_c/K_c = \iota_c(\tilde{\tau}_d p_d^r - f(z)) + \iota_{u_c}(u_c - \bar{u}_c) - \iota_b(b_f^s - \bar{b}_f^s) - \iota_r(i_f - \hat{p} - \bar{i}_f) + \phi, \tag{18}$$

$$I_s/K_s = \iota_\rho(\rho^m - \rho_0^m) + \iota_{u_s}(u_s - \bar{u}_s) - \iota_b(b_f^s - \bar{b}_f^s) - \iota_r(i_f - \hat{p} - \bar{i}_f) + \phi, \tag{19}$$

$$I^p/K = \left(p_e\dot{E} + \dot{B}_f^r + p(\mathcal{I} - \dot{N}) - \dot{M}_f\right)/pK, \tag{20}$$

$$I/K = \min\{I^d/K, I^p/K\}. \tag{21}$$

[4] Details on expectation formation of fundamentalists and technical traders are briefly presented in the appendix.

Equations (18) and (19) determine the investment by firms in clean capital, I_c/K_c, and standard capital, I_s/K_s respectively. They are both positively driven by the current capacity utilization in its deviation from normal capacity utilization, and they are negatively driven by the firms' actual debt-to-capital ratio b^s_f and its deviation from its target value $\bar{b}^s_f$, and by the real loan rate (the real borrowing cost). However, investment demand in clean capital is also driven by the difference between the effective real price of fossil-fuel ($\tilde{\tau}_d\ p^r_d$), with $\tilde{\tau}_d \equiv (1+\tau_d)$, and some function of fossil-fuel intensity of production $f(z)$, which we consider to be of a simple linear form $f(z) = \phi_3\ z$. This is a formalized representation of the idea that improving fossil fuel efficiency z is more profitable the more expensive fossil fuels are and/or the worst the fossil fuel efficiency is. Investment demand in standard capital, on the other hand, is additionally driven by the state of confidence or the investment climate ρ^m, which in its deviation from its steady-state value zero drives business fixed investment (representing the so-called animal spirits of investors). Equation (20) depicts the potential or maximum investment, which reflects the firms' budget constraints. Thus, investment is (occasionally) constrained by financing conditions, which are reflected by the potential investment rate I^p/K; firms must deviate from their investment plans due to credit rationing and their equity issuing policy.

We do not live in a Modigliani-Miller (1958) world; hence, the financial structure of a company still plays a crucial role. Firms can finance their investments through two main channels; namely, by issuing new equity $\dot{E}$ and/or by borrowing $\dot{B}^s_f$ from the banking sector.

$$\varepsilon = \frac{E}{(1-b^s_f)K},\ q = \frac{p_e E}{(1-b^s_f)(pK+pN+M_f)}, \tag{22}$$

$$\hat{\varepsilon} = (\eta(g^d_k - \phi)) + \eta_b(b^s_f - \bar{b}^s_f) + \eta_q(q-1), \tag{23}$$

$$\rho^e = \frac{pY^e - \delta pK - wL^d_f - i_f B^s_{f_i} - \tau_d F}{(1-b^s_f)(pK+pN+M_f)}. \tag{24}$$

Using the ratio $\varepsilon = \frac{E}{(1-b^s_f)K}$, firms choose their policy for issuing additional shares. The ratio ε evolves over time, according to (23), and it depends positively on three main drivers: the rate of investment in its deviation from the trend growth ϕ, firm's debt to capital ratio b^s_f, and on Tobin's average q (in its deviation from its fundamental value 1), which is defined by $q = \frac{p_e E}{(1-b^s_f)(pK+pN+M_f)}$ in the presence of debt. Issuance of new equity, in excess of the change in the capital stock owned by equity holders, is contingent on certain conditions. Namely, the state of investment opportunities (whether favorable or not), the rise of debt-to-capital ratio (as more equity financing is required to offset this tendency), and Tobin's average q, which reflects the ease of obtaining funds from equity financing. In this setting, from (11) and based on (24), the carbon tax influences the firm's profit which in turn influences the expected return on equities. Regardless of the firms' ability to finance the remaining portion of their planned investment expenditures with loans, they place significant importance to their equity issuance policy.

$$\dot{B}^r_f = \hat{B}^s_f B^s_f = d(b^s_f)B^s_f,\ b^s_f = \frac{B^s_f}{pK+pN+M_f},\ d \geq 0,\ d' \leq 0, \tag{25}$$

$$\dot{B}^s_f = pIs + pI_c + \dot{M}_f - p(\mathcal{I} - \dot{N}) - p_e\dot{E} \leq \dot{B}^r_f, \tag{26}$$

$$i_f = (1+\xi_f(u))i,\ \xi_f \geq 0,\ \xi'_f < 0. \tag{27}$$

Additionally, firms face financing restrictions for investment initiatives. An upper limit is established for the growth rate of bank loans, which tightens as the existing debt-to-capital ratio rises.[5] In the case of unconstrained firms, (26) should be regarded as a soft budget constraint, but in the event of credit rationing, it should be interpreted as a hard budget constraint. It has been implicitly assumed that corporations have a predetermined dividend policy in addition to their stock issuance policy. Given that unintended inventory changes ($\dot{N} - \mathcal{I}$) are assumed to be financed by loans, net expected sales ($pY^e - (\delta K + wL^d_f + r_f B_f + \tau_d F)$) are paid out as dividend to equity owners and therefore not available for investment financing. Firms' income and savings are given by $p\mathcal{I}$.

$$\dot{\rho}^m = \alpha_{\rho^m}\rho^m + \alpha_{\rho^e}\left[\rho^e - (r_l - \hat{p})\right] + \alpha_{\rho^e_e}\left(\rho^{ex}_e - \rho^{ex}_b\right) \tag{28}$$

Equation (28) describes the time rate of change of the state of confidence ρ^m, which is subject to optimistic (and accelerating) or pessimistic (and decelerating) forces depending on whether it is greater than or less than its benchmark value. This self-feeding process is constrained by the evolution of the equity premium and the real interest rate on long-term government bonds. An additional factor positively influencing the state of confidence is the difference between the expected returns on stocks and the expected returns on bonds.

2.4.2. Firms output adjustment

In conventional macroeconomic models, markets for goods are presumed to achieve instant equilibrium. Our model diverges by incorporating a Metzlerian inventory adjustment process, enriching the system's dynamics with two additional dimensions. This modification instills further buffers, potentially moderating the transition from a Keynesian regime to scenarios dominated by capital or labor shortage.

[5] This constraint is not binding with regard to the model's steady-state.

$$Y^d = C + I_s + \delta_s K_s + I_c + \delta_c K_c + G, \tag{29}$$

$$N^d = \alpha_{n^d} Y^e, \quad \mathcal{I} = \beta_n (N^d - N) + \phi N^d, \tag{30}$$

$$\dot{N} = Y - Y^d, \tag{31}$$

$$Y = Y^e + \mathcal{I}, \tag{32}$$

$$\dot{Y}^e = \beta_{y^e}(Y^d - Y^e) + \phi\, Y^e. \tag{33}$$

Firms' output decisions, denoted by Y, hinge on anticipated sales Y^e and targeted inventory alterations $\mathcal{I}$. These inventory adjustments are dictated by the desired stock levels N^d, scaled to expected sales. The reconciliation of inventory levels involves rectifying the divergence $(N^d - N)$ between targeted and actual inventories, supplemented by a term ϕ to account for trend growth. The final component of our model addresses the recalibration mechanism for sales expectations Y^e.

