Ecological Economics 230 (2025) 108519

Contents lists available at ScienceDirect

Ecological Economics

journal homepage: www.elsevier.com/locate/ecolecon

Analysis

Decoupling economic growth from energy use: The role of energy intensity in an endogenous growth model

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ARTICLE INFO

JEL classification:
O31
O44
Q20
Q43
Q55

Keywords:
Limits to growth
Endogenous growth
Exhaustible resources
Renewable energy
Product variety
Thermodynamic laws
Energy
Decoupling

ABSTRACT

We develop a theory of endogenous economic growth with explicit consideration of energy in the production process. Following basic thermodynamic considerations, energy is modeled as a (perfect) complement to machines. Long-run economic growth is driven by expanding product varieties. While energy flows on Earth are currently abundant, extrapolation of past consumption trends suggests that energy supply might be a binding constraint in a few centuries to millennia. We show that constant economic growth with bounded energy use is possible if the energy intensity of newly developed products declines at a constant, positive, and arbitrarily small rate. Hence, aggregate decoupling is possible even when no decoupling at the product level is possible. Aggregate decoupling is, however, not possible if there exists a strictly positive lower bound for the energy intensity of newly invented products. We further show that increasing energy prices decrease growth rates by reducing the incentive to innovate. Our results suggest that the energy intensity of structural change is decisive for future growth.

1. Introduction

Energy consumption and economic growth have been linked throughout history. Fig. 1 illustrates the rise in primary energy consumption by energy source and global GDP since 1800. Despite the development and exploration of new energy sources, the consumption of each individual fuel type has increased for the past 200 years, indicating that, at least historically, new energy sources are rather complements than substitutes to existing energy sources. These megatrends (King, 2021) raise questions about the dependence of economic growth on energy consumption and the impact of limited available energy resources on the economy.

In this paper, we build an endogenous growth model, which explicitly accounts for the role of energy in the production process, to study economic growth within thermodynamic limits. Our setup extends the expanding product variety model of Romer (1990) and Grossman and Helpman (1991) by energy inputs in the intermediate goods production. We highlight three key features, which are necessary to model long-run economic growth in accordance with thermodynamic laws.

First, energy is not a standard production factor that can be easily substituted. It is essential in some form for any production process or service provision. As Keen et al. (2019, p. 41) put it: "Labour without energy is a corpse, while capital without energy is a sculpture". Therefore, we model energy and machines as complements using a Leontief production function.

Second, energy efficiency for a specific machine or product cannot increase indefinitely. The conversion of energy into useful work is constrained by thermodynamic laws. Once the upper limit of energy efficiency for a specific product is reached, further gains can only occur through the invention of new varieties that are less energy-intensive than their predecessors. We explore this process using an expanding variety model, where new varieties progressively become less energy-intensive.

Third, we address the possibility of limited energy supply and its implications for economic growth. Without absolute decoupling of energy use from economic growth, energy demand grows exponentially and will eventually encounter physical limits. As energy becomes scarcer, rising energy prices reduce the growth rate of the economy.

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https://doi.org/10.1016/j.ecolecon.2025.108519
Received 7 November 2023; Received in revised form 13 December 2024; Accepted 2 January 2025
Available online 15 January 2025

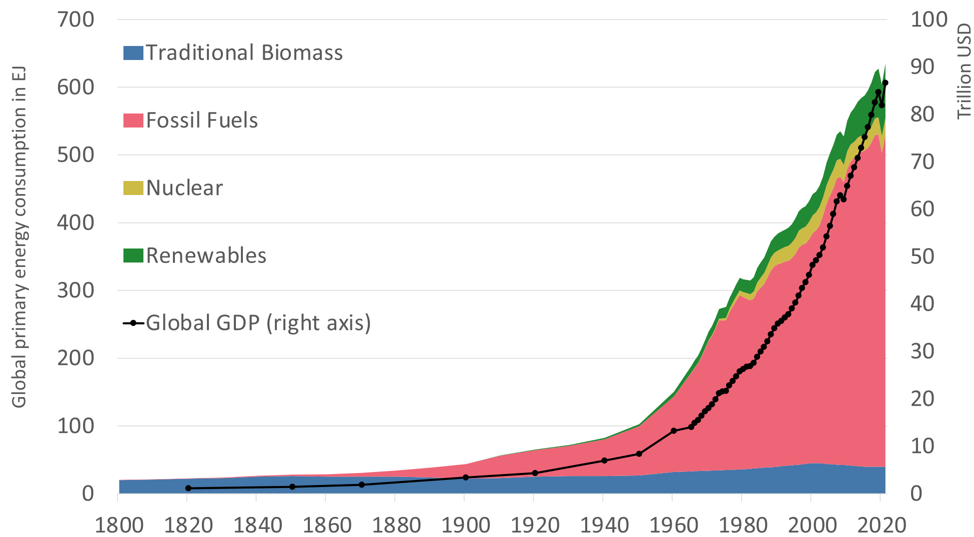

Fig. 1. Global primary energy consumption by source and global GDP. *Source:* Our World in Data based on Smil (2017), bp (2022) and Bolt and van Zanden (2020) for GDP, own representation.

We establish conditions under which decoupling of energy use from economic growth is possible, and when it fails.

Our results indicate that decoupling of energy use and economic growth depends crucially on the energy intensity of newly invented goods and services. When the energy intensities of new products do not decline constantly, economic growth rates are sensitive to the level of energy prices. As energy supply in the very long-run is constrained by inflowing solar radiation, energy prices increase due to scarcity pushing down the profits of innovators. Without innovation economic growth will cease and constant GDP levels will prevail. When, however, the energy intensity of new products declines at a constant rate, energy use can be decoupled from economic growth counteracting the effect of increasing energy prices.

On a product (or machine) basis, energy is always a perfect complement, and increases in energy efficiency are impossible. However, the invention of new products with (continuously) lower energy intensities can provide a sustained basis for long-run economic growth. This type of structural change is similar as in van Zon and Yetkiner (2003) and Bretschger and Smulders (2012). Our work departs from existing studies by not allowing for (limited) substitutability between energy and capital. If new products become less energy and material intensive, e.g. because they depend more and more on ideas, design, art, or intellectual works, aggregate energy intensity of the economy decreases. If there exists no lower bound for the energy intensity of new inventions, sustained economic growth is possible with any positive level of energy consumption. One of our key findings is that aggregate decoupling can occur, even when decoupling is impossible at the sectoral or product level. Yet, aggregate decoupling is not possible if there exists a strictly positive lower bound for the energy intensity of newly invented products.

The rest of the paper is structured as follows. Section 2 reviews the existing literature and discusses the importance of energy for economic growth. Section 3, then, introduces the extended product variety model with energy. Section 4 investigates the conditions under which long-run growth is possible and when it fails. Finally, Section 5 discusses the possibility of discontinued products, while Section 6 concludes.

2. Literature review and the role of energy

The question of sustained economic growth and limited or exhaustible resources was intensively studied in the wake of the oil crisis in the 1970s. Based on a quantitative Malthusian model of the global economy, the Club of Rome (Meadows et al., 1972) warned that the depletion of natural resources and increasing environmental pollution will lead to a collapse of the global economy. Using a neoclassical growth model, Dasgupta and Heal (1974) showed that the role of the elasticity of substitution between natural resources and capital is decisive for positive long-term growth with exhaustible or limited natural resources. When the elasticity of substitution exceeds unity, finite resources can be substituted by reproducible capital, thereby allowing economic growth to be decoupled from resource use. If the elasticity of substitution is below one, long-term consumption will converge to zero as natural resources are depleted. Stiglitz (1974) and Solow (1974) studied the particular case of unit elasticity. They showed that a constant consumption path can be maintained for a sufficiently high rate of technological progress or a sufficiently low income share of natural resources.

More recent work discussing the question of decoupling, focus on the accumulation of knowledge and ideas as the main driver of economic growth (Smulders, 1995). Endogenous growth models are fueled by either increases in quality or quantity through the implementation of a new idea. Bretschger (1998, 2005) discusses the role of substitutability, growth, and resource use in endogenous growth models. A key conclusion from the endogenous growth models including natural resources is that structural change from resource-intensive sectors to resource-efficient sectors can help decouple economic growth from resource use (Bretschger and Smulders, 2012). However, most of these models assume that knowledge or human capital is a substitute for energy or resources. Aghion and Howitt (1998, Chapter 5) show that growth with non-renewable resources is impossible in an AK-type growth model, but that introducing human capital as a substitute for the non-renewable resource can under certain conditions allow for decoupling.

While van Zon and Yetkiner (2003) develop a similar modification of the Romer model, they consider an elasticity of substitution between capital and energy of unity. Based on our thermodynamic argument made before, we explore the implications for growth when energy cannot be substituted by capital. For illustrative purposes, we model energy as a perfect complement to capital.

Casey (2024) studies the impact of climate change mitigation policies on energy use and builds a Schumpetarian growth model by also explicitly accounting for energy in the production function via a Leontief structure. However, as usual in Schumpetarian endogenous growth models, his setup is concerned with quality increases for the aggregated mass of products. We argue that these energy efficiency gains are bounded on a product basis and can, ultimately, only be realized for new varieties. Hence, we model the growth process as increasing the product variety allowing for declining energy intensities in heterogeneous products.

Our model setup differs from previous models in the literature in three ways: First, we explicitly account for energy as an essential input in production with limited substitution possibilities using a Leontief production function between energy and intermediates. Second, we stress the importance of limited energy intensity gains on a product basis. Thus, we use an expanding product variety setting, allowing for heterogeneous intermediate goods with respect to the energy intensity. Third, we are concerned with the possibility of limited energy supply in the long-run.

Other papers incorporate similar features to our model setup but focus either on R&D policies to foster environmentally friendly product lines (Ricci, 2007; Hart, 2004), the role of directed technical change (André and Smulders, 2014), or the role of research subsidies and emission pricing (Hart, 2019).

