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Climate Policy Under National Commitments and Global Economic Shocks

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ABSTRACT

Global economic shocks may affect countries' Nash equilibrium contributions to emissions reduction efforts. We study the effect of income uncertainty on emissions reduction under different scenarios of national commitment targets. Our results show that in the presence of uncertainty, the framing of emissions reduction targets has implications for total emissions reduction and ex-ante welfare. Under the plausible assumption of "prudence," higher global uncertainty tends to reduce equilibrium climate contributions if countries commit to a fixed cap for damage reductions before the income uncertainty is resolved. In contrast, if countries are committed to allocating a *proportion* of income to climate protection, ex-ante welfare may be higher than in the baseline case of no commitments.

JEL Classification: codes: Q54, H41, Q55

1 | Introduction

The early decades of the 21st century show that countries' national incomes are often exposed to global risks such as the COVID pandemic, or major warfare and its ensuing economic implications. At the same time, in the absence of any binding international agreements and a regulated policy space, countries face the task of reducing damages from carbon emissions, a global public goods problem, largely through noncooperative policies (Nordhaus 2018) in a time of high economic uncertainty.¹ Recommendations for national-level commitments often include damage reductions proportional to GDP, as seen in optimal carbon tax models (Golosov et al. 2014; van der Ploeg and Rezai 2021), or they may include fixed caps on emissions (Nordhaus 2018). Yet, it is unclear whether following one of these guidelines is better than another for reducing global emissions under uncertainty regarding national income. To study this issue, the current paper sets up a model of private provision of a public good to examine the effect of income risk

on non-cooperative contributions to emissions reduction under different scenarios of damage reduction commitments.

We compare the levels of emissions reduction under four different hypothetical scenarios. The first one is characterized by full ex-post flexibility, wherein the countries wait for future economic conditions to realize and then make an individual optimizing choice based on their known national income. This case would correspond to the baseline scenario, where countries' decision to contribute to damage reductions are unaffected by commitments and their contributions are allowed to be state-dependent. The second one is characterized by a fixed cap on emissions reduction independent of national income. A third one fixes national consumption and makes individual emissions reduction targets the residual variable. A fourth case fixes the relative shares of future national income used for national consumption and emissions reduction, making the relative shares independent of uncertain income. We analyze the respective Nash equilibria under the four different regimes and

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compare the equilibrium expected quantity of the contributions and the equilibrium welfare.²

A key finding of the article is that, under prudence, when countries set aside a fixed cap for emissions reduction before the income uncertainty is resolved, total emissions reduction are lower under uncertainty as compared to reductions under certainty. That is, if reducing damages from emissions requires setting a cap on emissions, countries acting independently will, in sum, contribute less to emissions reduction than if there were no such constraints. In contrast, total emissions reduction are higher under uncertainty than with certainty when countries are prudent and allocate a fixed amount to national consumption. Finally, the paper shows that the scenario for which countries must allocate a certain proportion of their income to damage reductions (instead of a fixed contribution amount) may result in a higher ex-ante welfare as compared to the baseline where contributions are fully flexible. Moreover, we find that ex-ante welfare is increasing under proportional commitments as the number of countries increases.

These results carry implications for voluntary, nonbinding national future-date emissions reduction targets, such as those expressed in Nationally Determined Contribution pledges (NDC) under the Paris Agreement.³ To achieve these goals, economists commonly propose a carbon tax that targets a fixed cap on damages from emissions (Boyce 2018). Our results show that under prudence, in this case, in the absence of international agreements, emissions reduction would be lower under income uncertainty. Our results are consistent with those in Lopez et al. (2022), who find that an increase in uncertainty is associated with an increase in pollution for the setting of an individual country (ignoring free-riding concerns associated with a game-theoretic framework). However, with prudence, the scenario of fixing national consumption levels leads to higher contributions under uncertainty, in contrast to the findings of Lopez et al. (2022). In our first contribution to the literature, we thus highlight that, in the presence of uncertainty, the formulation of future emissions reduction targets may have implications for total emissions reduction. Another common policy recommendation sets a (pigouvian) tax such that damages from emissions are proportional to income, as seen in optimal carbon pricing models (Golosov et al. 2014; Dennig et al. 2015). Our second contribution applies to this setting of emission intensity targets and shows that under income uncertainty, setting national emissions reduction targets in terms of commitments proportional to GDP may be beneficial as it leads to a higher ex-ante welfare as compared to the baseline scenario without any commitments to emission targets.

An alternative interpretation of our framework may be in terms of the theory of irreversible choice in the presence of uncertainty (Bernanke 1983). Major technological or investment choices made by countries may lead to irreversibilities and limit countries' ability to react flexibly to such global events, particularly with respect to their contributions to global climate policy. To elaborate on the types of rigidity, think of a country that could decide how many power plants for gas, oil, coal, or nuclear energy to construct or to close down, or whether to create an infrastructure based on e-mobility, invest in green energy, and so forth. The fact that some technological choices

may be binding for a long time is well-recognized.⁴ The empirical relevance of rigidities in their climate policy has been discussed in the context of choice of countries' energy industry. Thus, our framework also describes situations where choices of technology and capacity investments may often lead to commitment about the amount of feasible emissions reduction for decades into the future.⁵

Our analysis, however, comes with caveats—for simplicity, our model does not take into account a direct effect of income shocks on the climate: emissions may fall (rise) due to a fall (rise) in economic activity. Our theoretical model is also static, and abstracts away from the consideration that introducing the reduction commitments in practice may affect economic activity.⁶ Furthermore, the hypothetical scenarios of damage reduction commitments cannot be internationally enforced. Rather, they are policy prescriptions undertaken at the national level in the absence of any enforcing agreements, excluding the possibilities for any international coalition formation. We also make several simplifying assumptions, including the assumption that countries' utility functions are additively separable. Our results therefore exclude cross-elasticities between emissions reduction and national consumption that could potentially affect the results under uncertainty, as explored in Lopez et al. (2022). Lastly, our model does not consider the effect of climate policies on uncertainty, as explored in recent literature (Fischer and Springborn 2011; Annicchiarico et al. 2021; Espinosa et al. 2025).

Our article is related to the abstract theory of voluntary provision of a public good that emphasizes uncertainty and restrictions on the countries' strategy set. Some early papers like Dardanoni (1988) and Gradstein et al. (1992) focus on the general case of uncertainty faced by individuals when their utility function depends upon two goods. These papers are unconcerned with the implications of uncertainty for public good provision, though they cite private provision as an application of their framework. Gradstein et al. (1993) is one of the first papers in this literature to look at the situation where the price of the public good provided is subject to uncertainty. Other papers address the consequences of uncertainty regarding the contributions of other countries. In Cornes and Sandler (1984) and Sandler et al. (1987), countries are uncertain about what others are contributing, and may form beliefs about how their contributions affect the contributions of others.⁷ Keenan et al. (2006) study the symmetric Nash equilibrium under the same type of uncertainty, in contrast to the previous papers that focused on the single player's problem. A third source of uncertainty is incomplete information regarding fellow players. This kind of situation can arise due to countries having heterogeneous preferences for the public good, and beliefs about what the preferences of other countries may be (Maldonado and Rodrigues-Neto 2016). Unlike the above papers concerned with uncertainty regarding the characteristics of other players, the current research rather focuses on the consequences of group-wide uncertain future endowments and on scenarios of national-level target commitments, which has not been looked at so far in the literature.

Incomplete information may also play a role when countries' valuations of the public good are private information, such as

first studied by Gradstein et al. (1994) and more recently by Barbieri and Malueg (2008). Bac (1996) complements this literature with an analysis of a dynamic game where agents (countries) have private valuations for international environmental resources. While there is a large literature that explores the topic of private valuations of public goods,⁸ none of the papers mentioned consider the type of uncertainty arising because future incomes are unknown due to aggregate income uncertainty. The analysis in Robledo (1999) comes closest to the one in the current paper, in that he examines the decisions of two countries with uncertain future incomes who must make a decision about their contribution to a public good before this uncertainty is resolved. This corresponds to the “fixed contributions” case of our model. His focus, however, is on uncertainty as a variable, and the strategic role of higher or lower uncertainty in one player’s endowment for own and others’ contributions in the equilibrium.