2.5. Wage-price adjustments

The wage-price adjustment dynamics is of the kind considered e.g. by Chiarella and Flaschel (2000), Flaschel and Krolzig (2006) and Franke et al. (2006), $\hat{w}$ and $\hat{p}$ denote wage- and price-inflation, π^m represents the inflationary environment in which the economy is operating.

$$\hat{w}^b = \beta_{we}(e - \bar{e}) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^m + n_x \tag{34}$$

$$\hat{p} = \beta_{p1}(u - \bar{u}) + \beta_{p2}(\hat{\tilde{\tau}} + \hat{p}_d - \pi^m) + \kappa_p\left(\hat{w} - n_x\right) + (1 - \kappa_p)\pi^m \tag{35}$$

$$\dot{\pi}^m = \beta_{\pi_1^e}(\hat{p} - \pi^m) \tag{36}$$

The rate of growth in gross nominal wages, $\hat{w}^b$, shows a positive correlation with the deviation between the actual employment rate e and the employment rate at the non-accelerating inflation rate of unemployment (NAIRU), $\bar{e}$. This deviation is a key indicator of labor market demand pressures. The model further integrates two interlinked aspects to quantify cost pressures on wages. Firstly, it assumes that workers possess complete awareness of the immediate trends in price inflation $\hat{p}$. Secondly, based on this awareness, wage earners' cost pressures are linked to a specific measure of inflation rate π^m, reflecting the broader inflationary context. The variable π^m, representing this context, undergoes medium-term adaptive adjustments, as marked by its coefficient $\beta_{\pi_1^e}$ (see equ. (36)). Hence, cost pressure is calculated as a weighted mean of both $\hat{p}$ and π^m, signifying that workers' considerations extend beyond the immediate horizon (for $\kappa_w < 1$), incorporating the broader inflationary environment. This approach ensures that the model's responsiveness does not overly depend on the precision of short-term expectations.

In Equation (35), price inflation hinges on demand pressure, measured by the disparity between the actual rate of capacity utilization u and the firms' perceived normal level of capacity utilization $\bar{u}$.[6] Another pivotal determinant of price inflation, especially in the context of climate transition, is the fluctuation in the effective price of fossil fuels. Regarding cost pressure, firms are presumed to have limited yet perfect foresight concerning wage inflation. They compute a weighted average, factoring in the medium-term inflationary climate π^m, which also forms the basis of their mid-term expectations. The variable π^m, indicative of the inflationary environment, is modified exclusively on the basis of goods' price inflation. This suggests that over the medium term, wage and price inflations display minimal variance, owing to the adaptive recalibration of π^m. Such a mechanism maintains consistency in the model's law of motion, without substantially altering its structural or dynamic properties.

2.6. Fiscal and monetary authorities

The public sector in our model comprises the fiscal and the monetary authority. The fiscal side is modeled as

$$T^n = \tau_w w L^d + \tau_c\left[\rho^e\left(1 - b_f^s\right)\left(pK + pN + M_f\right) + \mathrm{i}B_c^s + B^l\right] + \tau_d\; Y/z + Y_m^n \tag{37}$$

$$G = \tilde{g}Y^e, \tag{38}$$

$$L_g^d = L_g^w = \alpha_g G, \tag{39}$$

$$S_g^n = T^n - \left(\mathrm{i}B_g^s + B^l\right) - pG, \tag{40}$$

$$\dot{B}_g^s = \alpha_{gb}(-S_g^n), \quad p_b\dot{B}^l = (1 - \alpha_{gb})(-S_g^n), \quad B_g = B_g^s + p_b B^l, \tag{41}$$

$$\hat{g} = -\beta_{gd}\left(\frac{d_g}{\bar{d}_g} - 1\right) - \beta_{gd}(\hat{B}_g - (\phi + \bar{\pi})) \pm \beta_{gu}(u - \bar{u}) \pm \beta_{ge}(e - \bar{e}), \quad d_g = \frac{B_g^s + p_b B^l}{pY^e} \tag{42}$$

Wage income taxes are raised with rate τ_w on net wages w with respect to wage income. The capital income tax rate τ_c is applied to dividend income and interest income of asset holders. The carbon tax τ_d is applied on the degree of fossil-fuel intensity of output

[6] Here u represents the sum of clean and standard capital utilization $u \equiv u_c + u_s = k_{cs}u + (1 - k_{cs})u$.

Y; the dirtier the output production, the higher the revenue from the carbon tax. Finally, the untaxed interest income of the central bank and its branches is assumed to always be transferred to the government, and thus also appears in the income account of the government. The total revenue from all taxes T^n is used to cover the expenditures of the government, G. Government expenditures for goods and services are both assumed to be constant fractions of expected sales. Government savings are obtained by deducting from T^n the different government expenditures (i.e., goods and services expenditures, interest payments, and transfers). In general, these savings will be negative, necessitating debt financing.[7] Government allocates its nominal debt financing requirements between short- and long-term debt in constant proportions.

The third equation in (41) describes total government debt $\left(B^s_g + p_b B^l\right)$. The final equation (42) provides the fiscal policy rule for government expenditures which may be of Keynesian or orthodox type. In either case, we assume that there exists a target debt to expected sales ratio $\bar{d}_g$ such that government expenditures are reduced when the actual ratio $d_g = \frac{B^s_g + p_b B^l}{pY^e}$ is above this target ratio and vice versa. In addition, the proportion g of government expenditures in expected sales is also reduced when the percentage increase in government debt $\hat{B}_g$ is above the nominal steady-state growth rate of the economy (and vice versa).

The monetary side is modeled as:

$$\dot{\mathrm{i}} = \beta_{\mathrm{ii}}(\mathrm{i}^0 - \mathrm{i}) + \beta_{\mathrm{i}p}(\hat{p} - \bar{\pi}) + \beta_{\mathrm{i}g}(g_{k_s} - \phi), \tag{43}$$

$$\dot{M}_f = \dot{B}^s_{fm}, \quad \dot{M}_c = \dot{B}^s_{cm}, \quad \dot{M} = \dot{M}_f + \dot{M}_c = \dot{B}^s_m \tag{44}$$

$$B^s_m = B^s_{fm} + B^s_{gm} = B^s_f + B^s_g - B^s_c, \tag{45}$$

$$Y^n_m = \mathrm{i}_f B^s_f + \mathrm{i} B^s_g - \mathrm{i} B^s_c = \left(\mathrm{i}_f - \mathrm{i}\right) B^s_f + \mathrm{i} B^s_m. \tag{46}$$

In our analytical framework, the monetary authority, adopting a Keynesian perspective, acknowledges the economic stimulus driven by increases in investment. This leads to an appropriate approach to interest rate adjustments, with more restraint in raising rates under conditions of low investment activity as opposed to scenarios with high investment levels. The authority's interest rate policy, guided by the Taylor rule, hinges on three key elements. The first involves interest rate smoothing, and the third captures the business cycle phase, specifically marked by the divergence of the standard capital investment growth rate from its trend growth rate. The second element of the Taylor rule is structured to modulate the accelerating inflation rate $\hat{p}$ towards a set target rate $\bar{\pi}$, with a defined adjustment speed $\beta_{\mathrm{i}p}$.

Given the emphasized role of firms' investment behavior, the monetary authority assigns a higher priority to the investment-related term of the Taylor rule, particularly over its inflation objective. As a result, this third aspect of the Taylor rule (43) assumes greater prominence. This assumption posits that inflation originates primarily from the real economy, as delineated in the wage-price module. By integrating the current investment behavior and factor utilization rates as supplementary constraints, the Taylor rule can be strategically utilized to temper economic activity during periods of heightened inflation. Thus, inflation is interpreted not as a monetary phenomenon, but as one managed through the modulation of the real sector's activities.

The dynamics of money supply are encapsulated in equation (44), executed through an accommodating strategy that involves the central bank's acquisition of corporate bonds and short-term government bonds from asset holders. The banking system absorbs all short-term corporate and government bonds not held by these asset holders. The banks' interest revenue is derived from the spread between the loan rate i_f and the short-term interest rate "i", applied to the bank's loan portfolio, coupled with their interest earnings from short-term bonds. Following Chiarella et al. (2014), banks function as extensions of the central bank rather than as profit-oriented entities, and thus, expenses related to banking services and the transfer of profits from the banking to the government sector are not considered.