Hart (2004) and Ricci (2007) both develop models in which they focus on the direction of R&D and how regulation and taxation might incentivize socially optimal research activities. Hart (2004) builds a vintage model with negative external effects of production. The invention of new, more productive, and cleaner vintages increases output and decreases damages from production, but might be below the socially optimal level because of market power and the fact that the social benefit of lower damages is external to the producers. Ricci (2007) extends the Schumpetarian model of endogenous growth by considering pollution from production. In his setup, pollution enters production in a Cobb–Douglas style production function. Each innovation improves the productivity of the intermediate good and decreases the pollution intensity. In contrast to Ricci (2007) we assume that energy and intermediate goods are complements, whereas we then study improvements in energy intensity in an expanding product variety model.

These studies show that decoupling natural resource use from economic growth is possible if either reproducible or human capital is a substitute for natural resources or resource efficiency grows sufficiently large. Both conditions, however, likely violate crucial thermodynamic laws when applied to the use of energy as an input for specific goods or services (Ockwell, 2008; Ayres, 1998).

We want to emphasize that energy is not an ordinary production factor. At least in some form energy is essential for any production process and for any provision of services, so that it cannot be substituted by human or man-made capital. We study decoupling within thermodynamic laws when we cannot substitute away from an essential and finite production factor, namely energy, for which efficiency gains on a product basis are bounded.

Consider the case of electricity. According to the first law of thermodynamics, the energy contained in electricity cannot exceed the energy input required for its generation, whether derived from fossil fuels, nuclear energy, or solar radiation. The ratio of the energy output in the form of electricity to the energy input is referred to as conversion efficiency. While technological advancements and capital-intensive processes can improve this efficiency, such as by reducing the amount of gasoline needed to drive 100 km by means of more efficient engines that convert more energy into motion and less into heat, the first law of thermodynamics dictates that conversion efficiency cannot surpass unity. Consequently, the energy efficiency of an existing process cannot be improved indefinitely.

A comprehensive meta-study on decoupling finds that primary energy consumption can only be decoupled from GDP to the extent that conversion efficiency from primary to useful energy is improved (Haberl et al., 2020, p. 32). Hence, greater capital investment and technological progress alone cannot sustain unlimited increases in the availability of useful energy.

This line of argumentation can be expanded to various machines and services that create (physical) work. Cullen and Allwood (2010) provide an overview of the theoretical energy efficiency limits for various devices like lighting devices, engines for creating motion, combustion devices for heat, etc. While there exists a large potential for energy efficiency increases of up to 90% of current global energy demand — these increases are strictly bounded level effects and cannot help decouple energy input from physical work. As the creation of consumption goods and services is an entropy increasing process which requires physical work, the limits to substitutability and energy efficiency increases apply as well (Daly, 1987). If all energy were exhaustible, positive consumption would therefore not be conceivable in the long-run.

Table 1
Average annual growth rates of energy and GDP in % p.a. up until 2018.
Source: Our World in Data based on Smil (2017) and bp (2022) for energy data and MADDISON-2020 database (Bolt and van Zanden, 2020) for real GDP.

Time period	Energy	Energy per capita	GDP per capita
Since 1820	1.69	0.67	1.33
Since 1900	2.28	0.90	1.65
Since 1970	2.01	0.47	1.97
Since 1990	1.74	0.41	2.22
Since 2010	1.54	0.35	1.81

While renewable energy could ensure a positive level of consumption, it could not provide an alternative base for sustained economic growth. Current global energy consumption of 635 EJ in 2021 is roughly 0.016% of the total incoming solar energy (see Table 2), but even incoming solar energy is by no means infinite. Historical growth rates of energy use have decreased to 1.54% over the last decade (Table 1). But even when accounting for population growth and assuming that the growth rate of aggregate energy use per person will continue to decline, total energy demand would surpass incoming solar energy fluxes within the next three to four millennia, assuming a growth rate of 0.25%. The current estimated maximum of technical feasibility is several magnitudes lower. Although more than 1,000 years might seem substantial, this period is relatively short when compared to major technological breakthroughs in human history, such as the Neolithic revolution (10,000 years ago), the invention of writing systems (5,400 years ago), paper (2,100 years ago), the printing press (570 years ago), or electricity (400 years ago). The historical perspective of current demand growth, along with future extrapolations, is illustrated in Fig. 2.

Without absolute decoupling of energy use from economic growth, energy demand grows exponentially and will at some stage hit physical boundaries. This also applies to other types of energy besides solar: fossil energy, geothermal energy or nuclear energy seem to be plentiful with respect to current energy consumption — with exponential growth, they are all scarce resources. Table 2 estimates the number of years until energy demand reaches renewable energy supply and until non-renewable resources are completely exhausted for various assumptions on energy demand growth rates. The table depicts physical availabilities of energy which provide a physical upper bound of energy sources. The technical and economic potential is usually several orders of magnitude lower (Moriarty and Honnery, 2012) but also subject to large uncertainties. Nevertheless, the key message of focusing on the maximum physical energy potential is marked: Without absolute decoupling, long-term wealth will be constrained by the amount of incoming solar energy.

3. The expanding variety model with energy

We extend the endogenous growth model with expanding product variety (Romer, 1990; Grossman and Helpman, 1991) by explicitly considering energy as an essential input.

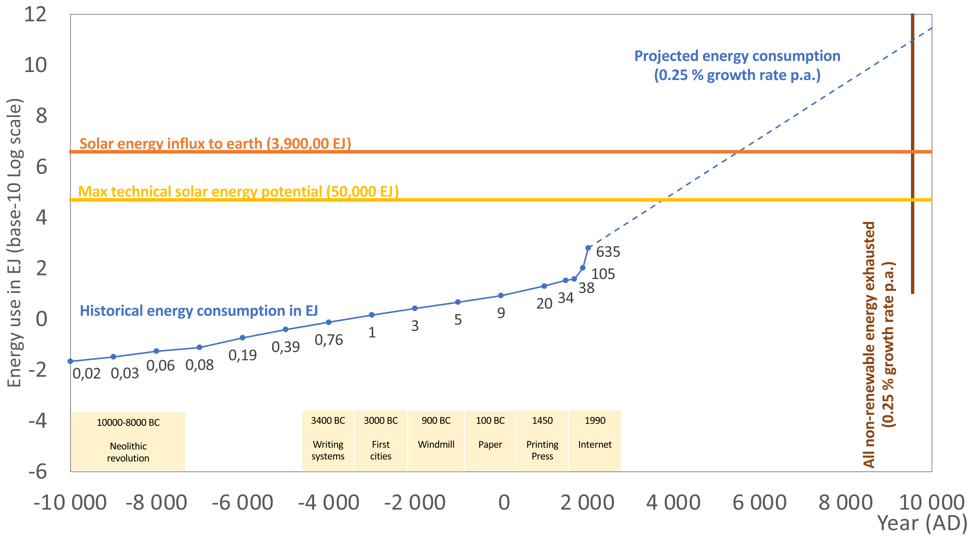

Fig. 2. Energy use and key innovations over large time scales. *Source:* Own illustration based on historic energy data from Fischer-Kowalski et al. (2014), bp (2022), Smil (2017) and Table 2 with projections for 0.25% growth rates of energy use.

Table 2
Physical potential and duration of various energy sources.

Energy type	Maximum availability [EJ]	Consumption in 2021 [% max. availability]	Years until max. achieved/exhausted with demand growth rate, p.a. [%]				
			2.00	1.50	1.00	0.50	0.25
Renewable energy							
Max technical solar energy potential	49,837	1.27	220	293	438	888	1,747
Solar energy influx to Earth	3,900,000	0.016	440	586	877	1,749	3,493
Exhaustible energy							
Fossil (reserves and resources)	528,100	0.12	144	174	223	328	449
All Carbon in Earth crust	168,000,000	0.0004	432	556	791	1,440	2,601
All Uranium in Earth crust	6.0E+12	1.1E−08	961	1,260	1,845	3,542	6,799
All Thorium in Earth crust	2.1E+13	3.0E−09	1,025	1,345	1,972	3,796	7,307
Geothermal potential	1.0E+13	6.4E−09	987	1,294	1,896	3,644	7,003
All exhaustible combined	3.7E+13	1.7E−09	1,054	1,382	2,028	3,909	7,531

Notes: For renewable energy: Duration until maximum availability of energy is reached assuming constant demand growth rate; for exhaustible energy: Duration until energy resources are depleted assuming constant demand growth rate. *Sources:* Solar energy: (Moriarty and Honnery, 2012, p. 247) and UNDP (2000, p. 163); Fossil energy refers to estimated reserves plus resources (BGR, 2021, p. 62); Geothermal energy refers to the total heat content of the Earth (Rybach, 2021, p. 15); Energy from exhaustible resources (Carbon, Uranium, and Thorium) in Earth crust are estimated from density estimates in Lide (2012) and multiplied with the specific energy densities. Other renewable energy sources (wind, ocean, hydro, and biomass) are several orders of magnitude lower than solar energy (Moriarty and Honnery, 2012).