Finally, our paper also contributes to the literature in environmental economics on the role of uncertainty in helping or hindering solutions to global commons problems. An extensive strand of this literature (see e.g., Pindyck 2012; Kunreuther et al. 2013; Drouet et al. (2015), and references therein) have analyzed many sources of uncertainty, including mitigation costs, climate change dynamics, and economic damages. As for GDP uncertainty, relevant papers include Gillingham et al. (2018), Nordhaus (2018), and Liu and Song (2022), who, for the majority of the analysis, assume a “learn, then act” structure, that is, when climate policy is then decided on a state-contingent basis, our “most flexible” case. Importantly, Lopez et al. (2022) analyze the role of income uncertainty and theoretically demonstrate that uncertainty may have negative effects on voluntary contributions that can be significant. Their model is richer than ours in some dimensions (form of the utility and production functions, for instance), but does not explicitly consider the Nash Equilibrium of a public good game, nor the different forms that national commitments can take. Ulph (2004), Kolstad (2005), and Kolstad (2007) study the formation of international environmental agreements (IEAs) when there is uncertainty about the environmental costs of pollution abatement. They assume that countries are risk-neutral. In contrast, Bramoulle and Boucher (2010) consider the case when countries are risk-averse and find that uncertainty may increase the number of signatories to IEAs. Bramoulle and Treich (2009) study the setting where individuals face the same risk of damage from pollution and find that uncertainty reduces pollution and improves individuals’ welfare. Barrett (2013), building on a discrete public good model with threshold uncertainty (see, e.g., Nitzan and Romano (1990)), shows that the effects of uncertainty in the threshold after which damages become catastrophic are of much greater importance than the effect of uncertainty in the level of the damages. Using a threshold formulation, Bochet et al. (2019) study a modified Nash demand game where agents can demand shares of a common pool resource, and the threshold level beyond which the amount of the resource available collapses to zero is subject to uncertainty. Kuusela and Laiho (2020) model a two-stage game where countries choose to strategically acquire information regarding the damages from pollution in the first stage, when this damage is uncertain, before committing to pollution abatement in

the second stage. Schumacher (2015) and, more recently, Banerjee and Gravel (2020) look at how individuals’ beliefs may affect their contributions to a public good when there is uncertainty. Auerswald et al. (2018) take account of the uncertainty about the impact of climate change and allow countries to choose mitigation effort as well as adaptation effort. Another strand of the literature studies the global climate problem in the face of incomplete information and how institutional provisions affect the likelihood of reaching an agreement. Konrad and Thum (2014) study the role of unilateral commitment on environmental policies when countries negotiate about a cooperative environmental agreement when incomplete information prevails, showing that unilateral commitment reduces the probability of reaching an agreement. Konrad and Thum (2018) study the effect of the Clean Development Mechanism that was an essential element of the Kyoto Protocol for the chances of reaching a cooperative environmental agreement. This rich body of literature so far has not looked at comparing different national commitment scenarios under uncertainty, as in the setting of our paper.

In summary, while the article is related to the strands of literature on private provision of a public good (with complete and incomplete information) as well as the literature on environmental economics, the setting of comparing national-level commitment targets under income uncertainty has not been considered so far. Moreover, we consider the non-cooperative solution rather than the case of coalition formation through signatory countries, as commonly considered in environmental economics literature. Our analysis of the determinants of non-cooperative agreement is therefore also relevant for research on cooperative outcomes, as the non-cooperative equilibrium is the “default” of such cooperative agreement and therefore affects the bargaining game between the countries.

We proceed as follows. Section 2 formally presents the framework of the four different scenarios of voluntary commitments that countries may follow. Section 3 derives the equilibrium outcomes of the four games. Section 4 presents a comparison of expected levels of emissions reduction in each of the cases studied and some implications for ex-ante welfare. Section 5 analyzes extensions and Section 6 concludes. All proofs are in the Appendix.

2 | A Formal Framework

Consider an uncertain world with n symmetric countries $i \in \{1, \dots, n\} = \mathcal{N}$. The assumption of symmetry is important. It allows us to abstract from possible differences between countries in terms of size, risk-attitudes, preferences, technology constraints, and so forth to highlight how, under income uncertainty, different possible commitment regimes on countries affect the voluntary provision game.⁹ Uncertainty is about the state of the world. There are S states of the world, indexed by $s \in \{1, \dots, S\} = \mathcal{S}$. Probabilities p_s are assigned to the states. Each country has an income that is state-dependent and equal to m_{is} , where symmetry implies $m_{is} = m_s$ for all $i \in \mathcal{N}$. The value of m_s can be seen as the random output based on the production facilities of the

country and a global systematic random component, such as uncertain universal technological progress, major global crises such as the financial crisis or the pandemic crisis, or other major global risks that are looming and affect the path of economic development and general prosperity. Since the risks assumed are global risks, we do not focus on countries' subjective perceptions regarding these risks (which may differ for different countries). In contrast, we assume that all countries face the same probabilities of global risk. We number states such that $\min_{s \in \mathcal{S}} m_s = m_1$, that is, the state with the smallest income is state $s = 1$. Countries know m_s for each state and the probability distribution $(p_1, \dots, p_S)$. Let average income be $\bar{m} = \sum_{s \in \mathcal{S}} p_s m_s$. We consider several games which differ in whether countries know the true state s before making decisions and in how countries' choices are restricted.¹⁰

Countries use part of their incomes for reducing carbon emissions. The reduction in emissions is the global pure public good. The amount of income not contributed towards emissions reduction is consumed in the respective country and has no international spillover. Drawing on the theory of non-cooperative voluntary contributions to a public good, we refer to it as the country's "private consumption." Countries' payoffs are measured in expected utility (of their representative citizen). Utility is additively separable between the two goods: in a given state,

$$U_{is} = u(m_s - g_{is}) + v(G_s), \quad (1)$$

where g_{is} is the contribution of country i to emissions reduction and G_s is the aggregate amount of contributions to emissions reduction (or the public good) that emerges from the individual contributions to the reduction. This assumes the form of an additive aggregation function

$$G_s = \sum_{i \in \mathcal{N}} g_{is}. \quad (2)$$

The function v captures the damages that climate change creates and how these damages can be reduced by the total amount of CO₂ reduction, as in Nordhaus (2018). Over the course of the article, we use the terms "mitigation of damages from emissions" and "(resources contributed to) emissions reduction" interchangeably—both terms would be valid interpretations of countries devoting financial resources to lowering emissions. We assume u and v are three times continuously differentiable, strictly increasing and strictly concave.

The assumptions that utility is separable in private consumption and emissions reduction and that all countries have the same utility functions u and v are strong. Separability helps us interpret our results in terms of the risk properties of the two utility components: if the functions u and v differ in their curvature, then the two consumption components differ in their risk-absorbing capacities, with implications for the comparison of different commitments from our analysis. The assumption of symmetry across countries helps us make our main points in the easiest possible setup. However, we discuss the consequences of asymmetries in Section 5. Furthermore, we will assume that u and v display "prudence," i.e., $u''' > 0$, $v''' > 0$.¹¹

We assume that countries maximize their own expected utility

$$\sum_{s \in \mathcal{S}} p_s (u(m_s - g_{is}) + v(G_s)). \quad (3)$$

Finally, in all our analysis we characterize and focus on interior solutions for all countries. We study four games that differ in the definition of countries' action spaces, each corresponding to a different type of commitment to contributions towards emissions reduction.

Game 1 (Full ex-post flexibility). Each country observes m_s and chooses the amount $g_{is} \in [0, m_s]$ of contributions towards emissions reduction, which implicitly also determines $x_{is} = m_s - g_{is}$, the private consumption in this country in this state of the world. For any given state, this framework is equal to the standard problem of voluntary public good provision. This case corresponds to the baseline, where countries' emissions reduction targets require no upfront commitments. We denote the equilibrium amount of private good for country i in state s for Game 1 by x_{is}^I , the country's equilibrium contribution to emissions reduction in state s as g_{is}^I , and the equilibrium amount of the total public good in state s by $G_s^I = \sum_{i \in \mathcal{N}} g_{is}^I$.

Game 2 (Fixed absolute contributions to climate protection). Each country must pledge a state-independent future contribution $g_i \in [0, m_1]$ to contributions towards emissions reduction, the public good (recall: $\min_{s \in \mathcal{S}} m_s = m_1$ by appropriate state numbering). Negative contributions are not feasible, and contributions cannot exceed the country's income. National consumption in the country is then $x_{is} = m_s - g_i$. It is the residual of national income and absorbs all national income risk. This game would correspond to the scenario where there is a fixed cap on emissions, and therefore, fixed requirements for contributions towards emissions reduction. We denote the equilibrium amount of private good for country i in state s for Game 2 by x_{is}^II , the country's equilibrium contribution to emissions reduction as g_i^II , and the equilibrium amount of the total public good by $G^II = \sum_{i \in \mathcal{N}} g_i^II$.

Game 3 (Fixed private consumption). In this scenario, each country must make commitments on future private consumption $x_i \in [0, m_1]$ before learning the state of the world. The interval of possible x_i accounts for the non-negative values of both a country's contribution to emissions reduction as well as the country's private consumption. Game 3 is the mirror image of Game 2. Private consumption and contributions to emissions reduction switch roles: future national consumption becomes an income-independent quantity and all income-risk bearing is placed on contributions to emissions reduction. This scenario would correspond to a stringent commitment in which countries set targets for national consumption (for internal reasons linked to political stability, say) and devote all remaining resources, if any, to emissions reduction. We denote the equilibrium amount of private good for country i for Game 3 by x_i^III , the country's equilibrium contribution to emissions reduction in state s as g_{is}^III , and the equilibrium amount of the total public good in state s by $G_s^III = \sum_{i \in \mathcal{N}} g_{is}^III = \sum_{i \in \mathcal{N}} m_s - x_i^III$.