The central bank's interest rate policy supplants the conventional LM-curve's negative correlation between price levels and nominal interest rates with a positive association between inflation rates and nominal interest rates. This approach offers a more direct counteraction to inflation, potentially enhancing economic stability, especially in scenarios where money demand is sensitive to interest rates.

3. Intensive form and steady-state considerations

3.1. Intensive form

We derive in this section the intensive form of the model. As a first step, we focus our attention on the real side of the economy. Later in the numerical section, we examine in a step-wise fashion the additional feedback channels from the assets market, the banking, and policy sectors. We examine the interaction of income distribution (47) and growth (48) with an interest rate policy rule that primarily attempts to control inflation (49) and the inflation environment (50), and with quantity adjustments on the market for Metzler-type goods (51). To provide a medium-term orientation for firms' investment decisions, we also employ a measure of the operating investment climate of the economy (53).

We proceed on the premise that these actual dynamics are fundamentally unstable, affected by centrifugal forces around the steady-state, but are frequently tamed by a fundamental behavioral nonlinearity of macrodynamics, namely a more or less rigidly

[7] Contrary to the firms' sector, which confronts a ceiling on availability of bank loans, the government sector is assumed to not be subject to credit rationing.

kinked money-wage Phillips curve resulting from downward wage rigidity. With a Taylor rule operating on inflation and the state of the business cycle, the $9D$ dynamics discussed in this paper are advanced enough to analyze the real sector of the economy. The economy's business cycle incorporates Goodwin-Rose accumulation dynamics, a Friedmanian unemployment inflation interaction, and a Metzler inventory accelerator mechanism, which are complemented by the application of fiscal and monetary policy rules and climate inertia processes in the wage-price spiral as well as in the growth process.[8]

To simplify the model analysis and for ease of exposition, we make a number of specific assumptions; for instance, as a measure of total capital stock of firms we simply employ pK instead of $pK + pN + M_f$. The laws of motion to be considered are unit wage costs $\upsilon = \omega/x$, full employment labor intensity (measured in output units) $l = xL/K$, the short-term rate of interest i, the inflationary climate π^m, sales expectations per unit of capital $y^e = Y^e/K$, inventories per unit of capital $\nu = N/K$, the investment climate variable ρ^m, the fossil-fuel intensity of production z and the carbon tax τ_d:

$$\hat{\upsilon} = \kappa\left[(1-\kappa_p)\beta_{we}(e-\bar{e}) + (\kappa_w - 1)\left(\beta_{p_1}(u-\bar{u}) + \beta_{p_2}(\hat{\tilde{\tau}}_d + \hat{p}_d - \pi^m)\right)\right] \tag{47}$$

$$\hat{l} = \phi - g_k, \tag{48}$$

$$\dot{\mathrm{i}} = \beta_{\mathrm{ii}}(\mathrm{i}^0 - \mathrm{i}) + \beta_{\mathrm{i}p}(\hat{p} - \bar{\pi}) + \beta_{\mathrm{ig}}(g_k - \phi), \tag{49}$$

$$\dot{\pi}^m = \beta_{\pi_1^e}(\hat{p} - \pi^m) \tag{50}$$

$$\dot{\nu} = y - y^d - g_k \nu \tag{51}$$

$$\dot{y}^e = \beta_{y^e}(y^d - y^e) + (\phi - g_k)y^e, \tag{52}$$

$$\dot{\rho}^m = \alpha_{\rho^m}\rho^m + \alpha_\rho\left(\rho^e - (\mathrm{i}_l - \hat{p}) - \rho^m\right) \tag{53}$$

$$\dot{z} = \phi_1 \; z \; g_{k_c} \tag{54}$$

$$\dot{\tilde{\tau}}_d = g_{\tau_d} \; \tilde{\tau}_d, \tag{55}$$

with $\kappa = (1 - \kappa_p\kappa_w)^{-1}$.

In order to obtain an autonomous system of differential equations from the above laws of motion, we provide the definitions for u, e, $\tilde{\tau}_d$, y, y^d, and their components. The laws of motion shown above as well as the subsequent laws of motion rely on the following definitions and algebraic equations:

$$\begin{aligned}
e &= \frac{l_f^d}{l}, \quad u = \frac{y}{y^p}, \quad \tilde{\tau}_d = (1+\tau_d), \\
g_{k_s} &= \imath_r\rho^m + \imath_{u_s}(u_s - \bar{u}_s) + \phi, \; g_{k_c} = \imath_c(\tilde{\tau}_d p_d^r - \phi_3 \; z) + \imath_{u_c}(u_c - \bar{u}_c) + \phi, \\
g_k &= k_{cs}(g_{k_c} - g_{k_s}) + g_{k_s}, \; \left(k_{cs} = \frac{K_c}{K_c + K_s}\right) \\
\dot{k}_{cs} &= k_{cs}(1 - k_{cs})(g_{k_c} - g_{k_s}), \\
y &= (1 + \phi\alpha_\phi)y^e + \beta_n(\alpha_\phi y^e - \nu), \\
y^d &= c_w(\cdot) + c_c(\cdot) + (1 - k_{cs})(g_{k_s} + \delta_s) + k_{cs}(g_{k_c} + \delta_c) + g, \\
c_c(\cdot) &\equiv 0, \; c_w(\cdot) \equiv c_{wy}y_w^D, \; y_w^D = (1 - \tau_w)\upsilon y, \\
\rho^e &= y^e - \delta - \upsilon y, \quad (\delta = \delta_s = \delta_c) \\
\tilde{g} &= \tau_w \upsilon y, \\
\hat{p} &= \kappa\left[\left(\beta_{p_1}(u - \bar{u}) + \beta_{p_2}(\hat{\tilde{\tau}}_d + \hat{p}_d - \pi^m)\right) + \kappa_p\beta_{we}(e - \bar{e})\right] + \pi^m
\end{aligned}$$

The steady-state values of the dynamical system are (for $\bar{u} = \bar{e} = 1$):

$$\begin{aligned}
&l^o = y^p, \quad y^{eo} = y^p/(1 + \phi\,\alpha_{n^d}), \quad \nu^o = \alpha_{n^d}\, y^{eo}, \\
&\upsilon^0 = \frac{y^{eo} - (\phi + \delta)}{y^p}, \quad \rho^{eo} = y^{eo} - \delta - \upsilon^0 y^p, \\
&\pi^{mo} = \bar{\pi}, \quad \mathrm{i}^o = \rho^{eo} + \bar{\pi} = \phi + \bar{\pi}, \quad z^o = \frac{1}{\phi_3}\left(\frac{\phi}{\imath_c} + p_d^r\tilde{\tau}_d\right)
\end{aligned}$$

Under this fundamental $9D$ system, investment decisions are currently financed solely through the issuance of new equity (and surplus profits). In addition, the government operates with a balanced budget and no debt. There is no money holding now engaged in the functioning of the economy. The banking system determines (the change in) both the short-term and implicit long-term interest rates, with no explicit treatment of open market activities required to achieve both objectives.

[8] The lowercase letters indicate that the corresponding extensive form variable is now expressed in per unit of capital form.

3.2. Labor market policy

We begin our analysis by focusing on a simple subsystem of the real economy. We isolate the dynamics of real wages of workers in the labor market in their interaction with the goods market dynamics. For reasons of simplicity, we assume that asset holders' interest income is all saved $s_c = 1$. We study the stability of the basic system without the involvement of the inventory adjustment dynamics, no labor supply, and no carbon tax policy yet. Hence, we assume $\beta_{we} = 0$, $\beta_v = 0$, $\beta_{p_2} = 0$, and $\tau_d = 0$. Later, we gradually augment the model to study the implications of the labor supply and carbon policy on the real-side of the economy.