3.1. Output sector

Output of the representative firm is given by

$$Y(t) = AL^{1-\alpha}\int_0^{N(t)} z_j(t)^\alpha \, dj, \tag{1}$$

with A a parameter on the overall level of productivity, L the (exogenous) labor input, $z_j(t)$ the jth intermediate composite, $N(t)$ the number of varieties and $\alpha < 1$. Contrary to conventional endogenous growth theory, we consider $z_j(t)$ to be a composite of the original intermediate, $x_j(t)$, and energy, $E_j(t)$, that is necessary for the use or employment of $z_j(t)$. We assume for simplicity that all products are used at their thermodynamic efficiency limit and that energy is a perfect complement to $x_j(t)$. Hence, using $x_j(t)$ requires $\varepsilon_j x_j(t)$ units of energy; if less energy is used, $z_j(t)$ decreases linearly in energy input.[1] We can therefore describe $z_j(t)$ by the Leontief production function[2]

$$z_j(t) = \min\left\{\frac{E_j(t)}{\varepsilon_j}, x_j(t)\right\}. \tag{2}$$

Profits of the final good producer are given by

$$\pi_Y(t) = Y(t) - w(t)L - \int_0^{N(t)} \Big(P_j(t)x_j(t) - Q(t)E_j(t)\Big)\, dj, \tag{3}$$

with $w(t)$ the wage and $P_j(t)$ the price for the jth intermediate good $x_j(t)$. Households own the energy resource, $E(t)$, and sell it at price $Q(t)$ to the firm. With the Leontief production function, the optimal

[1] This formulation permits, in principle, substitution between the intermediate goods composite and either labor supply L or the productivity parameter A. However, as discussed in the previous section, unlimited improvements in A conflict with thermodynamic principles, as they would imply infinite energy efficiency gains for existing products. Therefore, we treat A as a constant scaling factor. Similarly, increases in labor input L would require additional energy, which we do not explicitly model, and such increases would not achieve per-capita decoupling. Ultimately, our model set-up focuses on growth that stems from the invention of new varieties.

[2] This assumption disregards any increases in energy efficiency for a specific product or machine. Using a more general function like a constant elasticity of substitution function would allow for a more flexible modeling of substitution possibilities. However, as the elasticity of substitution has to be below one, there is only a limited increase in energy efficiency possible until an upper bound has been reached. This more general modeling framework would increase analytical complexity but has no implications for the growth dynamics of the model because energy efficiency increases on a product basis are bounded.

use of energy per variety $E_j(t)$ is directly related to $x_j(t)$, such that $E_j(t) = \varepsilon_j x_j(t)$ and aggregate energy use is given by

$$E(t) = \int_0^{N(t)} E_j(t)\, dj = \int_0^{N(t)} \varepsilon_j x_j(t)\, dj. \tag{4}$$

Optimal labor input is determined by

$$w(t) = \frac{\partial Y(t)}{\partial L} = (1-\alpha)Y(t)/L. \tag{5}$$

Finally, the first-order condition for intermediates demand follows from maximizing Eq. (3) with respect to $x_j(t)$ and results in

$$x_j(t) = L\left(\frac{A\alpha}{P_j(t) + \varepsilon_j Q(t)}\right)^{\frac{1}{1-\alpha}}. \tag{6}$$

Since we are interested in the implications of limited energy supply, we assume that aggregate energy demand might be bounded by $\bar{E}$, so that it always holds that

$$E(t) \leq \bar{E}. \tag{7}$$

If energy demand is not bounded by the maximum available energy supply $\bar{E}$, energy is abundant and the price of energy $Q(t)$ is equal to zero. Once the limit becomes binding, there exists a scarcity price for energy greater than zero, so that $Q(t) > 0$. Thus, the following scarcity equation has to hold at all times

$$Q(t)(\bar{E} - E(t)) = 0. \tag{8}$$

3.2. Intermediate goods producers

Intermediate goods producers are monopolists of each invented blueprint j and set prices $P_j(s)$ at time s to maximize the net-present value of profits:

$$\max_{P_j(t)} V_j(t) = \max_{P_j(t)} \int_t^\infty \pi_j(s) e^{-\int_t^s r(u)du} ds \tag{9}$$

subject to the demand functions Eq. (6), with operating profits $\pi_j(s) = (P_j(s) - 1)x_j(s)$, (possible changing) interest rate $r(u)$ and by assuming production costs that are normalized to one. The optimization problem is a static one as there are no intertemporal constraints; we, therefore, omit the time variable s when explicitly stating it is not necessary. Intermediate goods are made out of final output and rented out from the household to the intermediate goods producers. Monopoly profits are maximized if the intermediate goods producers price the intermediate good at

$$P_j(t) = \frac{1 + (1-\alpha)\varepsilon_j Q(t)}{\alpha}. \tag{10}$$

By substituting Eq. (10) into Eq. (6) we receive the resulting demand for intermediates according to

$$x_j(t) = L\left(\frac{A\alpha^2}{\varepsilon_j Q(t) + 1}\right)^{\frac{1}{1-\alpha}}. \tag{11}$$

Energy demand per variety is then given by

$$E_j(t) = \varepsilon_j x_j(t) = \varepsilon_j L\left(\frac{A\alpha^2}{\varepsilon_j Q(t) + 1}\right)^{\frac{1}{1-\alpha}}. \tag{12}$$

It decreases with higher energy prices and is influenced by two counteracting effects for energy intensity improvements. First, energy use for every new variety decreases due to improved energy efficiency; second, there is a rebound effect, as higher production of $x_j(t)$ increases energy demand. The dominance of either effect depends on the condition $\alpha(1 + Q(t)\varepsilon_j) < 1$. For example, in the case without a binding energy limit, the energy price is zero, the condition $\alpha < 1$ holds, and energy use for every new product decreases as energy intensity improves.

Aggregate intermediates are given by

$$X(t) = \int_0^{N(t)} x_j(t) dj = \int_0^{N(t)} L\left(\frac{A\alpha^2}{\varepsilon_j Q(t) + 1}\right)^{\frac{1}{1-\alpha}} dj. \tag{13}$$

Operating profits in the intermediate sector are then

$$\pi_j(t) = (1-\alpha)\alpha A L\left(\frac{A\alpha^2}{\varepsilon_j Q(t) + 1}\right)^{\frac{\alpha}{1-\alpha}}. \tag{14}$$

One can see directly that higher energy prices, $Q(t)$, reduce the profits in the intermediate sector.

3.3. Households

Households maximize intertemporal utility over per-capita consumption with the iso-elastic utility function

$$u(c(t)) = \frac{c(t)^{1-\sigma} - 1}{1-\sigma} \tag{15}$$

and a pure time preference rate ρ. The budget constraint of the household is given by

$$c(t) = w(t)L + \int_0^{N(t)} r_j(t)K_j dj + Q(t)E(t) - \dot{K}(t) \tag{16}$$

with $K(t) = \int_0^{N(t)} K_j dj$ denoting assets. Households own the energy resource $E(t)$ and sell it at price $Q(t)$ to the firm. They maximize utility by choosing the consumption level $c(t)$ and the frontier capital stock $K_{N(t)}$ subject to the budget constraint Eq. (16). Hence, optimal saving $\dot{K}(t)$ is determined by the Euler equation

$$r_{N(t)} = \rho + \sigma g_c(t), \tag{17}$$

with $g_c(t) = \dot{c}(t)/c(t)$. Note that the interest-rate is dependent on the frontier technology $N(t)$ and by this also time dependent. The detailed derivation can be found in the Appendix.

3.4. Research and development

We model R&D in a simple but adapted fashion according to Barro and Sala-i-Martin (2004, Chapter 6).

Research firms are assumed to freely enter the market by paying a product specific but time-invariant research and development cost η_j. This assumption on the R&D process implies that a fraction of final output is used as an input to R&D. Specifically, R&D relies on inputs from non-consumed output, denoted $R(t) = \eta_{N(t)}\dot{N}(t)$. Thus, the economy has to spent $\eta_{N(t)}$ for the invention of every new product $\dot{N}(t)$. Basic arbitrage then requires that the value of firm j, $V_j(t)$, has to be equal to the product specific R&D cost η_j, so that it holds that

$$\eta_j = V_j(t) = V_j. \tag{18}$$

From this follows directly that the value of firm j is constant over time, since $\frac{\partial V_j}{\partial t} = \dot{V}_j = 0$.

Taking the time-derivative of Eq. (9), we receive an expression for the possibly time-dependent interest-rate

$$r_j(t) = \frac{\pi_j(t)}{V_j(t)} + \frac{\dot{V}_j(t)}{V_j(t)}, \tag{19}$$

which depends on variety j. Plugging in Eq. (14) and using that $\dot{V}_j = 0$, the product-specific interest-rate is given by

$$r_j(t) = \frac{\pi_j(t)}{\eta_j} = \frac{(1-\alpha)\alpha A L}{\eta_j}\left(\frac{A\alpha^2}{\varepsilon_j Q(t) + 1}\right)^{\frac{\alpha}{1-\alpha}}. \tag{20}$$

3.5. Equilibrium

By inserting Eq. (20) into Eq. (17) and plugging in $j = N(t)$, we can derive the consumption growth rate of the economy given by

$$\gamma(t) = g_c(t) = \frac{1}{\sigma}\left(\frac{(1-\alpha)\alpha AL}{\eta_{N(t)}}\left(\frac{A\alpha^2}{\varepsilon_{N(t)}Q(t)+1}\right)^{\frac{\alpha}{1-\alpha}} - \rho\right). \quad (21)$$

The growth rate of consumption is higher for a greater willingness to save, expressed by lower preference parameters of the household, ρ and σ. The growth rate also raises with better technology A or lower frontier R&D costs $\eta_{N(t)}$. For $\varepsilon_{N(t)}Q(t) = 0$, the growth rate takes the usual form as in the standard model of expanding product variety (Barro and Sala-i-Martin, 2004, p. 296). We can now directly see the impact of energy costs and energy intensity on the growth rate:

Lemma 1. *The higher the energy price $Q(t)$ or the higher the energy intensity of the frontier product $\varepsilon_{N(t)}$, the lower the growth rate of the economy.*

The basic intuition behind this result is that profits of innovative firms in Eq. (14) are reduced if energy prices are high. This reduces the incentive to innovate. The expansion of product variety which drives economic growth will therefore be slower. Furthermore, we also assume that $\gamma(t) \geq 0$ at all times. $\gamma(t) < 0$ would imply that the profits of the intermediate goods producers become negative leading to no incentive to innovate anymore. The number of varieties would stay constant, so that $\gamma(t) = 0$.