Game 4 (Fixed proportional contributions). Each country must devote a proportion $\gamma_i m_s$ of income to contributions towards

emissions reduction, and this contribution share γ_i is independent of the state of the world. Contribution shares are from the interval $\gamma_i \in [0, 1]$. In Game 4, a country reacts to high or low national income along both dimensions—much like in Game 1—but in fixed proportions—unlike in Game 1 in which the country can react in a way that is fully individually optimal. This scenario corresponds to the policy recommendation where future commitments are proportional to state-specific income, as described in optimal carbon tax models (Golosov et al. 2014; Dennig et al. 2015; van der Ploeg and Rezai 2021). We denote the equilibrium amount of private good for country i in state s for Game 4 by $x_{is}^{IV} = (1 - \gamma_i^*)m_s$, the country's equilibrium contribution to emissions reduction in state s as $g_{is}^I = \gamma_i^*m_s$, and the equilibrium amount of the total public good by $G_s^{IV} = \sum_{i \in N} g_{is}^{IV} = \sum_{i \in N} \gamma_i^* m_s$.

The above completes the descriptions of four variants of a voluntary contribution game in the presence of global income uncertainty and commitments. In the next section, we solve for the equilibria in these games and compare them. The Nash equilibrium in all four games will be generically inefficient, given the positive externality of contributions. But the constraints of how countries can adjust to the varying budget make it less clear what the overall effects for expected public-good provision and welfare will be.

3 | Equilibrium for the Four Games

3.1 | Full Flexibility

We start with the benchmark setup (Game 1). Each country can observe the state of the world, forms expectations about what the contributions of the other $n - 1$ countries will be, and chooses its own contributions as an optimal reply to these Nash conjectures. Let the state of the world be s . As states of the world are mutually exclusive events, any state that materializes leads to a standard public-good contribution game; in other words, the fact that another state $s' \neq s$ could have emerged is irrelevant for the choices and payoffs emerging in s . Formally, country i solves the problem of maximizing (1) for all possible states, resulting in S Nash levels of contributions towards emissions reduction $G_1^I, \dots, G_S^I$. In a symmetric equilibrium, $g_{is}^I = G_s^I/n$ for any $i \in N$. We then obtain the following:

Proposition 1. *The condition that determines the interior symmetric equilibrium provisions of contributions towards emissions reduction and individual countries' private consumptions for Game 1 is*

$$u'\left(m_s - \frac{1}{n}G_s^I\right) = v'(G_s^I) \text{ for all } s = 1, \dots, S. \quad (4)$$

This equilibrium exists for state s if $u'(0) > v'(nm_s)$ and $v'(0) > u'(m_s)$. When this symmetric equilibrium exists, it is the unique interior equilibrium.

The condition in Equation (4) for the interior equilibrium implies that for an individual (contributing) country in any state of the world s , at the Nash equilibrium, the marginal utility

from private consumption must equal the marginal utility from the contributions towards emissions reduction in that state. Multiplying the S marginal conditions in Equation (4) by the probabilities of the different states of the world and summing them up yields, using symmetry,

$$\sum_{s \in S} p_s u'\left(m_s - \frac{1}{n}G_s^I\right) = \sum_{s \in S} p_s v'(G_s^I), \quad (5)$$

which states that, overall, the individual country has the same expected marginal utility from private consumption and contributions towards emissions reduction. Note that the condition reduces to the optimal allocation choice of a player who makes optimal consumption choices in a two-dimensional consumption space for a budget m_s , equalizing marginal utility for each good in this state. If this occurs for all states of the world, then this also equalizes the expected utility of a marginal unit of expenditure on the two components of the utility function. The public good problem simply adds to this in the sense that other players also contribute an anticipated amount to contributions towards emissions reduction, and for a given player in a given state s this anticipated contribution sum is simply an expansion of the own budget by an additional amount $\sum_{j \in N \setminus \{i\}} g_{js}^I$, and so for all independent states $s \in S$.

Game 1 is the benchmark case used to study the three types of commitments to emissions reduction which we look at now.

3.2 | Fixed Contributions to Climate Protection

Suppose countries have to decide on their contributions to emissions reduction before knowing the realization of the state of the world (Game 2). Once the level of g_i is chosen before s is known, the private consumption in the country becomes the residual $x_{is} = m_s - g_i$, and differs for the different income states $s = 1, \dots, S$. In a symmetric equilibrium, $g_i^II = G^{II}/n$ for any $i \in N$, where G^{II} is the equilibrium total public good provided. The following describes the equilibrium:

Proposition 2. *The condition that characterizes the unique symmetric interior equilibrium provisions of the level of contributions towards emissions reduction and individual countries' consumption for Game 2 is*

$$\sum_{s \in S} p_s u'\left(m_s - \frac{G^{II}}{n}\right) = v'(G^{II}). \quad (6)$$

This equilibrium exists if $\sum_{s \in S} p_s u'(m_s) < v'(0)$ and $\sum_{s \in S} p_s u'(m_s - m_1) > v'(nm_1)$. When this symmetric equilibrium exists, it is the unique interior equilibrium.

The condition at the interior equilibrium can be interpreted as follows: for any country at the Nash equilibrium, the expected marginal utility from private consumption must equal the ex-ante marginal utility from contributions towards emissions reduction. Note that, despite the need to calculate expectations in the left-hand side of condition (6), from a formal point of view the essential structure of the framework of Bergstrom et al.

(1986) (or the more elegant framework by Buchholz 1990) is similar to that of our model. We also note that each component x_{is} of the vector of state-contingent consumption levels is fully informative for all its components: for instance, a given private consumption level in state s implies that the private consumption level in state s' is $x_{is'} = x_{is} + (m_{s'} - m_s)$. This link reduces the problem to a standard choice, much like in Bergstrom et al. (1986), where the marginal utility of private consumption is replaced by the marginal expected utility of private consumption and where the country has only one degree of freedom of choice.

It is interesting to assess the implications of riskiness of income in this context. To this end, we compare our results with a game without uncertainty, that is, a game with only one state of the world in which income is the “certain income” $\bar{m}$, with $\bar{m} = \sum_{s \in S} p_s m_s$.

Proposition 3. *Let $u''' > 0$ (prudence). In Game 2, (fixed commitments to emissions reduction) systematic income risk reduces the equilibrium provision to emissions reduction compared to the game without uncertainty.*

Intuitively, with no income risk then both the private and public good consumption components are certain. If income becomes uncertain, then it matters that the commitment to an income-independent contribution to emissions reduction shifts all this risk to the private-consumption utility component. Countries make provisions for this risk and take precautions. If they are ‘prudent’ in the private consumption utility component (convexity of u'), they shift resources from the safe use to the risky utility component; therefore, the equilibrium contributions to lowering emissions are smaller under uncertainty than under certainty. Proposition 3 has implications for national future-date emissions reduction targets that, following policy recommendations, may target a fixed limit on emissions (Zhou and Wang 2016; Nordhaus 2018): it shows that voluntary contributions towards emissions reduction under income uncertainty would be lower. Lopez et al. (2022) find a related result for an individual country, without assuming national commitments to pollution reduction, and abstracting from strategic concerns in a game-theoretic framework where there is free-riding. They also find that uncertainty increases pollution levels (which is an alternative re-framing of lower emissions reduction).

3.3 | Fixed National Private Consumption Choices

Let us now reverse the assumptions about which type of consumption is less flexible due to long-term national commitments. Suppose countries must choose fixed levels of their private consumption not knowing the realization of the state of the world, and all income over and above that level of private consumption becomes contributions to emissions reduction (Game 3). In a symmetric equilibrium, $x_i^{III} = x^{III}$ for any $i \in N$. This implies $g_{is}^{III} = m_s - x^{III} = G_s^{III}/n$ for any $i \in N$ and $s \in S$. The following describes the equilibrium:

Proposition 4. *The condition that determines the interior symmetric equilibrium provisions of contributions towards*

emissions reduction and individual countries’ consumption for Game 3 is

$$u'(x^{III}) = \sum_{s \in S} p_s v'(nm_s - nx^{III}). \quad (7)$$

If $u'(m_1) < \sum_{s \in S} p_s v'(n(m_s - m_1))$ and $u'(0) > \sum_{s \in S} p_s v'(nm_s)$ this equilibrium exists. When this symmetric equilibrium exists, it is the unique interior equilibrium.

The equilibrium condition states that the expected marginal utility from contributions towards emissions reduction must equal the ex-ante marginal utility from private consumption for a country. The proof of existence and uniqueness uses again that global income risk is the same for all countries, which reduces the dimensionality of the problem. The existence condition requires that the marginal utility of the first unit of the private good is sufficiently high to prevent countries to expend all income on reducing emissions, and that the marginal utility of private good is sufficiently declining to make some contribution to emissions reduction worthwhile for them.

In analogy to Proposition 3, we can ask what is the implication of income uncertainty for the equilibrium contributions towards emissions reduction. All utility risk is attributed to the component that captures the benefit from emissions reduction. Compared to a situation without income risk, it is this utility component that becomes risky and absorbs all the risk. This motivates the following proposition, which holds under prudence.