The simple system can then be defined as:

$$\boxed{\begin{aligned} \hat{\upsilon} &= \kappa \left[(\kappa_w - 1) \left(\beta_{p_1} \left(\frac{y}{y^p} - \bar{u} \right) \right) \right], \\ \hat{y}^e &= \beta_{y^e} \left(\frac{y^d}{y^e} - 1 \right) + (\phi - g_k) \end{aligned}} \tag{56}$$

with

$$\begin{aligned} y^d &= c_w + g_k + \delta + \tilde{g}\ y^e, \\ c_w &= y_w^D = (1 - \tau_w)\upsilon\ y, \\ y &= (1 + \phi\ \alpha_n) y^e + \beta_n (\alpha_\phi y^e - v^o), \\ g_k &= k_{cs}(g_{k_c} - g_{k_s}) + g_{k_s}, \\ \upsilon &= \omega / x,\ \kappa = (1 - \kappa_p \kappa_w)^{-1},\ \tilde{g} = \tau_w \upsilon y \end{aligned}$$

We study the stability of this small system (υ, y^e) by means of the characteristics of the Jacobian. For ease of exposition, we assume the following parameter values: $\phi = 0.02$, $\delta = 0.05$, $y^p = 0.9$, $\kappa_p = 0.55$, $\kappa_w = 0.45$, $\iota_c = 0.3$, and $p_d^r = 3.75$. For comparison purposes, we keep the same values for the subsequent larger systems to the extent possible.

Proposition 1 *(Stability conditions). The system of differential equations reaches a locally asymptotically stable steady-state provided that the adjustment speed in the goods market, β_{y^e}, is moderate, and β_{p_1}, which reflects the price adjustment speed parameter, is sufficiently large.*

Proof. The stability of the system (close to equilibrium), can be analyzed by employing the "Routh-Hurwitz conditions".

The Jacobian $J(y^{e^*}, \upsilon^*)$ of the above system is:

$$\begin{bmatrix} \beta_{p1}(0.7309 - 0.813 y^{e^*}) & -0.813\ \beta_{p_1}\ \upsilon^* \\ 1.002\ \beta_{y^e}\ y^{e^*} & 0.096 - 0.213 y^{e^*} + \beta_{y^e}(-0.893 + 1.002\ \upsilon^*) \end{bmatrix}$$

y^{e^*} and υ^* represent the steady-state solutions of y^e and υ. The system is locally stable if the trace of the Jacobian, $\mathrm{Tr}[J(y^{e^*}, \upsilon^*)] < 0$ and the determinant $\mathrm{Det}[J(y^{e^*}, \upsilon^*)] > 0$; which translates to the simple conditions $0.0289\beta_{y^e} - 0.096 < 0$, and $0.6739\beta_{p_1}\beta_{y^e} > 0$ respectively. □

Numerically the dynamics of the above 2D system can be illustrated by the time series and phase diagram as shown in Fig. 1.

For $\beta_{y^e} = 0.4$ and $\beta_{p_1} = 2.5$, we have negative real parts of the complex eigenvalues $\{-0.0422 - 0.81985i, -0.0422 + 0.81985i\}$; hence, this system is stable since the steady-state will be reached even after a disturbing the system. The oscillation will quickly bring the system back to the approximate setpoint $\{0.92, 0.90\}$.

Let us now augment the subsystem and assume $\beta_{we} > 0$, then the simple system can then be defined as:

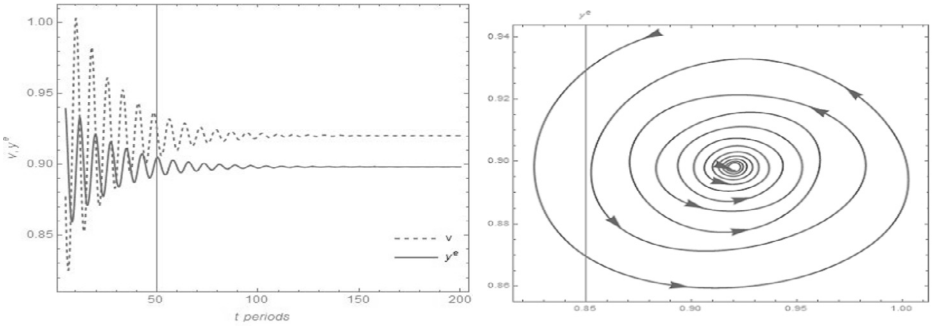
$$\boxed{\begin{aligned} \dot{\upsilon} &= \kappa \left[(1 - \kappa_p)\beta_{we}(e - \bar{e}) + (\kappa_w - 1) \left(\beta_{p_1} \left(\frac{y}{y^p} - \bar{u} \right) \right) \right] \upsilon, \\ \dot{y}^e &= \beta_{y^e} \left(y^d - y^e \right) + (\phi - g_k) y^e, \\ \dot{l} &= (\phi - g_k) l, \end{aligned}} \tag{57}$$

with, in addition to the prior definitions, $e = \frac{l^d}{l}$.

We study the stability of the system (υ, y^e, l) by means of the characteristics of the Jacobian.

Proposition 2 *(Stability conditions). The steady state of the system of differential equations (57) is locally asymptotically stable if conditions in Proposition 1 are met and in addition the wage adjustment parameter β_w is sufficiently small.*

Proof. The stability of the system (close to equilibrium) can be analyzed using the "Routh-Hurwitz conditions".



(a) Time Series **(b)** Converging Spiral

Fig. 1. Phase Diagram of $(v,\, y^e)$, for the parameter values: $\beta_{p_1} = 2.5$, $\beta_{y^e} = 0.4$, $y^p = 0.9$, $\tau_w = 0.3$.

The Jacobian $J(v^*, y^{e^*}, l^*)$ of the above system is:

$$\begin{bmatrix} 0.73\beta_{p_1} - 0.59\beta_{we} - \left(0.81\beta_{p_1} - 0.6\frac{\beta_{we}}{l^*}\right) y^{e^*} & \left(-0.813\beta_{p_1} + 0.6\frac{\beta_{we}}{l^*}\right) v^* & -0.6\beta_{we}\frac{v^*\ y^{e^*}}{l^2} \\ 1.002\beta_{y^e} y^{e^*} & \beta_{y^e}\left(1.002v^* - 0.89\right) - 0.213y^e + 0.096 & 0 \\ 0 & -0.334\ l^* & 0.3 - 0.334\ y^{e^*} \end{bmatrix}$$

The 3D system is locally stable if the trace of the Jacobian, $\mathrm{Tr}[J(v^*, y^{e^*}, l^*)] < 0$, the determinant $\mathrm{Det}[J(v^*, y^{e^*}, l^*)] < 0$, and $c(3) < 0$[9]; which translates to the conditions $0.0289\beta_{y^e} - 0.096 < 0$, $0.0529\beta_{we}\beta_{y^e} < 0$, and $0.01950\beta_{p_1}\beta_{y^e}^2 - 0.0647\beta_{p_1}\beta_{y^e} - 0.0159\beta_{we}\beta_{y^e}^2 - 6.245\times 10^{-18}\beta_{we}\beta_{y^e} < 0$ respectively. □

Fig. 2 below illustrates numerically the dynamics of the above 3D system, showing both the time series of the three variables $\{v, y^e, l\}$ and the phase plot of $\{v, y^e, e\}$. The system is stable so long as the adjustment parameter β_{we} (the reaction coefficient of employment rate e in wage Phillips curve) remains low.

So far we have analyzed a simple subsystem of the real side of the economy without a climate policy. Next, we look into the impact of a carbon tax on the dynamics of the labor market and expected sales. We allow both ι_c and β_{p_2} this time to be positive. While in the following sections we are studying the stability of the higher dimension system numerically, it is still possible to systematically analyze the stability of higher dimensional systems analytically by taking into account the already established stability conditions in the lower dimension subsystems, and then focus only on calculating the (well-behaved) determinants of the higher-order systems. In other words, the procedure consists of establishing a cascade of stable matrices (see. Chiarella et al. (2006)).