In a closed economy it has to hold that the market value of firms equals the households assets, so that

$$K(t) = \int_0^{N(t)} V_j dj = \int_0^{N(t)} \eta_j dj. \quad (22)$$

From the budget constraint of the household Eq. (16) we can then derive the aggregate macroeconomic balance of the economy. Household expenditures have to be equal to households income

$$c(t) + \eta_{N(t)}\dot{N}(t) = w(t)L + Q(t)E(t) + \int_0^{N(t)} r_j(t)K_j dj, \quad (23)$$

where $\eta_{N(t)}\dot{N}(t)$ are the expenditures for R&D, which are equal to the change in assets of the economy $\dot{K}(t) = \frac{\partial \int_0^{N(t)} K_j dj}{\partial t} = \frac{\partial \int_0^{N(t)} \eta_j dj}{\partial t} = \eta_{N(t)}\dot{N}(t)$. We can rewrite the integral in Eq. (23) by using the product specific interest-rate Eq. (20) and the fact that $K_j = \eta_j$ to receive

$$\int_0^{N(t)} r_j(t)K_j dj = \int_0^{N(t)} (1-\alpha)\alpha AL\left(\frac{A\alpha^2}{\varepsilon_j Q(t)+1}\right)^{\frac{\alpha}{1-\alpha}} dj = (1-\alpha)\alpha Y(t). \quad (24)$$

For the last equality insert the demand for intermediate goods from Eq. (11) into the final good production function Eq. (1).

Finally, inserting Eq. (5) in Eq. (23) and using the constant returns of scale property of the production function so that $\alpha^2 Y(t) - Q(t)E(t) = X(t)$, we can derive the macroeconomic balance of the economy given by

$$Y(t) = X(t) + c(t) + \eta_{N(t)}\dot{N}(t). \quad (25)$$

At all times final output has to be divided between intermediates good production, $X(t)$, consumption, $c(t)$, and the creation of $\dot{N}(t)$ new goods. Eq. (25) is a differential equation in the amount of varieties and can be rewritten by inserting the demand for intermediate goods from Eq. (11), so that

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{N(t)\eta_{N(t)}}\left(\int_0^{N(t)} L\left(\frac{A\alpha^2}{\varepsilon_j Q(t)+1}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha^2+\varepsilon_j Q(t)}{\alpha^2}\right) dj - c(t)\right). \quad (26)$$

Together with the energy demand of the economy Eq. (4), the scarcity constraint Eq. (8) and the consumption growth rate Eq. (21), the model is fully characterized by four equations in four unknowns: $E(t), Q(t), c(t)$, and $N(t)$. The full model is presented in Table 3.

4. Growth

This section discusses the growth process for four different cases. First, we discuss the case of abundant energy, in which the energy limit is not binding and show that the economy grows as in the standard expanding variety model. Second, we study the case of a binding limit on energy supply. Without improvements in energy intensity, decoupling is not possible and the economy converges to a steady-state with constant consumption, output and number of varieties. Third, if the energy intensity of newly invented products decreases, we show that decoupling is possible and that consumption, output and the number of varieties can grow unbounded even if the energy limit is binding. Finally, we study an economy in which improvements in energy intensity are possible, but approach a lower bound at some point. Again, decoupling is not possible and the economy converges to a steady-state with constant consumption, output and number of varieties once the lower bound of energy intensity is reached.

4.1. Growth with abundant energy

As long as the energy demand of the economy is not constrained by the maximum supply of energy, energy is abundant and the price of energy in our model economy is equal to zero. For $Q(t) = 0$, the demand for intermediate goods, the profit of intermediates and the entry costs of R&D become independent of j. The full model characterization from Table 3 reduces to

$$E(t) = \int_0^{N(t)} \varepsilon_j L\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} dj, \quad (27)$$

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{\eta}\left(\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} L\left(\frac{1-\alpha^2}{\alpha^2}\right) - \frac{c(t)}{N(t)}\right), \quad (28)$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left(\frac{(1-\alpha)L}{\alpha\eta}\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} - \rho\right). \quad (29)$$

Proposition 1. *If energy is abundant, so that it holds that $E(t) < \bar{E}$ and thus, the energy price $Q(t)$ is equal to zero, $g_c = g_N > 0$ is a balanced growth path of the economy.*

Proof. The growth rate of consumption Eq. (29) is constant such that we can solve for $c(t) = c_0 e^{\frac{1}{\sigma}\left(\frac{(1-\alpha)L}{\alpha\eta}\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} - \rho\right)t}$. Using this expression in Eq. (28) and solving the differential equation we receive $N(t) = \frac{\sigma c(t)}{\eta\rho + (1-\alpha)L(A\alpha^2)^{\frac{1}{1-\alpha}}(\alpha(\sigma-1)+\sigma)}$. Taking the log and the derivative with respect to time, we see that the number of varieties grows with consumption per capita, so that $g_c = g_N$. □

4.2. Growth with limited energy

As we have argued in Section 2, solar, fossil, geothermal and nuclear energy seem to be plentiful with respect to current energy consumption, however, with exponential growth, they are all scarce resources in the long-run. We now study our economy when energy is scarce, such that energy demand reaches maximum available energy supply, $E(t) = \bar{E}$. This also means that according to Eq. (8) the energy price is now greater than zero, $Q(t) > 0$.

To highlight the role of the energy intensity once the energy limit becomes binding, we first analyze the economy for a constant energy intensity $\varepsilon_j = \varepsilon \ \forall j$ and show that in this case decoupling of energy use and economic growth is not possible. Second, we study the case of decreasing energy intensities and show that if there is no lower bound for the energy intensity, energy demand is bounded and economic growth can be decoupled from energy use. Finally, if there is a small but positive lower bound for the energy intensity of new goods, decoupling is not possible.

Table 3
General Equilibrium.

Energy Demand	$E(t)=\int_0^{N(t)}\varepsilon_j L\left(\frac{A\alpha^2}{\varepsilon_j Q(t)+1}\right)^{\frac{1}{1-\alpha}}dj$
Consumption Growth	$\frac{\dot{c}(t)}{c(t)}=\frac{1}{\sigma}\left(\frac{(1-\alpha)\alpha AL}{\eta_{N(t)}}\left(\frac{A\alpha^2}{\varepsilon_{N(t)}Q(t)+1}\right)^{\frac{\alpha}{1-\alpha}}-\rho\right)$
Macroeconomic Balance	$\frac{\dot{N}(t)}{N(t)}=\frac{1}{N(t)\eta_{N(t)}}\left(\int_0^{N(t)}L\left(\frac{A\alpha^2}{\varepsilon_j Q(t)+1}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha^2+\varepsilon_j Q(t)}{\alpha^2}\right)dj-c(t)\right)$
Scarcity Constraint	$0=(\bar{E}-E(t))Q(t)$

4.2.1. *Constant energy intensity*

If energy is scarce, $E(t)=\bar{E}$, and the energy intensity of intermediates is constant, $\varepsilon_j=\varepsilon$, the model equations from Table 3 reduce to

$$\bar{E}=\varepsilon N(t)L\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{1}{1-\alpha}}, \tag{30}$$

$$\frac{\dot{N}(t)}{N(t)}=\frac{1}{\eta}\left|L\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{1}{1-\alpha}}\left(\frac{\varepsilon Q(t)+1-\alpha^2}{\alpha^2}\right)-\frac{c(t)}{N(t)}\right|, \tag{31}$$

$$\frac{\dot{c}(t)}{c(t)}=\frac{1}{\sigma}\left(\frac{(1-\alpha)\alpha AL}{\eta}\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{\alpha}{1-\alpha}}-\rho\right). \tag{32}$$

Proposition 2. *If the energy intensity of the economy is constant, $\varepsilon_j=\varepsilon$, and energy is scarce, $E(t)=\bar{E}$, the economy converges to a steady-state with zero-growth, $g_Q=g_c=g_N=0$.*

Proof. Rearrange Eq. (30) to receive $N(t)=\frac{\bar{E}}{\varepsilon L}\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{1}{\alpha-1}}$. Taking the log and the derivative with respect to time, we see that the growth rate of $N(t)$ is directly linked to the growth rate of the energy price $Q(t)$ via $g_N=\frac{1}{(1-\alpha)(1+\frac{1}{\varepsilon Q(t)})}g_Q$. Ultimately, higher energy prices reduce growth as in Proposition 1 and lead to negative growth of varieties and consumption since $\lim_{Q(t)\to\infty}\frac{\dot{N}(t)}{N(t)}=-\frac{c(t)}{\eta N(t)}$ and $\lim_{Q(t)\to\infty}\frac{\dot{c}(t)}{c(t)}=-\frac{\rho}{\sigma}$. Thus, $g_N>0$ and by this $g_Q>0$ decreases the growth rate up until the point of $g_Q=g_c=g_N=0$. From then on the steady-state with zero-growth is reached. □

Proposition 2 directly implies that if energy is limited and the energy intensity is constant, long-run economic growth is not possible. Economic growth leads to increasing energy prices, which in turn reduces the profit of the intermediate goods producer via Eq. (14) and by this the incentive to innovate. Profits eventually decrease to zero and the economy reaches it is steady-state with constant output, consumption and energy prices.