Proposition 5. *Let $v''' > 0$ (prudence). In Game 3 systematic income risk increases the expected equilibrium provision to the public good compared to the game without uncertainty.*

Proposition 5 is the mirror image to Proposition 3. Their joint message is that if all risk that comes with risky national incomes is completely absorbed by one of the two activities (private consumption or emissions reduction), then this implies that the country will shift more resources into this activity if the respective utility component (u or v respectively) is characterized by prudence. To facilitate this shift of resources, they have to cut back on the activity that is not risky.¹² Proposition 5 carries policy implications for the scenario where countries choose to fix consumption and keep emissions reduction as flexible as possible to income shocks. The results show that in this case, uncertainty would improve the outcome on emissions reduction as compared to certainty. This result contrasts with the findings of Lopez et al. (2022) as pointed out in the discussion to Proposition 3. Hence, Proposition 3 combined with Proposition 5 shows that the framing of national-level commitments may affect the aggregate resources devoted to emissions reduction under income uncertainty.

3.4 | Strict Proportionality

Game 4 corresponds to the scenario where, like in optimal carbon tax models (Golosov et al. 2014) or targets expressed in terms of emissions intensity, countries implement a policy

recommendation such that they choose what share of the income the country will expend on contributions towards emissions reduction, and the remaining share will be expended on private consumption. In other words, if each country decides which share γ_i of national income to contribute to emissions reduction in the different states, then this fixed share automatically determines the share of a country's national income which is privately consumed, for all possible states. Each of the n contributors maximizes

$$\max_{\gamma_i} \sum_{s=1}^S p_s u((1 - \gamma_i)m_s) + \sum_{s=1}^S p_s v\left(\gamma_i m_s + \sum_{j \neq i} \gamma_j m_s\right). \quad (8)$$

In a symmetric equilibrium, $\gamma_i^* = \gamma^*$ for any $i \in N$. The following describes the equilibrium:

Proposition 6. *The condition that determines the symmetric equilibrium contributions to emissions reduction levels and individual countries' consumption for Game 4 is*

$$\sum_{s=1}^S p_s m_s u'((1 - \gamma^*)m_s) = \sum_{s=1}^S p_s m_s v'(n\gamma^*m_s). \quad (9)$$

This interior equilibrium is unique if it exists, and boundary conditions for the existence of this equilibrium are: $\sum_{s=1}^S p_s m_s u'(m_s) < \bar{m} v'(0)$ and $\bar{m} u'(0) > \sum_{s=1}^S p_s m_s v'(nm_s)$.

Intuitively, condition (9) shows that, at the (interior) Nash equilibrium, the expected marginal utility from contributions towards emissions reduction must be the same as the expected marginal utility from private consumption for each country. Proportional contributions allow countries to place some risk on both types of goods.

In analogy to Propositions 3 and 5, we can ask what is the implication of income uncertainty for the Nash equilibrium contributions to overall damage reduction from emissions. Unlike the previous cases, in Game 4 systematic income risk may increase or decrease the expected equilibrium provision to emissions reduction compared to certain income, even when both u and v display prudence, regardless of whether u is “more” or “less” concave than v . The following example illustrates this:

Example 1. Constant relative risk aversion utility. Let

$$u(x) = \frac{x^{1-\eta_c} - 1}{1 - \eta_c} \text{ and } v(y) = \frac{y^{1-\eta_G} - 1}{1 - \eta_G}, \quad (10)$$

where the relative risk aversion coefficient for u (respectively, v) is η_c (η_G), with $\eta_c, \eta_G > 0$. Let the number of agents be 2. Let the uncertain income case be characterized by $p_1 = 1/3$, $m_1 = 2$, $p_2 = 2/3$, $m_2 = 5$. Let the certain income be characterized by $\bar{m} = p_1 m_1 + p_2 m_2 = 4$. Let γ^* be the symmetric equilibrium choice of share of income devoted to emissions reduction in the equilibrium with income uncertainty and $\bar{\gamma}^*$ be the chosen share in the absence of income risk. Table 1 summarizes the comparison of γ^* and $\bar{\gamma}^*$ for several values of η_c and η_G .

TABLE 1 | Proportional contributions to emissions reduction with and without income uncertainty.

	γ^*	$\bar{\gamma}^*$
1) $\eta_c = 0.3, \eta_G = 0.3$	0.3333	0.3333
2) $\eta_c = 0.4, \eta_G = 0.3$	0.4016	0.4009
3) $\eta_c = 0.3, \eta_G = 0.4$	0.2766	0.2772
4) $\eta_c = 0.8, \eta_G = 0.4$	0.4990	0.5000
5) $\eta_c = 0.4, \eta_G = 0.8$	0.2213	0.2207

The above example shows that, in the case in which countries commit a fixed proportion of their income to emissions reduction, depending upon the utility function's characteristics, the percentage of the national income contributed to emissions reduction may increase or decrease under uncertainty. As Table 1 shows, income uncertainty may leave contributions unchanged, as in line 1), increase them, as in lines 2) and 5), or decrease them, as in lines 3) and 4). Moreover, it appears that the proportion rising or falling is not related to whether η_c is higher than η_G —that is, the concavity of the function u relative to v (see lines 2) and 4)). Increases and decreases in contributions can occur both when u is “more” concave than v ($\eta_c > \eta_G$) and when u is less concave than v ($\eta_c < \eta_G$). Example 1 is therefore mildly encouraging: it demonstrates that, depending upon the preferences of countries, policies with emissions intensity targets may improve public good provision under uncertainty.

4 | Comparisons Under Uncertainty

For the same aggregate income uncertainty, the equilibria in the four games generally differ. We first compare the expected quantity of emissions reduction.

4.1 | Expected Emissions Reduction Amounts

Each country's consumption pattern changes due to the outcome of income uncertainty. The differences in shape of u and v determine how uncertainty affects the allocation choices in each of the four equilibria, as analyzed in the previous section. This makes a full comparison for the four equilibria for general u and v cumbersome and dependent on the details of the income distribution. Some insight into the structural properties can be gained, however, if we know properties of the function $\phi(x)$, defined as

$$\phi(x) \equiv [u']^{-1}(v'(x)).$$

The following results uses properties of ϕ to compare the equilibrium outcomes of Games 1–3.

Proposition 7. (i) *The equilibrium quantity of emissions reduction in Game 2 is smaller than the expected equilibrium emissions reduction for the state-contingent case in Game 1 if u is characterized by prudence (i.e., $u'''(x) > 0$) and $\phi(x)$ is concave.* (ii) *The expected equilibrium emissions reduction in Game 3 is larger than the expected equilibrium emissions reduction for the*

state-contingent case in Game 1 if v is characterized by prudence (i.e., $v'''(x) > 0$) and $\phi(x)$ is convex.

Proposition 7 helps us understand the combined effect of commitment and income risk, comparing Games 1–3. In Game 1 there are no commitment, so both private consumption and emissions reduction are in equilibrium affected by income risk. Games 2 and 3 are more extreme in how the income risk is absorbed than in Game 1. In Game 2, all income risk is absorbed by private consumption; in Game 3, all income risk is absorbed by emissions reduction. The sufficient condition on the concavity or convexity of $\phi(x)$ is key. Its interpretation is easiest in the case of the constant relative risk-aversion utility in Example 1, as one can show that $\phi(x) = x^{\frac{\eta_G}{\eta_c}}$, so $\phi(x)$ is concave [convex] if $\eta_c \geq (\leq) \eta_G$.¹³ As we compare Game 1 to Game 2, more risk is shifted to the private consumption sector in the latter. If furthermore $\eta_c \geq \eta_G$ —that is, if the utility for private consumption displays a larger relative risk-aversion than that for emissions reduction—then we will see prudent players allocate more resources to this sector. The interpretation of the comparison between the equilibrium outcomes of Game 1 and Game 3 is similar, except that now the kind of commitment in Game 3 results in a larger shift of risk towards emissions reduction.

One may wonder about the comparison of Game 1 and Game 4. As in both these games both components of utility are in equilibrium affected by income risk, the comparison between equilibrium outcomes is very much dependent on the fine details of the income distribution and no general conclusion can be drawn, even under the assumptions used in Proposition 7.

4.2 | Ex-Ante Welfare

One can divide the schemes that we are considering into restrictive (Games 2, 3, and 4) and non-restrictive (Game 1), depending on whether the scenario requires a commitment from the part of countries or not. In other words, in Game 1 countries are allowed to adjust—independently in each state—their private consumption and their contributions to emissions reduction. In contrast, in Games 2, 3, and 4, countries are either constrained to a fixed level of contributions to reducing emissions, a fixed level of private consumption, or an across-states constant proportional division of income between private consumption and contributing to lowering emissions. It turns out that having a restrictive scheme may end up benefiting countries with respect to the non-restrictive scheme. On the one hand, the non-restrictive scheme always does better in an individualistic problem, simply because a constrained maximization problem cannot give a larger utility than an unconstrained one. On the other hand, when multiple agents are contributing to a public good, it is possible that a restriction imposed to an individual's choice may result in a larger public good contribution in some states, and in turn this may end up alleviating the free-rider problem and lead to a Pareto improvement with respect to a nonrestrictive scheme.