The higher system of equations can be expressed as:

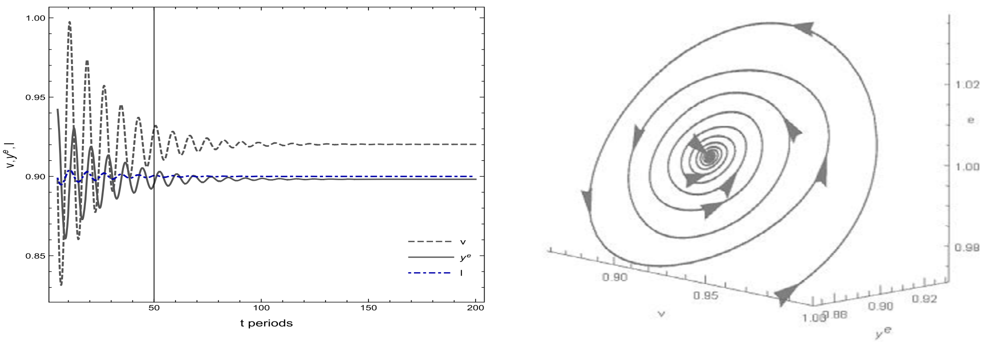
$$\boxed{\begin{aligned} \dot{v} &= v\ \kappa\left[(1-\kappa_p)\beta_{we}(e-\bar{e}) + (\kappa_w - 1)\left(\beta_{p_1}(u-\bar{u}) + \beta_{p_2}\hat{\tilde{\tau}}_d\right)\right], \\ \dot{y}^e &= \beta_{y^e}\left(y^d - y^e\right) + (\phi - g_k)y^e, \\ \dot{l} &= l(\phi - g_k), \\ \dot{z} &= \phi_1(\iota_c(\tilde{\tau}_d\ p_d^r - \phi_3 z))z, \\ \dot{\tilde{\tau}}_d &= g_{\tau_d}\tilde{\tau}_d, \end{aligned}} \tag{58}$$

with

$$g_k = k_{cs}(g_{k_c} - g_{k_s}) + g_{k_s}$$

Investment decisions in green capital, through g_k, now play a role in this subsystem. In turn, as we previously assumed, the growth rate of capital in the economy is affected by: the general environment (or sentiment) in the economy, which we consider in this special case to be at its steady-state value zero $\rho^{m0} = 0$; the current capacity utilization in its deviation from normal capacity utilization, which introduces a short-term component into the investment behavior of firms; and the difference between the effective real price of fossil-fuel ($\tilde{\tau}_d\ p_d^r$) and some function of fossil-fuel intensity of production $f(z)$, which we consider to be of a simple linear form $f(z) = \phi_3\ z$. What this last expression tells us, is that as long as the real effective carbon price is larger than firms' carbon

[9] $c(3) = \mathrm{Det}\begin{bmatrix} J_{11}+J_{22} & J_{23} & -J_{13} \\ J_{32} & J_{11}+J_{33} & J_{12} \\ -J_{31} & J_{32} & J_{32}+J_{33} \end{bmatrix}.$



(a) Time Series **(b)** Converging Spiral

Fig. 2. Phase Diagram of (υ, y^e), for the parameter values: $\beta_{p_1} = 2.5$, $\beta_{y^e} = 0.4$, $y^p = 0.9$, $\tau_w = 0.3$.

intensity of production, it will push the firms to invest in green capital. Additionally, the tax on fossil-fuel (the carbon tax), which enters the investment decision will also indirectly have an impact on the employment demand in the labor market, through ι_c. We have assumed that the growth rate of the price of the fossil-fuel p^r is equal to the economy's inflation rate π^m; hence, the real price of fossil-fuel p^r_b is constant.

A numerical illustration of the dynamics of the 5D system above can give us an idea about the stability of the subsystem. As previously stated, we employ identical parameter values and initial conditions as in the previous 2D and 3D systems (to the extent possible); this will help us illustrate more effectively the impact of the additional channels on the stability of the system. New parameter values, related to the additional laws of motion and state variables, have been assumed, namely: the real price of fossil-fuel $p^r_b = 3.75$. With regards to the growth rate of the carbon tax, $g_{\tau d}$, determining the tax schedule over time, we consider three scenarios: one with a low constant per period growth rate (i.e., $g_{\tau d} = 0.1\%$); one with a higher constant growth rate (i.e., $g_{\tau d} = 0.5\%$); and finally, one with a time varying growth rate.[10] That is, the carbon tax τ_d's growth rate increases in the beginning, then tips after a given period and subsequently decays steadily to zero over the remaining time period considered. This way we assume a scenario where carbon tax can be slightly more ambitious in the beginning and at the same time it does not increase indefinitely over time.

Fig. 3(a) below shows the evolution of the tax schedule τ_d and the fossil fuel efficiency technology z over time. A high growth rate of the carbon tax over time leads to higher fossil fuel efficiency. This is also reflected in the evolution of the share of clean capital k_{cs}, which grows faster the higher is the carbon tax rate (see Fig. 3(b)). In the absence of a climate policy, there is no incentive to invest in clean capital, and the share of clean capital k_{cs} remains constant, as shown in the same figure. Over the long time horizon, the employment rate and capacity utilization, when a carbon policy with a tax schedule is implemented, are lower compared to when there is no carbon tax.

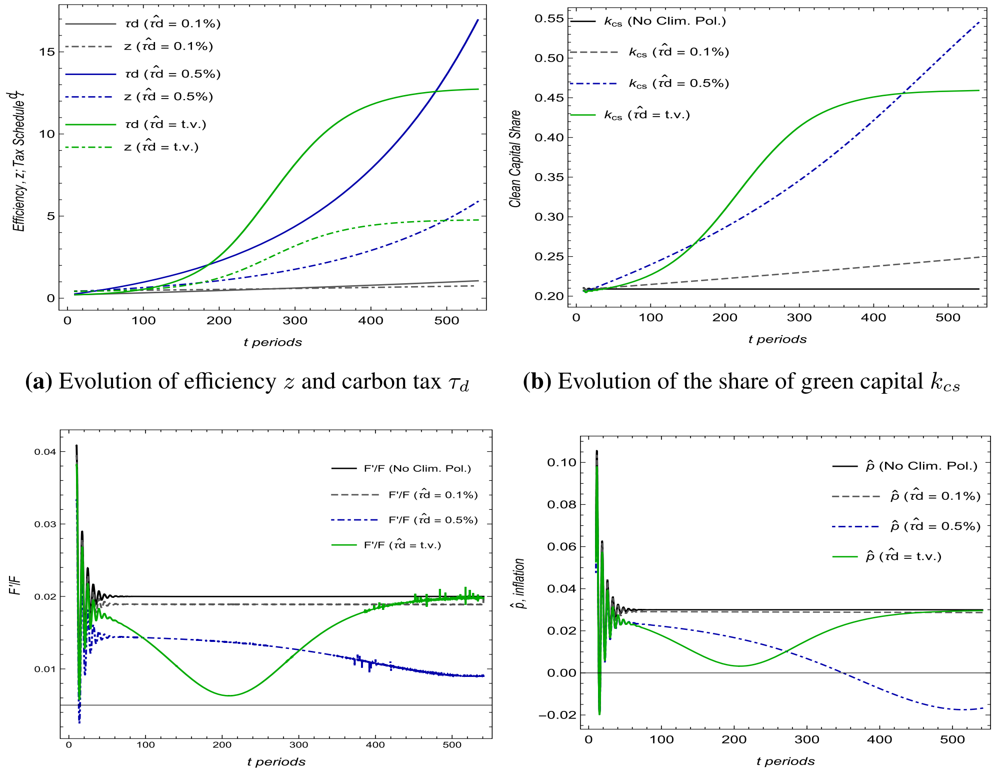
Introducing a carbon tax reduces the growth rate of fossil-fuel use, and the higher the exogenous rate of tax growth the lower the growth rate of fossil-fuel use. A similar observation is possible with respect to price inflation; a carbon tax causes deflationary pressure on prices, which increases (more deflation) the higher the carbon tax is, as shown in Fig. 3(d). This deflationary pressure is principally coming from a decrease in employment rate and capacity utilization. Low employment rate decreases consumption which in turn decreases expected sales. The stronger the tax measure the harder for the economic state variables to reach a constant steady-state out of the transient response. In the case of a time varying carbon tax though, where the tax growth rate goes to zero after a given period, the state variables revert back to a constant steady state.