Lemma 2. *(i) The elasticities of energy and output to energy prices are*

$$\zeta_{E,Q}=-\frac{1}{1-\alpha}\frac{\omega}{\alpha^2},\quad \zeta_{Y,Q}=-\frac{\alpha}{1-\alpha}\frac{\omega}{\alpha^2}=-\frac{1}{1-\alpha}\frac{\omega}{\alpha} \tag{33}$$

with $\omega:=QE/Y$ the energy expenditure share of the economy and $\omega\alpha^{-2}<1$. (ii) The elasticity with respect to the energy intensity ε equals the elasticity with respect to energy prices. (iii) The elasticity of total output to A is

$$\zeta_{Y,A}=\frac{1}{1-\alpha}. \tag{34}$$

Proof. For a constant energy intensity output and energy are given by Eq. (30) and $Y(t)=ALN(t)\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{\alpha}{1-\alpha}}$. (i) Take the log of Eq. (30) and $Y(t)$. The elasticities are $\zeta_{\bar{E},Q}:=\frac{\partial\ln(\bar{E})}{\partial Q(t)}Q(t)=-\frac{1}{1-\alpha}\frac{\varepsilon Q(t)}{1+\varepsilon Q(t)}$ and $\zeta_{Y,Q}=-\frac{\alpha}{1-\alpha}\frac{\varepsilon Q(t)}{1+\varepsilon Q(t)}$. With $\omega:=Q(t)E/Y(t)$, we can rearrange Eq. (30) and obtain $\varepsilon=\frac{E}{(\alpha^2-\omega)Y}$ with $\alpha^2>\omega$, which we can substitute to get the final results. (ii) Follows from (i) as Y and E are functions of $\varepsilon Q(t)$. (iii) Follows from $Y(t)$. □

A direct implication of Lemma 2 is that output responds stronger to relative changes in energy prices $Q(t)$ (and, thus, also ε) than to relative changes in TFP, A, if and only if the energy expenditure share ω exceeds the capital income share α. For $\alpha<1/2$, follows further that $0<-\zeta_{Y,Q}<1$, and, thus, output is inelastic in energy prices. Energy demand is inelastic if $Q(t)$ is very small and elastic if $Q(t)$ is very high. While higher energy prices do not affect the energy intensity of intermediate goods because of the Leontief production function, they reduce energy intensity per final output $\Theta:=Y/E=\frac{\alpha^2\varepsilon}{1+\varepsilon Q(t)}$. We now turn to investigate the effect on growth rates.

Lemma 3. *(i) A proportional change in $A,Q(t)$ and ε affects growth rates as follows:*

$$\frac{\partial g_c}{\partial A}A=\Gamma,\quad \frac{\partial g_c}{\partial Q(t)}Q(t)=\frac{\partial g_c}{\partial\varepsilon}\varepsilon=-\frac{\omega}{\alpha}\Gamma \tag{35}$$

with $\Gamma:=\frac{\alpha AL}{\eta\sigma}\left(\frac{A\alpha^2}{\varepsilon Q(t)+1}\right)^{\frac{\alpha}{1-\alpha}}>0$ (ii) A proportional change in A affects growth rates higher than a proportional change in energy prices $Q(t)$ if and only if $\alpha>\omega$.

Proof. Using (32) we get $\frac{\partial g_c}{\partial A}A=\frac{\alpha AL}{\eta\sigma}\left(\frac{\alpha^2 A}{Q(t)\varepsilon+1}\right)^{\frac{\alpha}{1-\alpha}}$ and $\frac{\partial g_c}{\partial Q(t)}Q(t)=-\frac{LQ(t)\varepsilon}{\eta\sigma}\left(\frac{\alpha^2 A}{Q(t)\varepsilon+1}\right)^{\frac{1}{1-\alpha}}$. The response to ε is equivalent to the case for $Q(t)$ due to (32). With $\varepsilon=\frac{E}{(\alpha^2-\omega)Y}=\frac{\omega}{Q(t)(\alpha^2-\omega)}$ (the latter equality uses $\omega=EQ/Y$), we get the final result. □

Lemmas 2 and 3 suggest that the relative sensitivity of level and growth effects to changes in energy prices and total factor productivity is similar. Level and growth effects respond stronger to changes in energy prices (than TFP) when the energy expenditure share exceeds the capital income share. This result can explain the economic take-off during the coal-fired industrial revolution: Expenditure shares on energy (including energy contained in food and fodder) were higher than 60% before 1700, declining to approximately 10% in the early 20th century (Fizaine and Court, 2016). Thus, access to cheap energy could have fueled economic growth much stronger during the early days of the industrialization than in recent decades when energy expenditure shares were very low. With low energy expenditure shares, however, changes in TFP through improved institutions or health become more decisive for creating wealth. Stern and Kander (2012) come to the same conclusion for the Swedish economy. They build an augmented Solow growth model with exogenous technological progress and energy as input factor and find that when energy services are scarce they strongly constrain output growth. We show that their result is a special case in a more elaborate model when there is no horizontal innovation with endogenous growth.

We can now solve for the steady-state values of the model. For $g_c=0$ we can rearrange Eq. (32) for the steady-state constant energy price

$$\bar{Q}=\frac{\alpha\eta\rho}{(1-\alpha)L\varepsilon}\left(\frac{(1-\alpha)\alpha AL}{\eta\rho}\right)^{\frac{1}{\alpha}}-\frac{1}{\varepsilon}. \tag{36}$$

Plugging Eq. (36) into Eq. (30) we can solve for the steady-state constant amount of varieties

$$\bar{N}=\frac{\bar{E}}{\varepsilon L}\left(\frac{(1-\alpha)\alpha AL}{\eta\rho}\right)^{\frac{1}{\alpha}}. \tag{37}$$

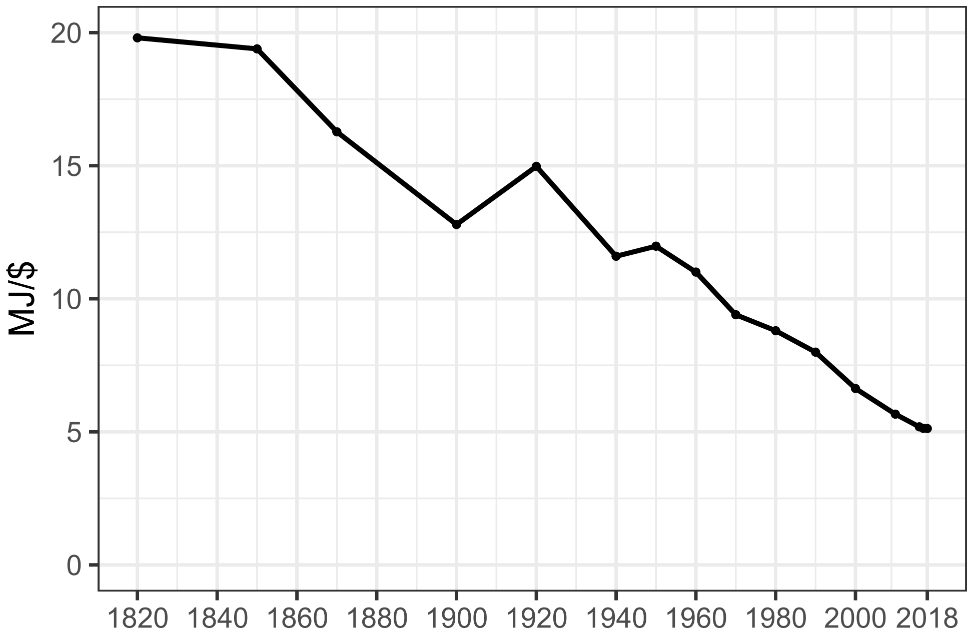

Fig. 3. Global energy intensity from 1820 to 2018 in MJ/$. *Source:* Smil (2017), bp (2022) and Bolt and van Zanden (2020) for GDP, own representation.

Subsequently, we can solve for the constant steady-state consumption level by inserting Eqs. (36) and (37) into Eq. (31) for $g_N = 0$

$$\bar{c} = \bar{E}\left(\frac{\eta\rho}{(1-\alpha)\alpha\varepsilon L}\left(\frac{(1-\alpha)\alpha A L}{\eta\rho}\right)^{\frac{1}{\alpha}} - \frac{1}{\varepsilon}\right), \tag{38}$$

and for the steady-state output level by inserting Eqs. (11), (36) and (37) into Eq. (1)

$$\bar{Y} = \frac{A\bar{E}}{\varepsilon}\left(\frac{(1-\alpha)\alpha A L}{\eta\rho}\right)^{\frac{\alpha}{1+\alpha}}. \tag{39}$$

One can directly see that the steady-state levels of consumption and output are proportional to the maximum available supply of energy $\bar{E}$. The more energy the economy has at its disposal, the higher the long-run levels of output and consumption. However, for constant energy intensities decoupling of economic growth and energy use is not possible — human wealth is determined by the influx of solar energy.

4.2.2. Decreasing energy intensity

Until now we assumed a constant energy intensity, tying economic growth and energy consumption together. However, empirically we observe a drastic decline in energy intensity over the last two hundred years. Fig. 3 displays the world energy intensity in MJ/$ for this time period. Whereas the world needed 20 MJ in 1820 to produce one dollar of GDP, it only took five MJ in 2018. Additionally, advanced economies experienced substantial structural changes from (energy intensive) manufacturing to less energy intensive service sectors. To capture this development, we now move to the case where the energy intensity ε_j is heterogeneous and decreases exogenously with new product inventions. This describes structural change towards more knowledge-based and service-based products that require less energy per $ of output.

We assume that new products arrive, on average, at a declining energy intensity rate, according to $\varepsilon_j = \varepsilon_0 e^{-\beta j}$ with ε_0 the energy intensity of the first product. Thus, the energy intensity of new products decreases, in relative terms, by β. We now have two growth effects at work, a horizontal via increasing product variety $N(t)$ and a vertical via decreasing energy intensity ε_j. Thus, our model combines horizontal growth effects as in the models of Romer (1990) and Grossman and Helpman (1991) as well as vertical growth effects as in the Schumpeterian growth model of improving quality (Aghion and Howitt, 1992).

While Howitt (1999) and Peretto and Connolly (2007) develop endogenous growth models with horizontal and vertical innovation in which long-run growth is determined by the vertical channel alone, long-run growth in our model is determined by the horizontal growth effect.