We formally make our point in the context of strict proportionality (Game 4). As happens for the emissions reduction

level, a full comparison of welfare between strict proportionality and state-contingent provision for general u and v is cumbersome, and the result will depend on the details of the income distribution. Therefore, we illustrate our point that restrictions caused by national commitments may be welfare-improving by assuming specific functional forms for utility and comparing the non-restrictive scheme with state-contingent public good provision against the restrictive scheme with proportional contributions. In particular, we reprise the pair of utility functions from Example 1 and obtain the following result.

Proposition 8. Let $u(x) = \frac{x^{1-\eta_c}-1}{1-\eta_c}$ with $\eta_c > 0$ and $v(G) = \frac{G^{1-\eta_G}-1}{1-\eta_G}$ with $\eta_G > 0$.

1. If $n > 1/(1 - \eta_c)$, then there exists some $\bar{\eta}_G \in (0, 1)$ such that, for any $\eta_G < \bar{\eta}_G$, the ex-ante expected utility in the equilibrium of the game with strict proportionality (Game 4) is larger than that of the state-contingent game (Game 1).
2. If $n < 1/(1 - \eta_c)$, then there exists some $\bar{\eta}_G \in (0, 1)$ such that, for any $\eta_G < \bar{\eta}_G$, the ex-ante expected utility in the equilibrium of the game with strict proportionality (Game 4) is smaller than that of the state-contingent game (Game 1). Furthermore, if $n = 1$, then the ex-ante expected utility in the equilibrium of the game with strict proportionality (Game 4) is smaller than that of the state-contingent game (Game 1) for any η_c and η_G with $\eta_c \neq \eta_G$.

The result described in Part 2. of Proposition 8 for $n = 1$ is intuitive. When there is only one agent the only force at play is the loss of flexibility in the individual maximization problem. But if $n > 1$, then the utility in the restrictive Game 4 can be larger than in the nonrestrictive Game 1, as part 1. of Proposition 8 shows. For instance, consider a two-state model with $p_1 = p_2$, $m_1 = 2$, $m_2 = 10$, and fix $\eta_c = 3/5$. For $n = 3$, numerical simulations show that the ex-ante expected utility in the equilibrium of the game with strict proportionality (Game 4) is larger than that of the state-contingent game (Game 1) for $\eta_G < 0.07928$, illustrating that, when $n > 1$, the distortions induced by the proportionality restriction can alleviate the free rider-problem.

One can show that an increase in n increases ex-ante expected equilibrium utility in both the state-contingent and in the proportional game, because individual contributions fall (and so private consumption rises) and total contributions to emissions reduction (public good provision) increases. More relevant to Proposition 8 is the comparison of these equilibrium utilities. One can rewrite the condition in Part 1. as an upper bound for the relative risk aversion parameter η_c : $\eta_c < 1 - \frac{1}{n}$. Therefore, the larger n , the larger the set of values of η_c for which the free-rider problem is alleviated by the proportional restriction. Furthermore, for any fixed $\eta_c \in (0, 1)$ there exists an $\bar{n} < \infty$ such that the sufficient condition in Part 1. of Proposition 8 holds and the utility in the restrictive game is larger than in the non-restrictive game. These results accord with the intuition at the beginning of this section: restricting individual choice for everybody may end up alleviating the free-rider problem, which is especially severe if the number of potential contributors is large.

5 | Extension: Asymmetric Countries

The conclusions of the paper have so far assumed that all countries have the same utility functions. This may appear as a restrictive assumption since countries in reality may differ in their valuations of the benefit from emissions reduction and in consequence may also differ in their contributions to mitigating the damage from emissions. In this section, we will explore the implications of relaxing the assumption of symmetry on the results of the paper to allow for countries to differ in their valuations of the private good (u_i) and the public good (v_i). In a given state, we allow the utility of a country as stated in condition (1) to change to $U_{is} = u_i(m_s - g_{is}) + v_i(G_s)$. As it turns out, under the assumption that the set of contributors does not change across income certainty and uncertainty, the results of Proposition 3 extend to the case of countries with asymmetric utilities, as noted below:¹⁴

Observation. Assume the set of contributors does not change when comparing Game 2 (fixed commitments to emissions reduction) with income uncertainty and without uncertainty. Let all countries' (asymmetric) utilities for private consumption display prudence. In Game 2, systematic income risk reduces the total equilibrium contributions to emissions reduction compared to the game without uncertainty.

We therefore note that our results in Proposition 3 are robust under the caveat that the set of contributors does not change across income certainty and uncertainty. We observe however that under asymmetric utilities, while total contributions decrease under uncertainty for Proposition 3 as compared to certainty, individual countries may contribute more under uncertainty than certainty, as shown in the example below:

Example 2. Consider the utility functions in Example 1 but assume there are two countries; for Country 1 we have

$$\eta_c = \eta_G = 0.8,$$

while for Country 2 we have

$$\eta_c = 0.3 \text{ and } \eta_G = 0.5.$$

Finally, assume that $p_1 = 1/3$, $p_2 = 2/3$, $m_1 = 2$, and $m_2 = 5$.

Numerical simulations show that $g_1^H \approx 1.17$ and $g_2^H \approx 0.68$, so that $G^H \approx 1.85$, while $\bar{g}_1 \approx 1.83$ and $\bar{g}_2 \approx 0.35$, so that $\bar{G} \approx 2.18$.

Example 2 shows that, in accordance with Proposition 3, the total emissions reduction is lower because of income uncertainty. But Country 2, which is relatively less sensitive to risk, ends up contributing more. This is because Country 1, which is more sensitive to risk, massively reduces its contribution to emissions reduction because of uncertainty, and Country 2 has to make up some of the difference because countries' contributions are strategic substitutes.

It is not difficult to establish, under similar conditions where the set of contributors does not change, that our results for Proposition 5 and Proposition 7 also go through for the case of

asymmetric utilities. We also find numerical examples that show a similar conclusion for individual countries as shown in Example 2 for Proposition 5 and both parts of Proposition 7: while our results go through for the overall effect on emissions reduction, it is possible for *some* individual countries to deviate from the overall pattern in their contributions.

We can therefore draw two main conclusions from the analysis of asymmetric utilities: first, that the main messages of the paper are unchanged if we allow for heterogeneity in countries' preferences, and second, that if countries have heterogeneous preferences, then for *some* countries contributions may go in the direction opposite to the overall contributions in the presence of commitments. We note at the same time that the assumption that the set of contributors does not change is important in this context: given that the existence of an interior equilibrium is more complex to establish under asymmetric preferences, the comparison of equilibrium outcomes is therefore complicated in this case by the very real possibility that the set of contributors might change across certainty and uncertainty.¹⁵

6 | Conclusion

In a closely interconnected world economy, the possibility of events such as world financial crisis, pandemics, or disruptive technological developments can cause global and correlated income risk for countries. The following questions emerge: how do countries address such uncertainties when making contributions to emissions reduction, and could their equilibrium contributions depend on the framing of the national commitments? This paper determines the different Nash provision equilibria in the presence of global income risks, and compares how the different voluntary national commitments to limiting damages from emissions affect the equilibrium contribution outcomes. The analysis takes the fully flexible allocation between contributions to emissions reduction and purely national concerns as a benchmark, and assumes that countries' preferences are characterized by prudence. Under fixed absolute financial commitments to (emissions) damage reduction, income risk will, under plausible conditions, lead to lower contributions to emissions reduction. On the other hand, we also find that income risk leads to higher contribution efforts to reducing damage from emissions when countries fix private consumption and invest all remaining resources towards emissions reduction. Our results on the scheme with proportional commitments are also positive, as uncertainty may increase contributions to emissions reduction and counteract the free-rider problem.

These results have policy implications for the framing of national commitments to emissions reduction—contributions proportional to GDP may be welfare-improving if all countries commit to them. An important caveat of our analysis is that, for simplicity, our theoretical model does not take into account a direct effect of income shocks on the climate: emissions may fall (rise) due to a fall (rise) in economic activity. Moreover, we note the consideration that a reduction in emissions may not translate one-to-one to an immediate lowering of warming (Dietz and Venmans 2019). Our results are also from a strictly non-

cooperative equilibrium analysis are not to be understood one-to-one as predictions for climate policy. In climate policy, countries negotiate with each other and the results are not strictly non-cooperative. However, as often emphasized in other contexts, the prediction of the non-cooperative Nash equilibrium outcome is also relevant for climate negotiations insofar as it (co-)determines the threat point of such negotiations, and it is the “default” in case such negotiations fail. National commitments to emissions reduction are one of the necessary first steps to achieving targets on climate policy, and our paper highlights the need for understanding the welfare implications of voluntary climate commitments as a first step in the direction to strengthen such future commitments. The results of the paper also lead to other interesting questions, such as a consideration of the case where countries are allowed to have private beliefs about the probability of income shocks. The study of public good contributions under commitments in the situation where countries hold heterogeneous beliefs about the gravity of imminent climate change is a promising topic for future research in this field.

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Data Availability Statement

The authors have nothing to report.