For the system to remain stable, under a climate policy, the adjustment speed β_{we} and β_{p_2} are low. Also under the parameter values chosen, the carbon price responsiveness ι_c should remain low as well. A small change in the wage adjustment parameter β_{we} or a small change in the parameter ι_c of the carbon price responsiveness can affect the stability of the system. The stability of the system is obtained by a relatively low initial carbon tax τ_d, and low growth rate of the tax rate $\hat{\hat{\tau}}_d$.

A careful study of the dynamics of this sub-system suggests that implementing a carbon tax on fossil fuel use can be achieved without disrupting the real-side of the economy, provided the tax rate starts at a low level and gradually increases over time. However, a higher growth rate of the carbon tax, $\hat{\hat{\tau}}_d$, will lead to a prolonged increase in labor intensity, ultimately reducing employment and causing the economy to deviate from its natural equilibrium. Consequently, pursuing ambitious climate policies too rapidly may destabilize the economy, suggesting a more moderate green transition.

Our analysis thus far has primarily focused on the real-side of the economy, examining the impact of a carbon tax and its growth rate on key economic indicators such as expected sales and real wages. While these insights provide valuable perspectives, they overlook the intricate interplay between various sectors and markets, particularly the financial market and the government sector. In the subsequent section, we expand our scope to encompass these interconnected elements, gaining a more comprehensive understanding of how carbon tax policies and their dynamic nature influence the overall economic stability and growth trajectory.

[10] In our model, a time period represents one month; therefore, a monthly growth rate of 0.5% is equivalent to 6.16% annual growth rate.



(a) Evolution of efficiency z and carbon tax τ_d **(b)** Evolution of the share of green capital k_{cs}

(c) Growth rate of fossil fuel use **(d)** Evolution of price inflation

Fig. 3. Time series of τ_d, z, k_{cs}, F'/F (fossil fuel use growth), and $\hat{p}$ (price inflation), for the parameter values: $\beta_{p_1} = 2.5$, $\beta_{y^e} = 0.4$, $\iota_c = 0.3$, $\tau_w = 0.3$.

4. Higher dimensional considerations

4.1. Monetary policy and investment dynamics

In this section, we augment the model by incorporating the financial market and examine the impact of monetary policy on firms' investment behavior. Monetary policy affects economic activity primarily through two channels. The first channel entails banks' loan rates, which impact excess profitability that is used to define the investment climate or the amount of confidence that drives investment activity. The investment behavior is also influenced by the portfolio choice in the economy, which in particular dictates the pricing of equity securities and therefore affects the expected rate of return on such investments. Regarding the first channel, debt serves as a disciplinary mechanism by constraining firms' investment decisions. Additionally, interest payments to banks are a factor that affects firms' budget equation, thereby diminishing their pure profits.

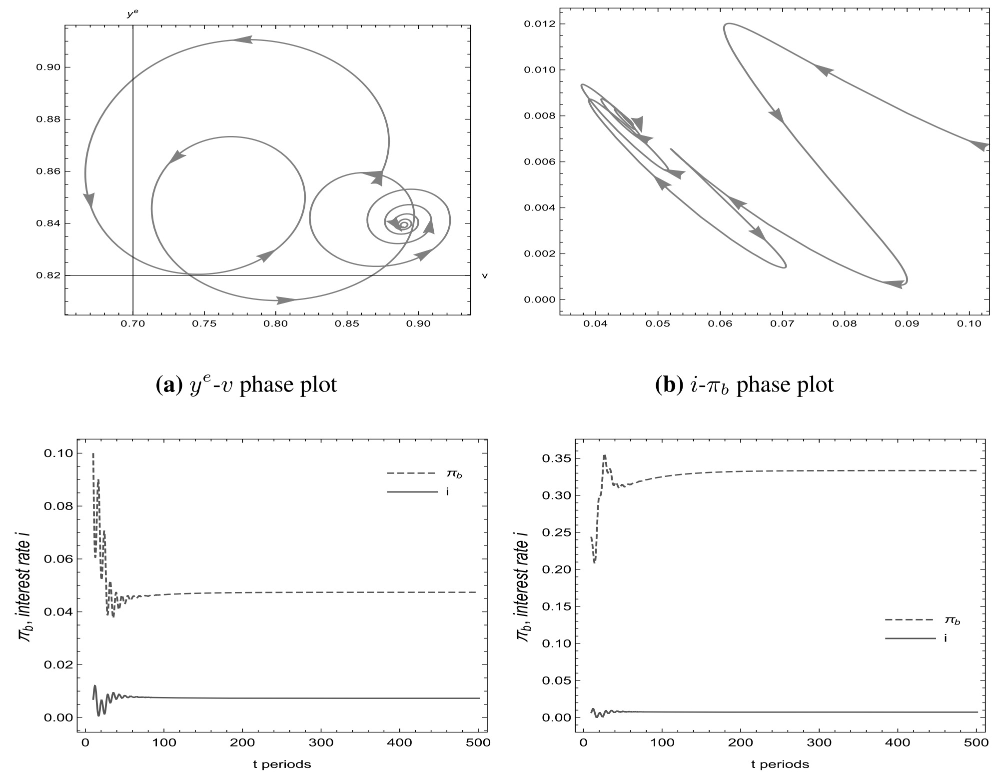
Capital allocation is now categorized into two distinct types: desired investment and constrained investment. The discrepancy between the debt to capital ratio that firms aim for and the actual ratio has an impact on the desired investment in this new configuration. The rate of investment is determined by the lower value between the desired rate and the rate that can be funded through firms' income, equity issuance, and bank loans. In this framework, the loan rate is established by a markup applied to the interest rate set by the central bank. It is posited that this markup is adversely associated with the condition of the business cycle, since it is anticipated that the costs of evaluating loan applicants are inversely connected to the state of the business cycle.

So in addition to the law of motions describing the real side of the economy (see above), we add the law of motions describing the financial sector. We also assume in the following two subsections the base case scenario for the carbon tax policy; meaning a low growth rate $g_{\tau_d} = 0.1\%$ and an initial tax rate $\tau_d = 20\%$. As in Chiarella et al. (2014) the laws of motion for the fundamentalists' and chartists' expectations:

$$\dot{\pi}_{ef} = \beta_{\pi_{ef}} \left(-\frac{\dot{b}_f^s}{1 - b_f^s} + \hat{p} + g_k - \phi - \pi_{ef} \right), \tag{59}$$

$$\dot{\pi}_{ec} = \beta_{\pi_{ec}} \left(\beta_{qc} \left(\frac{\tilde{q}}{q_c} - 1 \right) - \frac{\dot{b}_f^s}{1 - b_f^s} + \hat{p} + g_k - \phi - \pi_{ef} \right), \tag{60}$$

$$\dot{q}_c = \beta_{qc} (\tilde{q} - q_c), \tag{61}$$



(a) y^e-v phase plot

(b) i-π_b phase plot

(c) Expectation π_b, interest rate i time series

(d) Expectation π_b, interest rate i time series

Fig. 4. Phase Diagrams of (v-y^e and i-π_b phase plot), for the parameter values: $\beta_{p_1} = 2.5$, $\beta_{y^e} = 0.4$, $\iota_c = 0.3$, $\tau_w = 0.3$.

$$\dot{\pi}_{bf} = \beta_{\pi_{bf}} \left(-\frac{\dot{\mathrm{i}}}{\mathrm{i} + \xi} - \pi_{bf} \right), \tag{62}$$

$$\dot{\pi}_{bc} = \beta_{\pi_b} (\beta_{p_b} \left(\frac{p_b}{p_{bc}} - 1 \right) - \pi_{bc}), \tag{63}$$

$$\dot{p}_{bc} = \beta_{pb}(p_b - p_{bc}), \tag{64}$$

$$\dot{b}^s_f = g_k + \beta_{m_f}(\alpha_{m_f} y^e - m_f) + (\phi + \bar{\pi}) m_f - \phi \tilde{q} - \alpha_d \rho^e (1 - b^s_f) - (\hat{p} + g_k) b^s_f, \tag{65}$$

$$\dot{m}_f = \beta_{m_f}(\alpha_{m_f} y^e - m_f) + (\phi + \bar{\pi}) m_f - (g_k + \hat{p}) m_f \tag{66}$$

