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Trading budget deficits for public investment: Optimal deficit rules for present-biased governments☆

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A B S T R A C T

We develop a simple two-period principal-agent model in which a present-biased government, the agent, chooses public investment levels given a deficit rule imposed by the principal. The principal sets a deficit cap to curb current debt-financed consumption. However, this also reduces the government's long-term investment. We characterize the optimal deficit rule that balances these opposing effects. Our analysis yields three key insights. First, a deficit rule is always a second-best instrument: it reduces public deficits but also inefficiently suppresses public investment. Second, a decrease in the government's present bias and an increase in the productivity of public investment entail an increase in the optimal deficit cap. Third, we compare the welfare effects of three deficit rules: a balanced budget rule, the absence of a rule, and a benchmark deficit rule that limits its deficits to the level chosen by the social planner. For moderate present bias, the absence of a deficit rule yields higher welfare than a balanced budget, but it is consistently dominated by the benchmark rule. However, with substantial present bias the balanced budget rule delivers higher welfare than the other rules.

1. Introduction

The number of countries that adopted fiscal rules to restrict the government's ability to run deficits has increased substantially since the 1990's (Eyraud et al., 2018; Yared, 2019). Yet, these rules are controversial. Supporters highlight their positive effects on fiscal rigor. Opponents claim they reduce governments' room for maneuver and ultimately welfare. Historically, fiscal rules gained prominence in the 1990s, not as a novel invention, but as an increasingly adopted policy response to the rise in debt-to-GDP ratios observed in many advanced economies since the mid 1970s (Yared, 2019). In an overview article, Yared (2019) argues that this trend could not be explained by standard normative macroeconomic theories of tax smoothing (Barro, 1979; Lucas and Stokey, 1983), safe asset provision (Woodford, 1990; Aiyagari and McGrattan, 1998), or dynamic inefficiency (Diamond, 1965; Blanchard, 1985). Instead, it should be attributed to changes in political factors. He further explains how the competing explanations brought about by a large body of political economy literature all have in common that governments behave as if they had present-biased

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preferences.[1] This is a powerful and widespread narrative (see e.g., Eyraud et al., 2018; Bachmann, 2024), appealing due to its simplicity and intuitiveness. We perceive it as the dominant narrative to justify the introduction and continuation of fiscal rules. However, it is incomplete. In a little noticed reaction to an influential model of public debt and political turnover by Tabellini and Alesina (1990) that illustrates the benefits of fiscal rules, Peletier et al. (1999) show how the introduction of a fiscal rule does not only harness government debt but also hampers government investment. Their analysis hinges on the simple idea that "budgetary institutions matter, not only for deficits, but also for public investment" (Peletier et al., 1999, p.1378).

In the meantime, this *deficit–investment trade-off* of fiscal rules is well established in the policy debate. For instance, in an issue dedicated to "The Coming Debt Crisis", the Economist writes: "Hard constitutional limits provide much stronger protection against profligacy. Yet they also reduce flexibility: Germany's debt brake has hampered its ability to invest in infrastructure and spend on defence [...]. If a fiscal rule is so tight that it prevents a country from harnessing the advantages of debt—being able to spread the cost of crises or infrastructure through time, or stimulating in response to a deep recession—then it can hurt as much as it helps." (The Economist, 2025)

Moreover, the deficit–investment trade-off has been the subject of several empirical investigations. Two recent literature reviews conclude that there is no conclusive evidence of any relationship between fiscal rules and public investment (Potrafke, 2025; Blesse et al., 2026). However, once we restrict our attention to strict fiscal rules (that is, without escape clauses or special treatment of investment expenditure), there is evidence of a negative relationship (see Section 2).

Yet, surprisingly, this deficit–investment trade-off has gained little attention in the theoretical literature. We address this gap by asking: How should an optimal deficit rule be designed to optimally balance overspending and underinvestment? What are the comparative-static properties of such a rule with respect to the degree of present bias and the productivity of public investment? And how do simple rules with low informational requirements, such as a balanced budget rule or the absence of a rule, perform in the presence of the deficit–investment trade-off?

We answer these questions by developing a simple two-period principal-agent-like model. Our contribution is threefold. First, we provide a simple formalization of the deficit–investment trade-off and use it to analytically characterize the optimal deficit rule. Second, we derive comparative statics on two key parameters: the degree of present bias and the productivity of public investment. Third, we use our model to rank three simple deficit rules by welfare and identify the regimes in which each dominates.

Three papers come closest to our setup. Beetsma and Debrun (2007) develop a theoretical model of the revised EU Stability and Growth Pact that touches on many aspects of the deficit–investment trade-off. However, the richer model setup veils the underlying mechanism and neither includes comparative statics, nor do the authors compare different policy alternatives. Boyer et al. (2024) study a political-economy framework in which parties compete for electoral support through targeted redistribution. Their focus lies on electoral incentives and redistributive policies. Janeba (2025) also studies the deficit–investment trade-off in a similar model, but does not analyze how to optimally balance this trade-off.

The structure of this article is as follows: In the next section, we locate our contribution within the fragmented theoretical literature on fiscal rules. In Section 3, we describe the model and the deficit–investment trade-off for present-biased governments. Section 4 characterizes the optimal deficit rule, which balances the positive welfare effect of reducing excessive consumption today with the welfare cost of reduced investment. We show that the optimal deficit rule is a second-best instrument. Furthermore, we analyze how the optimal deficit cap reacts to changes in the present bias and the productivity of public investment. In Section 5, we acknowledge the high informational requirements of the optimal deficit rule and compare the performance of three simple deficit rules before concluding in Section 6.