Endnotes

¹ A list of major global risks and how experts assess their probability and impact is in the World Economic Forum's *Global Risks Report* (Forum 2021). According to Baker et al. (2016) (who develop the World Uncertainty Index, an index of economic policy uncertainty based on newspaper coverage frequency), global uncertainty levels have been rising since the 1960s. Ahir et al. (2021) report that global uncertainty levels as measured by the World Uncertainty Index, though declining post the coronavirus crisis, remain high.

² It has also been noted that major prosperity shocks can affect the attention given to the problem of global warming. For example, Botzen et al. (2021) and Evensen et al. (2021) point to a crowding out of awareness of impending risks from the climate change problem. Our analysis is not about attention issues, but about equilibrium behavior among fully rational and well-informed governments.

³ For instance, in their September 2022 NDC update, the UK pledges to reduce “economy-wide greenhouse gas emissions by at least 68% by 2030, compared to 1990 levels,” and, in their April 2021 NDC after rejoining the Paris agreement, the US sets “an economy-wide target of reducing its net greenhouse gas emissions by 50%–52% below 2005 levels in 2030 (Nationally Determined Contributions Registry 2023). Although countries participating in the Paris Agreement are legally required to have an NDC, they are free to set their goals and are not legally required to actually meet these targets (Falkner 2016).

⁴ Pizer and Kopp (2005), for example, refer to Dixit and Pindyck (1994) and Pindyck (1995) and state: “The potential exists for environmental protection activities to involve irreversible costs [so that] the cost of a policy is raised by the forgone option value associated with waiting.”

⁵ As Pindyck (1995, 2) puts it: “First, policies aimed at reducing ecological damage impose sunk costs on society. These sunk costs can take the form of discrete investments; for example, coal burning utilities might be forced to install scrubbers, or firms might have to scrap existing machines and invest in more fuel-efficient ones. Or they can take the form of flows of expenditures, e.g., a price premium paid by a utility for low-sulfur coal. In either case, such sunk costs create an opportunity cost of adopting a policy now, rather than waiting for more information about ecological impacts and their economic consequence.”

⁶ For instance, including carbon pricing may cause economic downturns (Ernst et al. 2023).

⁷ See also Cornes and Sandler (1985), Shogren (1987), Shogren (1990) and Sugden (1985).

⁸ See Bag and Roy (2008), Bag and Roy (2011) for sequential settings, Barbieri and Malueg (2010) and Menezes et al. (2001) for discrete public goods, and Barbieri and Maleug (2016) for a comparison of the discrete and continuous public good settings, among others.

⁹ We discuss asymmetries in more detail in Section 5. Symmetry across countries also removes issues of distributional effects from the picture that would be interesting to study, but orthogonal to the questions addressed here.

¹⁰ The focus on global risks is important. As is well-known, idiosyncratic risks are largely eliminated in the model of voluntary contributions. This is an implication of Warr (1983).

¹¹ In the framework of expected utility as per Von Neumann and Morgenstern (1944), the concept of higher-order risk attitudes was first introduced by Leland (1968), and subsequently formalized as prudence by Eeckhoudt and Kimball (1992) and Kimball (1990) to explain precautionary savings. A prudent individual is characterized by a positive third derivative of the utility function. While less common than risk aversion, prudence is a widely-accepted measure of precautionary attitudes even outside of savings behavior, and can be understood as characterizing a decision maker who prefers to combine both “good and bad outcomes” rather than face “all good” or “all bad” outcomes (Eeckhoudt et al. 2020).

¹² In our symmetric model what is true for one country is true for all countries; see however Section 5 for the effects of asymmetries.

¹³ More generally, one can show that $\phi(x)$ is concave (convex) if $v'''(x)(u''(\phi(x)))^2 \geq [\leq] u'''(\phi(x))(v''(x))^2$.

¹⁴ The proof is available from the authors upon request.

¹⁵ Differences across countries in endowments can be handled in a similar way, as long as endowments remain perfectly correlated across states.

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Appendix A

Proof of Proposition 1. The equilibrium analysis can be carried out for each s independently. For each state s the equilibrium outcome is a special parametric case of Bergstrom et al. (1986) and Buchholz (1990) for which existence and uniqueness is well known. For an interior symmetric equilibrium, (4) is the first-order condition. Therefore, if we show existence and uniqueness of a solution to (4), we have also shown that there exists a unique equilibrium, which happens to be symmetric. The left-hand side of (4) is continuous and strictly increasing in $G_s^I \in [0, nm_s]$ and the right-hand side of (4) is continuous and strictly decreasing in $G_s^I \in [0, nm_s]$. Accordingly, this interior equilibrium exists for state s if the left-hand side is smaller than the right hand side for $G_s^I = 0$ (i.e., $u'(m_s) < v'(0)$) and if the left-hand side is strictly larger for $G_s^I = nm_s$ (i.e., $u'(0) > v'(nm_s)$). $\square$

Proof of Proposition 2. We first characterize a symmetric equilibrium, then show existence and uniqueness of this equilibrium among symmetric equilibria, and then show that an asymmetric equilibrium cannot exist. For a symmetric equilibrium it must hold that $g_i^H = G^H/n$ for all $i = 1, \dots, n$. Hence, the first-order condition for

$$\max_{g_i} \sum_{s \in S} p_s u(m_s - g_i) + v\left(\sum_{j \in N} g_j\right)$$

can be written as (6). Recall $m_1 = \min_{s \in S} \{m_s\}$, such that feasible quantities of total contributions towards emissions reduction are in $[0, nm_1]$. The left-hand side of (6) is a continuous and strictly monotonically increasing function in G^H . The right-hand side of (6) is a continuous and strictly monotonically decreasing function in G^H . This equation therefore has one and only one solution for interior $G^H \in (0, nm_1)$ if $\sum_{s \in S} p_s u'(m_s) < v'(0)$ (left-hand side smaller than right-hand side at $G^H = 0$) and $\sum_{s \in S} p_s u'(m_s - m_1) > v'(nm_1)$ (left-hand side larger than right-hand side at $G^H = nm_1$). It remains to show that an interior equilibrium, if it exists, has to be symmetric. Suppose $G^H \in (0, nm_1)$ is the equilibrium quantity of total contributions towards emissions reduction in an interior equilibrium. Note that, by the requirements of Game 2, country i 's choice of g_i is the same for all states s . Suppose now $g_i^H > g_{i'}^H$ for some i and i' . Then $x_{i1}^H < x_{i'1}^H$, and as $x_{is}^H = x_{i1}^H + (m_s - m_1)$, it follows that $x_{is}^H < x_{i's}^H$ for all $s \in S$. Accordingly,

$$\sum_{s \in S} p_s u'(x_{is}^H) > \sum_{s \in S} p_s u'(x_{i's}^H). \quad (\text{A1})$$

Optimality of these x_{i1}^H and $x_{i'1}^H$ requires that

$$v'(G^H) = \sum_{s \in S} p_s u'(x_{is}^H) \text{ and } \sum_{s \in S} p_s u'(x_{i's}^H) = v'(G^H), \quad (\text{A2})$$

and Equations (A1) and (A2) are incompatible with each other. This completes the proof. $\square$

Proof of Proposition 3. Let $\bar{G}$ be the equilibrium quantity of total contributions towards emissions reduction of the game without uncertainty, that is, when income is known to be $\bar{m} = \sum_{s \in S} p_s m_s$, there is only one state, and each country maximizes (1). We note:

$$v'(\bar{G}) = u'\left(\bar{m} - \frac{\bar{G}}{n}\right) = u'\left(\sum_{s \in S} p_s \left(m_s - \frac{\bar{G}}{n}\right)\right).$$

The first equation is the (symmetric) equilibrium first-order condition, and the second equality follows from a straightforward transformation using the definition of $\bar{m}$. Now:

$$u'\left(\sum_{s \in S} p_s \left(m_s - \frac{\bar{G}}{n}\right)\right) < \sum_{s \in S} p_s u'\left(m_s - \frac{\bar{G}}{n}\right)$$

if $u''' > 0$ (prudence) by Jensen's inequality. Accordingly,

$$v'(\bar{G}) < \sum_{s \in S} p_s u'\left(m_s - \frac{\bar{G}}{n}\right). \quad (\text{A3})$$

The total public good quantity G^H solves (6), that is,

$$v'(G^H) = \sum_{s \in S} p_s u'\left(m_s - \frac{G^H}{n}\right) \quad (\text{A4})$$

To formally prove this claim that $G^H < \bar{G}$, assume by contradiction that $G^H \geq \bar{G}$. We then have by v' decreasing and Equation (A4) that

$$v'(\bar{G}) \geq v'(G^H) = \sum_{s \in S} p_s u'\left(m_s - \frac{G^H}{n}\right).$$

Now, by u' is decreasing and $G^H \geq \bar{G}$ we have

$$\sum_{s \in S} p_s u'\left(m_s - \frac{G^H}{n}\right) \geq \sum_{s \in S} p_s u'\left(m_s - \frac{\bar{G}}{n}\right).$$

Finally, by (A3) we have

$$\sum_{s \in S} p_s u'\left(m_s - \frac{\bar{G}}{n}\right) > v'(\bar{G}),$$

and the extremes of the chain of the last three displayed expressions are incompatible. $\square$