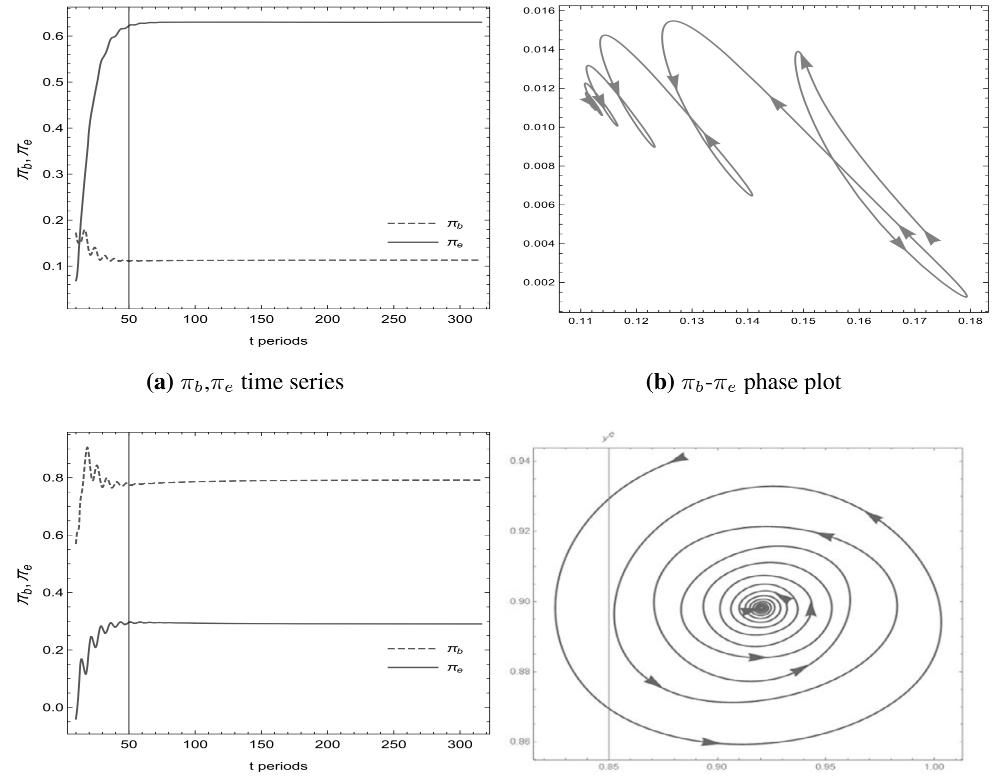
To focus on the credit channel in this special case of the general model, we assume $\hat{E} = \phi$ as equity issuing policy. In the present section, the current higher-dimensional system shows a greater tendency towards instability, primarily due to the introduction of additional feedback mechanisms via financial markets. To maintain a certain stability in this subsystem, the adjustment speed β_{ii} is set to be high, whereas β_{ip} is kept notably low, as outlined in the Taylor equation (Equation (43)). The capital gains expectations for long-term bonds held by heterogeneous investors are illustrated in Figs. 4(c) and 4(d). It is observed that as the percentage of investors who base their decisions on fundamental analysis increases (fundamentalist traders), the expected capital gains from long-term bonds tend to decrease.

4.2. Fiscal policy

In this final section of the numerical analysis, we account for variable bond-to-capital ratios in the growth of government debt, while simultaneously considering an active government expenditure program that proactively adjusts to the fluctuating phases of the economic cycle. First, we introduce the additional dynamic equations compared to the previous section, followed by the algebraic equations required for their solution. Subsequently, we present a selection of numerical solutions that capture the intricate dynamics of this higher-dimensional system.

$$\hat{g} = \beta_{gd} \left(\frac{d_g}{\bar{d}_g} - 1 \right) - \beta_{gd} \left(\frac{s^n_g}{b^s_g + p_b b^l} + (\phi + \bar{\pi}) \right) \pm (u - \bar{u}) \pm (e - \bar{e}), \tag{67}$$

$$\dot{b}^s_g = \dot{B}^s_g / pK - (\hat{p} + g_k) b^s_g, \;\; \dot{B}^s_g / (pK) = -\alpha_{g^b} s^n_g, \tag{68}$$



(a) π_b,π_e time series **(b)** π_b-π_e phase plot

(c) π_b,π_e time series **(d)** π_b-π_e phase plot

Fig. 5. Time-series of π_b-π_e and phase plot of π_b-π_e, for the parameter values: $\beta_{p_1} = 2.5$, $\beta_{y^e} = 0.4$, $\iota_c = 0.3$, $\tau_w = 0.3$.

$$\dot{b}^l = \dot{B}^l/pK - (\hat{p} + g_k)b^l,\ p_b\dot{B}^l/(pK) = -(1-\alpha_{g^b})s_g^n. \tag{69}$$

Including the fiscal authority in the dynamic model provides us with additional feedback effects and interesting results. Compared to the previous case, with only the monetary authority, the expected capital gain of equity is higher when the fiscal authority is present as opposed to the scenario with no fiscal authority. When there is a majority of fundamental traders in the economy, the expected capital gain of equity is larger than expected capital gain in bonds as illustrated in Fig. 5 (a). The opposite happens when there is a larger share of chartists (noise traders) as shown in Fig. 5 (b); this is probably due to the fact that the carbon tax directly affects the expected return on equity of the firm. The economy's stability is also less sensitive to small changes in the growth rate of the carbon tax $\hat{\tau}_d$ in the presence of the fiscal authority.

5. Conclusion

In this paper, we have employed the *KMGT* model, as developed by Charpe, et al. (2015), to study the feedback dynamics of implementing a carbon price and its implications on the utilization of fossil fuels by firms, with a particular focus on economic stability. Our study, employing the *KMG* framework, stands at the intersection of multiple sectors and markets, offering a comprehensive macrofounded approach to understand these complex interactions.

Central to our research was the investigation of the labor market's role in stabilizing an integrated economy. We began by exploring the interplay between the dynamics of workers' real earnings and the goods market. Through stability analysis, we delineated the conditions necessary for maintaining equilibrium in both the 2-dimensional and the 3-dimensional systems. The introduction of a carbon tax into our model opened a window into the potential disturbances climate policies could bring, suggesting that a carefully considered approach—characterized by moderate tax rates, gradual increases, and careful calibration of adjustment speeds for carbon pricing—is crucial for mitigating these effects.

Further enriching our model, we incorporated elements such as the financial market and various monetary and fiscal policies. This expansion, while offering a more holistic view, also highlighted the inherent instability brought on by the increased complexity of the system. To address this, we proposed varying adjustment speeds in the Taylor rule equation, noting a positive correlation between the prevalence of fundamentalist investors and economic stability.

Our focus on carbon tax as a singular climate policy instrument opens the door to interesting questions about the resilience of the economy to operate a fast shift in decarbonization. This study lays the groundwork for such an inquiry, testing the economy's ability to adapt swiftly and efficiently to significant environmental policy changes. Future research could benefit from exploring additional instruments, like green bonds, and the implications of a green transition in diverse labor markets, drawing on insights from studies like that of Charpe (2015). The *KMG* model, with its potential for various extensions, provides a fertile ground for further investigation into these critical aspects of the green transition.