We can calculate the long-run aggregate energy demand of the economy by inserting Eq. (11) and $\varepsilon_j = \varepsilon_0 e^{-\beta j}$ into Eq. (4) to receive

$$\begin{aligned} E(t) &= \int_0^{N(t)} \varepsilon_j x_j(t)\, dj \\ &= \int_0^{N(t)} L\varepsilon_0 e^{-\beta j}\left(\frac{A\alpha^2}{Q(t)\varepsilon_0 e^{-\beta j}+1}\right)^{\frac{1}{1-\alpha}} dj. \end{aligned} \tag{40}$$

The integral (40) can be solved for the aggregate energy demand of the economy to receive

$$E(t) = \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} L A^{\frac{1}{1-\alpha}}\left(\left(\varepsilon_0 Q(t) e^{-\beta N(t)}+1\right)^{\frac{\alpha}{\alpha-1}} - (\varepsilon_0 Q(t)+1)^{\frac{\alpha}{\alpha-1}}\right)}{\beta Q(t)}. \tag{41}$$

Proposition 3. *If the energy intensity of new products decreases by the rate* $\beta > 0$*, (i) total energy demand is continuously growing with the introduction of new varieties and (ii) converges to the maximum available supply of energy* $0 < \bar{E} < \infty$.

Proof. For (i), take $\frac{\partial E(t)}{\partial N(t)}$ which is strictly positive for any $N(t) \geq 0$ but converges to zero for $t \to \infty$. (ii) $\lim_{N(t)\to\infty} E(t) = \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} L A^{\frac{1}{1-\alpha}}\left(1-(Q(t)\varepsilon_0+1)^{\frac{\alpha}{\alpha-1}}\right)}{\beta Q(t)} = E^*$. Clearly, $0 < E^* < \infty$ if $Q(t)\varepsilon_0 > 0$, which only holds if $E^* = \bar{E}$ due to Eq. (8). □

From this proposition we can directly see that in the market equilibrium, the long-run energy price has to be constant, such that energy demand is equalized with the maximum available supply of energy. We can also study how aggregate energy demand develops if the energy price is altered, e.g. through a tax or subsidy:

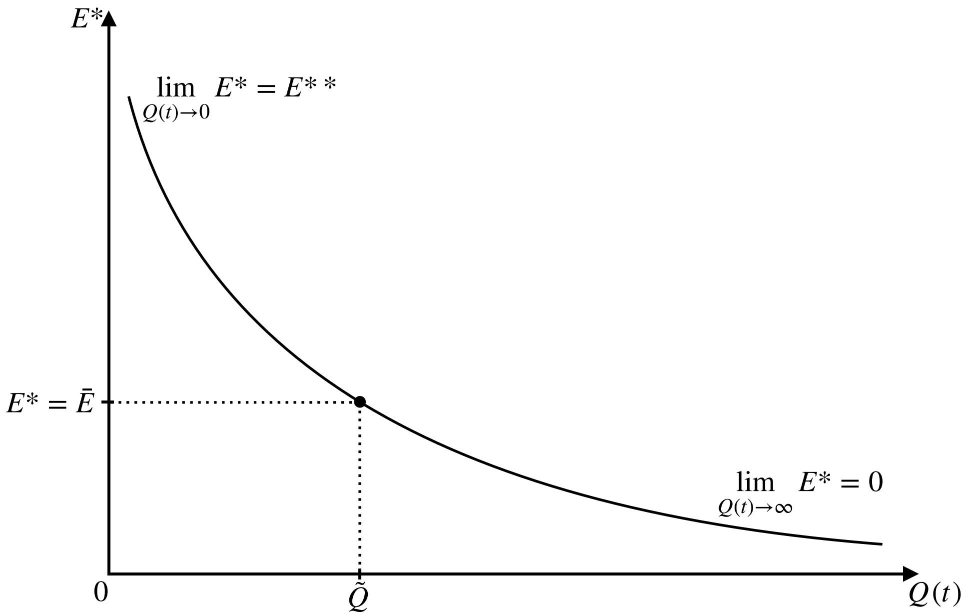

Fig. 4. Relationship between long-run energy demand, E^*, and energy price, $Q(t)$.

Corollary 1. *(i) If the energy price $Q(t)$ converges to infinity, long-run energy demand E^* becomes zero. (ii) If the energy price $Q(t)$ converges to zero, long-run energy demand is $E^{**} < \infty$ and bounded. (iii) Long-run energy demand falls monotonically in energy prices. (iv) The long-run energy price is constant.*

Proof. (i) Take $\lim_{Q(t)\to\infty} E^*$ with E^* from the proof of Proposition 3 which gives zero. (ii) $\lim_{Q(t)\to 0} E^* = \frac{L\epsilon_0 \left(A\alpha^2\right)^{\frac{1}{1-\alpha}}}{\beta} = E^{**}$. (iii) As $\frac{\partial E_j}{\partial Q(t)} = -\frac{L\epsilon_0^2 e^{-\beta j}\left(\frac{\alpha^2 A}{Q(t)\epsilon_0 e^{-\beta j}+1}\right)^{\frac{1}{1-\alpha}}}{(1-\alpha)\left(e^{\beta j}+Q(t)\epsilon_0\right)} < 0$, total derivative of total energy $dE(t)/dQ(t)$ as the integral over all varieties is negative and $dE^*/dQ(t) < 0$ as well. (iv) For large enough t and limited energy supply, we know that $E(t) = E^* = \bar{E}$, with E^* from Proposition 3. Taking the log and the derivative with respect to time of $E^* = \bar{E}$ we receive $\dot{Q}(t) = 0$. □

The long-run maximum available level of energy supply, $\bar{E}$, then determines the long-run constant energy price, $\tilde{Q}$, via

$$\bar{E} = \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} L A^{\frac{1}{1-\alpha}} \left(1 - \left(\tilde{Q}\epsilon_0 + 1\right)^{\frac{\alpha}{\alpha-1}}\right)}{\beta\tilde{Q}}. \quad (42)$$

A remarkable outcome is that by the specific choice of $\tilde{Q}$, e.g. via a tax on energy use, any energy demand greater than zero can be achieved — no matter how small $\bar{E} > 0$ might be. Fig. 4 depicts the relationship between long-run energy demand and the energy price.

For $t \to \infty$ the energy intensity for the frontier product, $\epsilon_{N(t)}$, converges to zero and, thus, long-run growth is only determined by increasing product variety. The same is true for product specific R&D costs, η_j. Since the profit of the frontier firm is independent of j for large enough t, also the product specific R&D costs become independent of j and converge according to $\lim_{t\to\infty} \eta_{N(t)} = \tilde{\eta}$. In the long-run the economic environment from Table 3 can be represented by Eq. (42) and

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{N(t)\tilde{\eta}}\left(\int_0^{N(t)} L\left(\frac{A\alpha^2}{\epsilon_j\tilde{Q}+1}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha^2+\epsilon_j\tilde{Q}}{\alpha^2}\right)dj - c(t)\right), \quad (43)$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left(\frac{(1-\alpha)\alpha A L}{\tilde{\eta}}\left(A\alpha^2\right)^{\frac{\alpha}{1-\alpha}} - \rho\right). \quad (44)$$

Proposition 4. *If the energy intensity of new products decreases by the rate $\beta > 0$, $g_c = g_N > 0$ is a balanced growth path of the economy, eventhough the maximum available energy supply is limited by $\bar{E}$.*

Proof. If the economy is on a balanced growth path we know that $\dot{g}_N = 0$ and that in the long-run it also holds that $\epsilon_{N(t)} \approx 0$, so that we can take the derivative with respect to time of Eq. (43) to receive $\int_0^{N(t)} L\left(\frac{A\alpha^2}{\epsilon_j\tilde{Q}+1}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha^2+\epsilon_j\tilde{Q}}{\alpha^2}\right)dj - c(t) = \frac{1-\alpha^2}{\alpha^2}N(t)L\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} - \frac{\dot{c}(t)N(t)}{\dot{N}(t)}$. We know from Eq. (43) that the LHS of this expression is equal to $\tilde{\eta}\dot{N}(t)$, so that $\tilde{\eta}\frac{\dot{N}(t)}{N(t)} = \frac{1-\alpha^2}{\alpha^2}L\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} - \frac{\dot{c}(t)}{\dot{N}(t)}$. Rearranging brings $\frac{c(t)}{N(t)} = \frac{g_N}{g_c}\frac{1-\alpha^2}{\alpha^2}L\left(A\alpha^2\right)^{\frac{1}{1-\alpha}} - \tilde{\eta}\frac{g_N^2}{g_c}$. If we are on a balanced growth path, the RHS is a constant since $\dot{g}_N = \dot{g}_c = 0$. Hence, $c(t)$ and $N(t)$ must grow at the same constant rate, $g_c = g_N$ given by Eq. (44). □

The key insight from this proposition is that sustained economic growth with finite energy supply is possible. While energy efficiency of existing products cannot be increased due to the assumed thermodynamic constraints, decoupling is possible if the economy invents new products that require less and less energy input. Interestingly, decoupling is possible for any $\beta > 0$ – even if the decrease in energy intensity for new products is extremely small. This also means that the energy share of the economy converges to zero for large enough t.

Hart (2018) also studies an expanding variety model with emphasize on the rebound effect and directed technical change. In his setup, improvements in energy intensity of high energy intensity products are subject to a rebound effect increasing total energy consumption rather than reducing it. We observe the same effect. In our setup improvements in energy intensity of new products are the source of economic growth enabling the economy to increase output. The Leontief production structure ties together intermediate production and energy consumption. However, through improving the energy intensity of new products the economy can create a new intermediate with higher value for less additional energy input. Total energy consumption increases but less and less so as new varieties demand less energy input, so that aggregate energy demand converges to a finite number.