Proof of Proposition 4. We first establish existence of the interior symmetric equilibrium. Let G_1^{III} be the quantity of contributions towards emissions reduction in state 1. For an interior symmetric equilibrium it must hold that $g_{is}^{III} = \frac{G_1^{III}}{n} + (m_s - m_1)$ for all $i = 1, \dots, n$ and $G_s^{III} = G_1^{III} + n(m_s - m_1)$. Using $x_i^{III} = m_1 - \frac{G_1^{III}}{n}$, the first-order condition in (7) can be written as

$$u'\left(m_1 - \frac{G_1^{III}}{n}\right) = \sum_{s \in S} p_s v'\left(G_1^{III} + n(m_s - m_1)\right). \quad (\text{A5})$$

The left-hand side of Equation (A5) is a continuous and strictly monotonically increasing function in $G_1^{III} \in [0, nm_1]$. The right-hand side of Equation (A5) is a continuous and strictly monotonically decreasing function in $G_1^{III} \in [0, nm_1]$. This equation has one and only one solution for interior $G_1 \in (0, nm_1)$ if $u'(m_1) < \sum_{s \in S} p_s v'(n(m_s - m_1))$ (left-hand side smaller than right hand side at $G_1^{III} = 0$) and $u'(0) > \sum_{s \in S} p_s v'(nm_s)$ (left-hand side larger than right hand side at $G_1^{III} = nm_1$).

It remains to show that an interior equilibrium, if it exists, has to be symmetric. Suppose $G_1^{III} \in (0, nm_1)$ is the equilibrium quantity of contributions towards emissions reduction in state $s = 1$. Note that, by construction of Game 3, country i 's choice of x_i^{III} is the same for all states s . Suppose now $x_i^{III} > x_{i'}^{III}$ for some i and i' . Then $g_{is}^{III} < g_{i's}^{III}$ for all $s \in S$. Standard first-order conditions for the

maximization of utility in (3) under the constraints imposed in Game 3 imply

$$\sum_{s \in S} p_s v'(G_s^{III}) = u'(x_i^{III}) < u'(x_{i'}^{III}) = \sum_{s \in S} p_s v'(G_s^{III})$$

which is impossible. This completes the proof. $\square$

Proof of Proposition 5. Let x^{III} be the symmetric equilibrium choice of private consumption in the equilibrium with income uncertainty and $\bar{x}$ be the private consumption in the game without uncertainty described in the Proof of Proposition 3. The respective first-order conditions imply

$$u'(x^{III}) = \sum_{s \in S} p_s v'(nm_s - nx^{III}) \quad (\text{A6})$$

and

$$u'(\bar{x}) = v'(n\bar{m} - n\bar{x}).$$

Now, by Jensen's inequality it holds that if v' is convex, that is, if $v''' > 0$ (prudence),

$$\sum_{s \in S} p_s v'(nm_s - n\bar{x}) > v'(n\bar{m} - n\bar{x}) = u'(\bar{x}).$$

Proceeding as in the Proof of Proposition 3, starting from $\bar{x}$, to fulfill Equation (A6) it is needed that u' goes up—that is, $x^{III} < \bar{x}$ — and $\sum_{s \in S} p_s v'(m_s - x^{III})$ must go down, which also happens by $x^{III} < \bar{x}$. So, the equilibrium private consumption is lower than $\bar{x}$ if income is uncertain in Game 3. A lower private consumption corresponds to a higher expected contribution to emissions reduction. This concludes the proof. $\square$

Proof of Proposition 6. Using (8), the first-order condition for the optimal γ_i for country i is

$$\sum_{s=1}^S p_s m_s u'\left((1 - \gamma_i^*)m_s\right) = \sum_{s=1}^S p_s m_s v'\left(\gamma_i^* m_s + \sum_{j \neq i} \gamma_j^* m_s\right) \quad (\text{A7})$$

This defines a system of n equations. Note that a solution to this system with $\gamma_i^* < \gamma_{i'}^*$ for some i and i' can be ruled out: suppose that such a solution exists. Then $\sum_{s=1}^S p_s m_s u'\left((1 - \gamma_i^*)m_s\right) < \sum_{s=1}^S p_s m_s u'\left((1 - \gamma_{i'}^*)m_s\right)$ and $\sum_{s=1}^S p_s m_s v'\left(\gamma_i^* m_s + \sum_{j \neq i} \gamma_j^* m_s\right) > \sum_{s=1}^S p_s m_s v'\left(\gamma_{i'}^* m_s + \sum_{j \neq i'} \gamma_j^* m_s\right)$, and this contradicts that both γ_i^* and $\gamma_{i'}^*$ fulfill Equation (A7). Hence, $\gamma_i^* = \gamma_{i'}^* \equiv \gamma^*$ in any candidate equilibrium, such that Equation (A7) can be written as

$$\sum_{s=1}^S p_s m_s u'\left((1 - \gamma^*)m_s\right) = \sum_{s=1}^S p_s m_s v'\left(n\gamma^* m_s\right). \quad (\text{A8})$$

The left-hand side of Equation (A7) is continuously and strictly monotonically increasing in γ^* for $\gamma^* \in [0, 1]$ from $\sum_{s=1}^S p_s m_s u'(m_s)$ to $\bar{m} u'(0)$, and the right-hand side of Equation (A8) is continuously and strictly monotonically decreasing from $\bar{m} v'(0)$ to $\sum_{s=1}^S p_s m_s v'(nm_s)$. Hence, a sufficient condition for existence and uniqueness of a γ^* that solves Equation (A8) is $\sum_{s=1}^S p_s m_s u'(m_s) < \bar{m} v'(0)$ and $\bar{m} u'(0) > \sum_{s=1}^S p_s m_s v'(nm_s)$. $\square$

Proof of Proposition 7. i) Let the equilibrium public good amount in Game 1 and state s be G_s^I . Using Proposition 1 we obtain

$$v'(G_s^I) = u'\left(m_s - \frac{G_s^I}{n}\right), \text{ for any } s \in \mathcal{S},$$

which can be rewritten as

$$u'^{-1}\left(v'(G_s^I)\right) = m_s - \frac{G_s^I}{n}, \text{ for any } s \in \mathcal{S}.$$

Multiplying by p_s , adding over states, and using the notation for ϕ , we obtain

$$\sum_{s \in \mathcal{S}} p_s \cdot \phi(G_s^I) = \sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G_s^I}{n}\right). \quad (\text{A9})$$

Let the equilibrium public good amount in Game 2 be G^II . Using Proposition 2, the first-order condition that characterizes equilibrium in Game 2 is

$$v'(G^II) = \sum_{s \in \mathcal{S}} p_s u'\left(m_s - \frac{G^II}{n}\right).$$

Using $u''' > 0$ and Jensen's inequality, the above-displayed equality implies

$$v'(G^II) > u'\left(\sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G^II}{n}\right)\right).$$

As u' is decreasing, so is u'^{-1} , and we have

$$u'^{-1}(v'(G^II)) < \sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G^II}{n}\right),$$

or

$$\phi(G^II) < \sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G^II}{n}\right). \quad (\text{A10})$$

The statement we want to prove is that ϕ concave implies $G^II < \sum_{s \in \mathcal{S}} p_s G_s^I$. Proceed by contradiction and assume

$$G^II \geq \sum_{s \in \mathcal{S}} p_s G_s^I. \quad (\text{A11})$$

We obtain

$$\begin{aligned} \phi(G^II) &< \sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G^II}{n}\right) \leq \sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G_s^I}{n}\right) = \sum_{s \in \mathcal{S}} p_s \\ &\cdot \phi(G_s^I), \end{aligned} \quad (\text{A12})$$

where the first inequality is Equation (A10), the second inequality follows from Equation (A11), and the equality is Equation (A9). The extremes of Equation (A12) give

$$\phi(G^II) < \sum_{s \in \mathcal{S}} p_s \phi(G_s^I).$$

Under the assumption that ϕ is concave, the above-displayed inequality gives us

$$\phi(G^II) < \sum_{s \in \mathcal{S}} p_s \phi(G_s^I) \leq \phi\left(\sum_{s \in \mathcal{S}} p_s G_s^I\right). \quad (\text{A13})$$

Since ϕ is increasing, by

$$\phi'(x) = \frac{v''(x)}{u''(\phi(x))},$$

the extremes of Equation (A13) give $G^II < \sum_{s \in \mathcal{S}} p_s G_s^I$ in contrast with the contradiction hypothesis Equation (A11). Thus, $G^II < \sum_{s \in \mathcal{S}} p_s G_s^I$ as we wanted to show.

ii) Recall that for the equilibrium of Game 1 (A9) holds. Let the equilibrium private consumption in Game 3 be x^III . This implies the equilibrium public good amount in Game 3 and state s is $G_s^III = n(m_s - x^III)$. Using Proposition 4, the first-order condition that characterizes equilibrium in Game 3 can be rewritten as

$$\sum_{s \in \mathcal{S}} p_s v'(G_s^III) = u'\left(m_s - \frac{G_s^III}{n}\right);$$

using $v''' > 0$ and Jensen's inequality, the above-displayed equality implies

$$u'\left(m_s - \frac{G_s^III}{n}\right) > v'\left(\sum_{s \in \mathcal{S}} p_s G_s^III\right).$$