Symbols

B	outstanding government fixed-price bonds (priced at $p_b = 1$)
C_i	real private consumption
E	number of equities
G	real government expenditure
I	net investment in fixed capital
$\mathcal{I}$	intended inventory changes
J	Jacobian matrix in the mathematical analysis
K	stock of fixed capital
L^d	employed workforce, i.e., number of employed people
L	labor supply, i.e., supply of total working hours per year
M	stock of money supply
N	inventories of finished goods
N^d	desired stock of inventories
S_i	total saving
T^n	total tax collections
W_c	real wealth of capitalists
e	employment rate
i	nominal rate of interest on government bonds
p	price level
p_{b^l}	price of long-term bond
p_e	price of equity
q	Tobin average q
u	rate of capacity utilization; $u = Y/Y^P$
y	output-capital ratio; $y = Y/K$
y^e	sales expectations per unit of capital; $y^e = Y^e/K$
$\tilde{g}$	government expenditures per unit of fixed capital; $\tilde{g} = G/K$
b_f^s	proportion of bank loans
b_g^s	government's short-term bonds supply
m_f^s	money supply
β_{p1}	reaction coefficient of u in price Phillips curve
β_{p2}	reaction coefficient of carbon tax τ_d in price Phillips curve
β_{we}	reaction coefficient of e in wage Phillips curve
$\beta_{\pi_1^e}$	adjustment speed in revisions of the inflation climate π^m
δ	rate of depreciation of fixed capitals
ε	equity per capital
κ	coefficient in reduced-form wage-price equations
κ_p	parameter weighting $\hat{p}$ vs. π in price Phillips curve
κ_w	parameter weighting $\hat{w}$ vs. π in wage Phillips curve
ν	inventories per unit of capital $\nu = N/K$
ω	real wage rate w/p
ϕ	trend growth rate
π^m	general inflation climate
ρ^e	expected rate of return
ρ^m	the investment climate
ξ_f	interest rate markup
τ_d	carbon tax
τ_w	tax rate on wages

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix A. Expectation formation of fundamentalists and technical traders

Market participants are divided into two distinct groups: fundamentalists and chartists. Fundamentalists base their investment decisions on long-term economic fundamentals, such as Tobin's q, while chartists rely solely on historical price trends. Fundamentalists anticipate capital gains to converge to the rate of growth of fundamental equity prices with a speed determined by $\beta_{\pi_{ef}}$. The evolution of fundamental equity prices is dictated by the interplay of investment rates, inflation, the rate of growth of equity supply and the growth rate of $(1 - b_{sf})$, the proportion of capital stock that is not financed by bank loans.

Fundamentalist Expectation

$$p_{ef} = \frac{(1-b_f^s)(pK+pN+M_f)}{E} \times q_f$$

$$q_f = \frac{p_{ef}E}{(1-b_f^s)(pK+pN+M_f)} = 1$$

$$\hat{p}_{ef} = \frac{d(1-b_f^s)/dt}{1-b_f^s} + \frac{dp/dt}{p} + \frac{d\left(K+N+\frac{M_f}{p}\right)/dt}{K+N+\frac{M_f}{p}} - \hat{E}$$

$$\dot{\pi}_{ef} = \beta_{\pi_{ef}}(\hat{p}_{ef} - \pi_{ef})$$

Chartist Expectation

$$q = \frac{p_e E}{(1-b_f^s)(pK+pN+M_f)}$$

$$q_c = \frac{p_{ec}E}{(1-b_f^s)(pK+pN+M_f)}$$

$$\hat{q}_c = \beta_{qc}\left(\frac{q}{q_c} - 1\right) = \beta_{qc}\left(\frac{p_e}{p_{ec}} - 1\right)$$

$$\hat{p}_{ec} = \hat{q}_c + \frac{d(1-b_f^s)/dt}{1-b_f^s} + \frac{dp/dt}{p} + \frac{d\left(K+N+\frac{M_f}{p}\right)/dt}{K+N+\frac{M_f}{p}} - \hat{E}$$

$$\dot{\pi}_{ec} = \beta_{\pi_{ec}}(\hat{p}_{ec} - \pi_{ec})$$

In the context of long-term bonds, the formation of capital gains expectations follows a similar procedure. The intrinsic value of a long-term bond is calculated by incorporating a risk premium, denoted as ξ_b, to the short-term interest rate i. Fundamentalists are presumed to base their expected capital gains on the rate at which these fundamental bond prices rise, adopting a regressive approach. As for chartists, their formation of expectations is partially responsive to the prevailing equilibrium bond prices p_b, adjusting to these with a time lag as dictated by the equation governing the lag-adjusted bond prices p_{bc}. Consequently, their capital gains expectations are aligned with the growth rate of these lag-adjusted bond prices p_{bc}, adjusted in an adaptive manner.

Fundamentalist Expectation	Chartist Expectation
$p_{bf} = \frac{1}{\mathrm{i}(1+\xi_b)}$	$\hat{p}_{bc} = \beta_{pb}\left(\frac{p_b}{p_{bc}} - 1\right)$
$\dot{\pi}_{bf} = \beta_{\pi_{bf}}(\hat{p}_{bf} - \pi_{bf}) = \beta_{\pi_{bf}}(-\hat{\mathrm{i}} - \pi_{bf})$	$\dot{\pi}_{bc} = \beta_{\pi_b}(\hat{p}_{bc} - \pi_{bc})$

References

Charpe, M., Flaschel, P., Krolzig, H.-M., Proaño, C.R., Semmler, W., Tavani, D., 2015. Credit-driven investment, heterogeneous labor markets and macroeconomic dynamics. J. Econ. Interact. Coord., 163–181, Springer.

Chiarella, C., Flaschel, P., 2000. The Dynamics of Keynesian Monetary Growth: Macro Foundations. Cambridge University Press.

Chiarella, C., Flaschel, P., Franke, R., 2005. Foundations for a Disequilibrium Theory of the Business Cycle: Qualitative Analysis and Quantitative Assessment. Cambridge University Press.

Chiarella, C., Flaschel, P., Franke, R., Semmler, W., 2006. A high-dimensional model of real-financial market interaction: the cascade of stable matrices approach. Quant. Empir. Anal. Nonlinear Dyn. Macromodels 277, 359–384. https://doi.org/10.1016/S0573-8555(05)77011-5, Emerald Group Publishing Limited, Leeds.

Chiarella, C., Flaschel, P., Groh, G., Semmler, W., 2000. Disequilibrium, Growth and Labor Market Dynamics: Macro Perspectives. Springer Science & Business Media.

Chiarella, C., Flaschel, P., Semmler, W., 2014. Reconstructing Keynesian Macroeconomics Volume 3: Macroeconomic Activity, Banking and Financial Markets. Routledge.

Flaschel, P., Franke, R., Semmler, W., 1997. Dynamic Macroeconomics: Instability, Fluctuation, and Growth in Monetary Economies. Mit Press.

Flaschel, P., Krolzig, H.-M., 2006. Wage price Phillips curves and macroeconomic stability: basic structural form, estimation and analysis. Contrib. Econ. Anal. 277, 7–47.

Franke, R., Flaschel, P., Proaño, C.R., 2006. Wage-price dynamics and income distribution in a semi-structural Keynes-goodwin model. Struct. Chang. Econ. Dyn. 17, 452–465. https://doi.org/10.1016/j.strueco.2006.08.001.

Goodwin, R.M., 1982. A Growth Cycle. Palgrave Macmillan UK, London, pp. 165–170.

Keynes, J.M., 1937. The general theory of employment. Q. J. Econ. 51, 209–223.

Metzler, L.A., 1941. The nature and stability of inventory cycles. Rev. Econ. Stat. 23, 113.

Modigliani, F., Miller, M.H., 1958. The cost of capital, corporation finance and the theory of investment. Am. Econ. Rev. 48 (3), 261–297.

Richters, O., 2021. Modeling the out-of-equilibrium dynamics of bounded rationality and economic constraints. J. Econ. Behav. Organ. 188, 846–866.

Rose, H., 1967. On the non-linear theory of the employment Cycle1. Rev. Econ. Stud. 34 (2), 153–173.

Tobin, J., 1969. A general equilibrium approach to monetary theory. J. Money Credit Bank. 1, 15–29.