4.2.3. Limits to energy intensity improvements

The equation for aggregate energy demand (40) further allows to identify necessary and sufficient conditions that make a decoupling of economic growth and absolute energy use impossible:

Corollary 2. *If there exists an arbitrarily small but strictly positive lower bound $\underline{\varepsilon} > 0$ for the energy intensity of new goods, which will be reached in finite time, such that $\varepsilon_j \geq \underline{\varepsilon}$ for all j and $\varepsilon_i > \varepsilon_j$ for $i < j$, energy use will not be bounded by above and decoupling of energy and economic growth is not possible.*

Proof. With (40) follows $E(t) \geq \int_0^{N(t)} \underline{\varepsilon} x_0 dj = \underline{\varepsilon} x_0 N(t)$ with x_0 the demand for the first intermediate (i.e. the intermediate with the highest energy intensity ε_0 which is given from (11)). For sustained economic growth, $N(t)$ grows to infinity, implying that energy demand will also grow without bound. □

The aggregate energy demand of the economy will always be greater or equal to the case in which even the first variety has the lowest possible energy intensity $\underline{\varepsilon}$. If there exists a maximum available amount of energy supply, $\bar{E}$, the inequality cannot hold for growing $N(t)$ and, thus, economic growth will cease.

A simple example of a function with a positive minimum energy intensity is $\varepsilon_j = \max\left\{\underline{\varepsilon}, \varepsilon_0 e^{-\beta j}\right\}$. In this case, even strongly declining energy intensity due to a high β will still imply unbounded energy use if $\underline{\varepsilon} > 0$. Thus, we would again end up in a zero-growth economy as in Section 4.2.1.

This specification is similar to the argument made in Meran (2023). He argues that there is a minimal material base to further economic growth resulting in a lower bound to the material intensity (Meran, 2023, p. 2). He then shows that perpetual growth, e.g. through the accumulation of knowledge, cannot be guaranteed given this lower bound. Corollary 2 shows that his result is a special case in our model once assuming a lower bound for the energy intensity of new products.

Assuming that $\varepsilon_j = \max\left\{\underline{\varepsilon}, \varepsilon_0 e^{-\beta j}\right\}$ we can again solve for the long-run steady-state of the economy. By inserting Eq. (11) into Eq. (4) we receive the aggregate energy demand of the economy

$$E(t) = \int_0^{N(t)} (\max\left\{\underline{\varepsilon}, \varepsilon_0 e^{-\beta j}\right\}) L \left(\frac{A\alpha^2}{(\max\left\{\underline{\varepsilon}, \varepsilon_0 e^{-\beta j}\right\}) Q(t) + 1} \right)^{\frac{1}{1-\alpha}} dj. \tag{45}$$

If we define $\underline{\omega}$ to be the variety at which $\varepsilon_{\underline{\omega}} = \varepsilon_0 e^{-\beta \underline{\omega}} = \underline{\varepsilon}$, we can split the integral to receive

$$E(t) = \int_0^{\underline{\omega}} \varepsilon_0 e^{-\beta j} L \left(\frac{A\alpha^2}{\varepsilon_0 e^{-\beta j} Q(t) + 1} \right)^{\frac{1}{1-\alpha}} dj + \int_{\underline{\omega}}^{N(t)} \underline{\varepsilon} L \left(\frac{A\alpha^2}{\underline{\varepsilon} Q(t) + 1} \right)^{\frac{1}{1-\alpha}} dj.$$

The first term is the energy demand of all varieties prior to variety $\underline{\omega}$ and can be rewritten to

$$\underline{\Omega}(t) = \int_0^{\underline{\omega}} \varepsilon_0 e^{-\beta j} L \left(\frac{A\alpha^2}{\varepsilon_0 e^{-\beta j} Q(t) + 1} \right)^{\frac{1}{1-\alpha}} dj = \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} L A^{\frac{1}{1-\alpha}} \left((\varepsilon_0 Q(t) e^{-\beta \underline{\omega}} + 1)^{\frac{\alpha}{\alpha-1}} - (\varepsilon_0 Q(t) + 1)^{\frac{\alpha}{\alpha-1}} \right)}{\beta Q(t)}. \tag{46}$$

The second term simplifies because we know that all varieties invented after $\underline{\omega}$ lead to $\varepsilon_j = \underline{\varepsilon}$, so that aggregate energy demand is given by

$$E(t) = \underline{\Omega}(t) + (N(t) - \underline{\omega}) \underline{\varepsilon} L \left(\frac{A\alpha^2}{\underline{\varepsilon} Q(t) + 1} \right)^{\frac{1}{1-\alpha}}. \tag{47}$$

In the long-run the economic environment from Table 3 can be represented by Eq. (47) and

$$\frac{\dot{N}(t)}{N(t)} = \frac{1}{N(t)\tilde{\eta}} \left(\int_0^{N(t)} L \left(\frac{A\alpha^2}{\varepsilon_j Q(t) + 1} \right)^{\frac{1}{1-\alpha}} \left(\frac{1 - \alpha^2 + \varepsilon_j Q(t)}{\alpha^2} \right) dj - c(t) \right), \tag{48}$$

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma} \left(\frac{(1-\alpha)\alpha A L}{\tilde{\eta}} \left(\frac{A\alpha^2}{\underline{\varepsilon} Q(t) + 1} \right)^{\frac{\alpha}{1-\alpha}} - \rho \right). \tag{49}$$

With a binding energy limit, such that $E(t) = \bar{E}$, we can rearrange Eq. (47) to receive

$$N(t) = \frac{\bar{E} - \underline{\Omega}(t)}{\underline{\varepsilon} L} \left(\frac{A\alpha^2}{\underline{\varepsilon} Q(t) + 1} \right)^{\frac{1}{\alpha-1}} + \underline{\omega}. \tag{50}$$

One can see directly that an increase in the energy price, $Q(t)$, also increases the number of varieties, $N(t)$, since $\frac{\partial \underline{\Omega}(t)}{\partial Q(t)} < 0$. Hence, the growth rate of the energy price, g_Q, is positively linked to the growth rate of varieties, g_N. Analogous to the proof of Proposition 2, economic growth via $g_N > 0$ automatically leads to increasing energy prices $g_Q > 0$. Increasing energy prices, then, decrease the consumption growth rate via Eq. (49) until $g_c = 0$. Once the rising energy price pushed down the growth rate to zero, the zero growth era begins and we can solve for the steady-state equilibrium of the economy. Rearranging Eq. (49) for $g_c = 0$ gives us the steady-state constant energy price of the economy

$$\underline{Q} = \frac{\alpha \eta \rho}{(1-\alpha) L \underline{\varepsilon}} \left(\frac{(1-\alpha)\alpha A L}{\tilde{\eta} \rho} \right)^{\frac{1}{\alpha}} - \frac{1}{\underline{\varepsilon}}. \tag{51}$$

For a constant energy price it also holds that $\underline{\Omega}(\underline{Q}) = \underline{\Omega}\ \forall t$. Inserting Eq. (51) into Eq. (50), we can derive the steady-state constant amount of varieties in the economy

$$\underline{N} = \frac{\bar{E} - \underline{\Omega}}{\underline{\varepsilon} L} \left(\frac{(1-\alpha)\alpha A L}{\tilde{\eta} \rho} \right)^{\frac{1}{\alpha}} + \underline{\omega}. \tag{52}$$

Note here, that the number of varieties increases with the maximum available amount of energy supply. Furthermore, the amount of varieties increases with $\underline{\omega}$. The more varieties are introduced until the variety from which no more improvements in energy intensity are possible is invented, the higher the overall amount of varieties that can be sustained in the economy. Nevertheless, if there exists a lower bound for the energy intensity, long-term wealth will be constrained by the amount of incoming solar energy.

5. Vanishing intermediate goods

This section discusses an extension of the model by allowing for vanishing intermediate products. Until now we assumed that every newly invented variety lives forever and is used in the production process together with its energy intensity. However, we now allow for a portion of varieties to be discontinued every period. We model this process by assuming that products vanish at a constant, exogenous rate. This accounts for the fact that outdated technologies such as steam engines and wired telephones are decommissioned and no longer in use. We find that the general results stay untouched. Vanishing products can help to decouple even stronger and decrease total energy demand to zero in the long-run. However, if the energy intensity is homogeneous, or has a lower bound, decoupling is not possible.

The total number of active varieties used in the production process is no longer equal to the total number of invented varieties $N(t)$ but is now the number of invented varieties minus the number of discontinued products $N(t) - \kappa(t)$. We assume that $\kappa(t)$ evolves according to $\kappa(t) = e^{\delta t} - 1$, such that products vanish at the rate δ. In $t = 0$ the number of active varieties in the economy is equal to the number of invented varieties N_0 because no product has vanished so far ($\kappa_0 = e^{\delta \cdot 0} - 1 = 0$).

Table 4
Implications for long-run growth for different assumptions on energy intensity and energy supply.