As u' is decreasing, so is u'^{-1} , and we have

$$m_s - \frac{G_s^III}{n} < u'^{-1}\left(v'\left(\sum_{s \in \mathcal{S}} p_s G_s^III\right)\right).$$

Multiplying by p_s , adding over states, and using the notation for ϕ , we obtain

$$\sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G_s^III}{n}\right) < \phi\left(\sum_{s \in \mathcal{S}} p_s G_s^III\right). \quad (\text{A14})$$

The statement we want to prove is that ϕ convex implies $\sum_{s \in \mathcal{S}} p_s G_s^III > \sum_{s \in \mathcal{S}} p_s G_s^I$. Proceed by contradiction and assume

$$\sum_{s \in \mathcal{S}} p_s G_s^III \leq \sum_{s \in \mathcal{S}} p_s G_s^I. \quad (\text{A15})$$

We obtain

$$\begin{aligned} \phi\left(\sum_{s \in \mathcal{S}} p_s G_s^III\right) &> \sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G_s^III}{n}\right) \geq \sum_{s \in \mathcal{S}} p_s \left(m_s - \frac{G_s^I}{n}\right) \\ &= \sum_{s \in \mathcal{S}} p_s \cdot \phi(G_s^I), \end{aligned} \quad (\text{A16})$$

where the first inequality is Equation (A14), the second inequality follows from Equation (A15), and the equality is Equation (A9). The extremes of Equation (A16) give

$$\phi\left(\sum_{s \in \mathcal{S}} p_s G_s^III\right) > \sum_{s \in \mathcal{S}} p_s \phi(G_s^I).$$

Under the assumption that ϕ is convex, the above-displayed inequality gives us

$$\phi\left(\sum_{s \in \mathcal{S}} p_s G_s^III\right) > \sum_{s \in \mathcal{S}} p_s \phi(G_s^I) \geq \phi\left(\sum_{s \in \mathcal{S}} p_s G_s^I\right). \quad (\text{A17})$$

Since ϕ is increasing, the extremes of Equation (A17) give $p_s G_s^{III} > \sum_{s \in S} p_s G_s^I$ in contrast with the contradiction hypothesis (A15). Thus, $p_s G_s^{III} > \sum_{s \in S} p_s G_s^I$ as we wanted to show. $\square$

Proof of Proposition 8. For state-contingent provision, by (4) we have

$$\frac{1}{(m_s - g_s)^{\eta_c}} = \frac{1}{(ng_s)^{\eta_G}}.$$

As $\eta_G \rightarrow 0$, the above implies that $g_s \rightarrow m_s - 1$. Thus, $\min_s \{m_s\} > 1$ ensures an interior solution in which, for η_G sufficiently close to 1, each agent privately consumes x_s which is arbitrarily close to 1 in each state. Therefore, as $\eta_G \rightarrow 0$, the equilibrium utility from private consumption approaches zero, and the ex-ante expected utility in the equilibrium of the state-contingent provision game approaches

$$EU_L^{SC} = n \left(\sum_{s=1}^S p_s m_s - 1 \right) - 1.$$

In the proportional contribution scheme, (9) yields that the optimal level of γ , denoted as γ^* , solves

$$\sum_{s=1}^S p_s m_s \frac{1}{((1 - \gamma^*)m_s)^{\eta_c}} = \sum_{s=1}^S p_s m_s \frac{1}{(\gamma^* n m_s)^{\eta_G}}.$$

As $\eta_G \rightarrow 0$, the above implies that γ^* converges to $\bar{\gamma}^*$, with

$$\bar{\gamma}^* = 1 - \left(\frac{\sum_{s=1}^S p_s m_s^{1-\eta_c}}{\sum_{s=1}^S p_s m_s} \right)^{\frac{1}{\eta_c}};$$

note as well that $\bar{\gamma}^*$ satisfies

$$(1 - \bar{\gamma}^*)^{1-\eta_c} \sum_{s=1}^S p_s (m_s)^{1-\eta_c} = (1 - \bar{\gamma}^*) \sum_{s=1}^S p_s m_s. \quad (A18)$$

The equilibrium expected utility in the proportional contribution scheme is

$$EU^P = \sum_{s=1}^S p_s \left(\frac{((1 - \gamma^*)m_s)^{1-\eta_c} - 1}{1 - \eta_c} + \frac{(n\gamma^* m_s)^{1-\eta_G} - 1}{1 - \eta_G} \right);$$

as $\eta_G \rightarrow 0$, EU^P above converges to EU_L^P , where

$$\begin{aligned} EU_L^P &= \frac{(1 - \bar{\gamma}^*)^{1-\eta_1} \sum_{s=1}^S p_s (m_s)^{1-\eta_c}}{1 - \eta_c} + n\bar{\gamma}^* \sum_{s=1}^S p_s m_s - \frac{1}{1 - \eta_c} - 1 \\ &= \frac{((1 - \bar{\gamma}^*) + (1 - \eta_1)n\bar{\gamma}^*) \sum_{s=1}^S p_s m_s}{1 - \eta_1} - \frac{1}{1 - \eta_c} \\ &\quad - 1. \end{aligned} \quad (\text{by (A.18)})$$

Simple algebra implies that $EU_L^P > EU_L^{SC}$ is equivalent to

$$(1 - (1 - \eta)n) \left((1 - \bar{\gamma}^*) \sum_{s=1}^S p_s m_s - 1 \right) > 0. \quad (A19)$$

We now show $(1 - \bar{\gamma}^*) \sum_{s=1}^S p_s m_s < 1$. We have

$$\begin{aligned} (1 - \bar{\gamma}^*) \sum_{s=1}^S p_s m_s &= \left(\frac{\sum_{s=1}^S p_s m_s^{1-\eta_c}}{\sum_{s=1}^S p_s m_s} \right)^{\frac{1}{\eta_c}} \sum_{s=1}^S p_s m_s \quad (\text{by (A.18)}) \\ &= \left(\sum_{s=1}^S p_s (m_s)^{1-\eta_c} \right)^{\frac{1}{\eta_c}} \left(\sum_{s=1}^S p_s m_s \right)^{1-\frac{1}{\eta_1}} \\ &= \left(\sum_{s=1}^S p_s (m_s)^{1-\eta_c} \right)^{\frac{1}{\eta_c}} \left(\sum_{s=1}^S p_s m_s \right)^{\frac{\eta_1-1}{\eta_c}} \\ &= \left(\frac{\sum_{s=1}^S p_s (m_s)^{1-\eta_c}}{\left(\sum_{s=1}^S p_s m_s \right)^{1-\eta_c}} \right)^{\frac{1}{\eta_c}} \\ &< 1, \end{aligned}$$

where the last inequality follows because Jensen's inequality yields

$$\sum_{s=1}^S p_s (m_s)^{1-\eta_c} < \left(\sum_{s=1}^S p_s m_s \right)^{1-\eta_c}.$$

Therefore, we have established $(1 - \gamma^*) \sum_{s=1}^S p_s m_s < 1$ so Equation (A19) gives us that $EU_L^P > EU_L^{SC}$ under the assumption of part 1. of the proposition. As we have a strict inequality between EU_L^P and EU_L^{SC} at the limit for $\eta_G \downarrow 0$, this strict inequality persists also for η_G in a right-neighborhood of zero; thus, we have proved that, if η_G is sufficiently close to zero, then the ex-ante equilibrium utility in the proportional game is strictly larger than that in the state-contingent game.

The proof of the first sentence of Part 2. follows along the same lines; one only needs to reverse the implication obtained from Equation (A19).

To conclude, we now prove the last sentence of Part 2. When $n = 1$, the model collapses to a nonstrategic, individualistic problem. Taking into account the budget constraint, in the state-contingent game the problem for the lone agent is

$$\max_{\{g_1, \dots, g_S\}} \sum_{s=1}^S p_s \left(\frac{(m_s - g_s)^{1-\eta_c} - 1}{1 - \eta_c} + \frac{(g_s)^{1-\eta_G} - 1}{1 - \eta_G} \right). \quad (A20)$$

Note that the objective function displayed in Equation (A20) is strictly concave in g_s . Given our focus on interior solutions, this implies that the strict maximum satisfies

$$(m_s - g_s)^{\eta_c} = (g_s)^{\eta_G} \quad \forall s, \quad (A21)$$

as simple differentiation shows.

In the game with strict proportionality, it is as if the lone agent is solving problem Equation (A20) with the additional set of S constraints

$$g_s = \gamma m_s \quad \forall s,$$

for some $\gamma \in (0, 1)$. Only when $\eta_c = \eta_G$ does Equation (A21) imply that the above displayed constraints are satisfied for some γ . Therefore, if $\eta_c \neq \eta_G$, then strict concavity of the objective function displayed in Equation (A20) implies that utility is strictly lower in the optimal solution of the game with strict proportionality than in the optimal solution of the state-contingent game. $\square$