Energy Supply	Energy intensity: ε_j	Implication for long-run growth
Abundant: $E(t) < \bar{E}$	Constant: $\varepsilon_j = \varepsilon$	Growth
	Decreasing: $\varepsilon_j = \varepsilon_0 e^{-\beta j}$	Growth
	Lower Bound: $\varepsilon_j = \max\{\underline{\varepsilon}, \varepsilon_0 e^{-\beta j}\}$	Growth
Limited: $E(t) = \bar{E}$	Constant: $\varepsilon_j = \varepsilon$	No Growth
	Decreasing: $\varepsilon_j = \varepsilon_0 e^{-\beta j}$	Growth
	Lower Bound: $\varepsilon_j = \max\{\underline{\varepsilon}, \varepsilon_0 e^{-\beta j}\}$	No Growth

The rest of the model stays untouched. New varieties have lower energy intensities, while early invented products with a high energy intensity are discontinued after some time.[3] The long-run growth rate of the economy is still given by Eq. (44) and we assume that it is always larger than the rate at which products vanish, i.e. $\tilde{\gamma} \geq \delta$. Otherwise, the amount of varieties declines to zero and the economy collapses. For decreasing energy intensities according to $\varepsilon_j = \varepsilon_0 e^{-\beta j}$, we can solve for the aggregate energy demand of the economy, which is now given by

$$E^V(t) = \int_{\kappa(t)}^{N(t)} \varepsilon_j x_j dj. \tag{53}$$

For $\kappa(t) = e^{\delta t} - 1$ and $N(t) > \kappa(t)$ we can solve the integral and receive

$$E^V(t) = \frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} LA^{\frac{1}{1-\alpha}} \left(\left(Q\varepsilon_0 e^{-\beta N(t)} + 1\right)^{\frac{\alpha}{\alpha-1}} - \left(Q\varepsilon_0 e^{\beta(1-e^{\delta t})} + 1\right)^{\frac{\alpha}{\alpha-1}} \right)}{\beta Q},$$

which is equal to Eq. (41) for $\delta = 0$.

Proposition 5. *If the energy intensity of new products decreases with the rate $\beta > 0$, new varieties come into existence at rate $\tilde{\gamma} > 0$ and old varieties vanish at rate $\delta > 0$, (i) total energy demand converges in the long-run to zero, (ii) while the economy grows with the constant rate $\tilde{\gamma}$.*

Proof. (i) Take $\lim_{t\to\infty} E^V(t) = 0$. (ii) The economy grows with the number of active varieties $N(t) - \kappa(t)$. The growth rate thus is $\frac{\frac{\partial(N(t)-\kappa(t))}{\partial t}}{(N(t)-\kappa(t))} = \frac{\tilde{\gamma} N_0 e^{\tilde{\gamma} t} - \delta e^{\delta t}}{-e^{\delta t} + N_0 e^{\tilde{\gamma} t} + 1}$. And $\lim_{t\to\infty} \frac{\tilde{\gamma} N_0 e^{\tilde{\gamma} t} - \delta e^{\delta t}}{-e^{\delta t} + N_0 e^{\tilde{\gamma} t} + 1} = \tilde{\gamma}$ for $\tilde{\gamma} > \delta$. □

The key insight from this proposition is that vanishing varieties and decreasing energy intensity enable the economy to decouple even stronger than in the case without vanishing varieties. Since the most energy intensive products vanish over time, only varieties with energy intensities close to zero are active in the long-run and total energy demand becomes zero. Furthermore, the economy grows constantly because the number of active varieties increases at a constant rate. However, Corollary 2 still holds. Once a lower bound for the energy intensity is reached, the same varieties with the same energy intensity are invented and decommissioned each period not affecting the average energy intensity of the economy. For sustained economic growth the number of varieties must grow, and with that also the energy demand grows without bound. Thus, for a limited amount of energy, decoupling is not possible and the economy ends up in zero-growth as in Section 4.2.1.

6. Conclusion

This paper combined insights from thermodynamics on energy conversion rates and available energy flows with an expanding product-variety endogenous growth model to assess the role of energy for economic growth. In particular, we set up a simple and transparent endogenous growth model that respects crucial thermodynamic laws to highlight the key mechanism of decoupling economic growth from energy use: decreasing energy intensity. Our insights provide various contributions to ongoing debates on possibilities to decouple economic growth from energy use and on the thermodynamic feasibility of sustained economic growth.

Thermodynamic laws suggest that the substitutability between energy and capital is limited for a *specific* machine or product. Hence, sustained increases in energy efficiency are only possible through the development of new products or varieties that are less energy intensive than previous ones. Whether there exists a thermodynamically-constrained minimum amount of energy for a new product is an open question: while products have some material or energy-related base, it is not clear whether there exists a lower bound. On the one hand, immaterial products, ideas, art, and knowledge are examples where energy requirements relative to the value of the output could possibly be arbitrarily small. On the other hand, Smil (2017) and Parrique et al. (2019) argue that absolute decoupling of energy use and economic growth does not seem plausible in the future, since also the service sector consumes energy and is rather a complement to the material economy than a substitute. Bogmans et al. (2020) find that energy demand will not saturate anytime soon but estimate an energy Kuznets-Curve indicating that energy saturation might come at very high-income levels. Hart (2018) shows a shift towards energy-intensive consumption has occurred over the industrial period and is even continuing in the most advanced economies.

The implications of the existence of a lower bound of energy content are tremendous: if energy intensity can converge to zero, decoupling of economic growth and energy use is possible. In that case, energy use can be maintained at any constant level without affecting asymptotic growth rates. If, however, the energy intensity of new products faces a non-zero lower bound, decoupling is not possible: The wealth of humanity converges to a constant level and will be determined by the influx of solar energy.

Our results further emphasize why the price of energy affects growth rates and innovation in the short-run. High energy prices reduce monopoly profits from newly invented products and are therefore an incentive to innovate less. This growth effect, however, is only temporary and diminishes over time: Either energy intensities converge to zero (and the price of energy becomes irrelevant), or if energy intensities do not converge to zero, the economy will eventually stagnate when energy demand hits physical supply and the energy price converges to a constant. Thus, our results stress the need to shift towards less energy intensive value creation and highlight the role of innovation and structural change. The possibility of vanishing products with high energy intensities to free up energy allows even stronger decoupling, however, if there again exists a lower bound to the energy intensities of new varieties, decoupling is not possible. Our main findings are summarized in Table 4.

Our approach could be extended by endogenizing the choice of the energy intensity ε or its decline rate β. This could be done by assuming that the energy improvement of a new product depends on research effort — or that energy intensities are an outcome of a probability distribution where only products with a sufficiently low energy intensity survive. However, we think that this would not change the main findings of our model. Endogenizing the choice of the energy intensity does not allow an analytically traceable solution anymore, while only providing few additional insides. The key mechanism of

[3] For the case of constant, homogeneous energy intensities, energy demand of the economy does not change, because products with the same energy intensity are discontinued and created each period. Hence, the average energy intensity of the economy does not change.

constantly declining energy intensities for new products remains intact, as well as the possibility of a lower bound preventing to decouple economic growth from energy use.

Additional empirical research could, on the one hand, provide insights on the energy intensity of new developed products. One could investigate vertical vs. horizontal improvements in energy efficiency, to see to what extent the energy intensity of new products behaves differently than energy improvements of existing products. This might help to understand if it is reasonable to assume ever declining energy intensities or if a lower bound exists. On the other hand, it could shed light on potential limits to energy use and how they differ between sectors. Demand for lighting, heating or cooling could be saturated once optimal per-capita levels are obtained for human well-being. In particular, unbounded income growth would not lead to unbounded demand growth for these energy-intensive services. Also, the transportation sector – at least for human travel – faces an upper bound which is given by the total amount of time a person could allocate to traveling and the most energy intensive way of traveling per unit of time. Whether there exists a reasonable upper bound for transporting goods, however, is less clear. Globalization, specialization and differentiation of value chains could link energy demand to economic growth.

Finally, a major driver for energy consumption could also be triggered by ongoing digitization and automatization of the economy: energy demand of communication networks, personal computers, and data centers grows substantially stronger than the energy demand of the aggregate economy (Van Heddeghem et al., 2014). A recent study by Lange et al. (2020) found that instead of saving energy, digitalization has brought additional energy consumption. If economic value creation consists more and more on computation-intensive digital services, energy demand could increase without bound. Similarly, if economic production is performed by robots that require energy or artificial intelligence even automates the production of new ideas (Aghion et al., 2017), energy demand could as well increase without bound. Thus, while human needs with respect to lighting, heating, cooling and travel could be met with a bounded energy per capita supply, digitization and automatization could turn out to be a major driving force for coupling economic growth to energy use.

CRediT authorship contribution statement

Tobias Bergmann: Writing – review & editing, Writing – original draft, Formal analysis. **Matthias Kalkuhl:** Writing – original draft, Supervision, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

Tobias Bergmann acknowleges funding from the PhD scholarship of the Heinrich Böll Foundation. We also thank three anonymous reviewers for valuable comments from which the paper benefited greatly.

Appendix. Households

Households maximize utility given the budget constraint

$$\max_{\{c(t),K_{N(t)}\}} \int_0^\infty e^{-\rho t}\ \frac{c(t)^{1-\sigma}-1}{1-\sigma}\ dt \tag{54}$$

$$s.t.\ c(t) = w(t)L + \int_0^{N(t)} r_j(t)K_j dj + Q(t)E(t) - \dot{K}. \tag{55}$$

Households choose consumption $c(t)$ and the frontier capital stock $K_{N(t)}$ at time t. The current value hamiltonian and the first order conditions are

$$\mathcal{H} = \frac{c(t)^{1-\sigma}-1}{1-\sigma} + \mu(t)\Big(w(t)L + \int_0^{N(t)} r_j(t)K_j dj + Q(t)E(t) - c(t)\Big), \tag{56}$$

$$\frac{\partial \mathcal{H}}{\partial c(t)} = c(t)^{-\sigma} - \mu(t) \stackrel{!}{=} 0 \Rightarrow \mu(t) = c(t)^{-\sigma}, \tag{57}$$

$$\frac{\partial \mathcal{H}}{\partial K_{N(t)}} = \mu(t) r_{N(t)} \stackrel{!}{=} \rho\mu(t) - \dot{\mu}(t) \Rightarrow r_{N(t)} = \sigma\frac{\dot{c}(t)}{c(t)} + \rho. \tag{58}$$

Rearranging and using $g_c(t) = \frac{\dot{c}(t)}{c(t)}$, we receive Eq. (17) from the main text

$$g_c(t) = \frac{1}{\sigma}(r_{N(t)} - \rho). \tag{59}$$

Data availability

Data will be made available on request.

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