2. Literature

The literature in the field of fiscal rules, public debt and its influence on investment is fragmented and can be organized into theoretical and empirical contributions.

A growing empirical literature on the effectiveness and consequences of fiscal rules is surveyed in Potrafke (2025). Blesse et al. (2026) provide a survey of the literature focused on the consequences on public investment. Both conclude that there is at best limited evidence for a negative effect of fiscal rules on public investment. However, the articles surveyed encompass analyses of all kinds of fiscal rules, also including investment clauses. There seems to be empirical evidence for a negative effect of more rigid fiscal rules on public investment. Poterba (1995) is an early example finding evidence for a deficit–investment trade-off in US states. He finds that states with pay-as-you-go requirements reduce capital spending by about 20%, while non-capital spending is reduced by 11%. Venturini (2020) presents quasi-experimental evidence for Italian municipalities, where balanced-budget rules (BBR) were imposed for municipalities above a certain population threshold. She finds a quantitatively relevant negative effect of the BBR on municipal long-term investment into human capital and infrastructure, while investment in welfare, productive activities support and productive services remains unaffected. Ardanaz et al. (2021) study a panel of 75 advanced- and emerging economies during fiscal consolidations and find that countries with flexible fiscal rules reduce their public investment much less than countries with strict fiscal rules. Wijsman and Crombez (2021) use European panel data and the European Comission's Fiscal Rules Strenght Index (FRSI) from 1997 to 2016 and find that an increase in the FRSI reduces public investment both as share of GDP and as share of total government primary expenditure. Janeba (2025) analyzes the impact of a ruling of the German constitutional court and argues, that

[1] See e.g., Laibson (1997) for a model of quasi-hyperbolic (or quasi-geometric) discounting that captures this type of preferences.

it constitutes an exogenous tightening of the government budget. He finds that the German government—bound by the so-called German "debt brake" that limited general government deficit to.35% of GDP—reduced the investment ratio in reaction to the ruling.

The theoretical literature on the costs and benefits of fiscal rules predominantly focuses on the trade-off between commitment and flexibility. While fiscal rules are designed to increase the commitment of a government that acts in a time-inconsistent manner, the same commitment (or rigidity) may be unwelcome in moments of crisis or economic downturns, for instance by preventing counter-cyclical fiscal policy. Amador et al. (2006) explore the commitment-flexibility trade-off in a general setting where agents face temptation in a consumption-savings model. Halac and Yared (2014) study the optimal level of discretion in a fiscal policy model with a time-inconsistent present biased government. Their analysis is extended to a setting with international spillovers in Halac and Yared (2018) and limited options to enforce the budget rule in Halac and Yared (2022). Dotti and Janeba (2023) use a two-period model to investigate how an optimal deficit rule should accommodate fiscal shocks in the presence of a present-biased government and propose a rule that encompasses a zero structural deficit. Azzimonti et al. (2016) analyze the commitment-flexibility trade-off in a quantitative model for a BBR. Although these studies share certain elements with our setup, they focus on the commitment-flexibility trade-off and exclude public investment or intertemporal public goods from their analyses.

Another strand of literature studies the interplay between public debt, investment, and fiscal rules. Most contributions focus on the so-called *golden rule of public finance*, a type of fiscal rule where admissible deficits are conditioned on the level of observed investment. In a seminal paper, Bassetto and Sargent (2006) investigate the effects of the introduction of a golden rule in a fully-fledged dynamic model and show that it significantly increases efficiency if Ricardian equivalence does not hold. Bom (2019) explores a negative wealth effect of public investment in the presence of a BBR. In their setting, the lower market value of firms caused by the increase in public capital, more than offsets the benefits of the increased investment and decreases the current generation's welfare. Introducing a golden rule increases welfare for all generations. In a model framework closely related to ours, Edenhofer et al. (2025) describe and analyze a version of the golden rule tailored to climate policy, one type of long term public investment. The authors show, how tying admissible deficits to the (marginal) benefits of realized emission reductions can correct incentives for a present-biased government and induce efficient climate policy with a rule that addresses past criticisms of the classical golden rule. In contrast, we do not analyze the effects of a golden rule in our model, since our contribution lies precisely in analyzing the deficit–investment trade-off for fiscal rules that cannot be conditioned on the level of public investment. In practice, the definition of investment is blurry and governments can use *creative accounting* (see e.g., Milesi-Ferretti, 2004; von Hagen and Wolff, 2006) to circumvent conditional rules, for example of the golden rule type. This was one of the reasons why Germany abandoned a golden rule for one, that is more similar to a deficit rule (see e.g., Feld, 2024).

The literature analyzing public investment in the presence of debt-ceilings or deficit caps, like a balanced budget rule, is very limited. To our knowledge, Peletier et al. (1999) are the first to highlight the deficit–investment trade-off but confine themselves to noticing that "a balanced-budget rule induces the median voter to invest too little" (Peletier et al., 1999, p.1380). In a quantitative model, Uchida and Ono (2021) study how a debt ceiling influences the distribution of the fiscal burden across generations. In their overlapping generations framework, a present bias is induced through the voting process that is influenced by short-sighted, egoistic, elderly voters. Through public education expenditures made by the government, Uchida and Ono (2021) include an intertemporal public investment decision of the type we consider. However, their sophisticated model makes it difficult to discern the mechanism we describe in this article. In particular, the authors do not study optimal deficit rules.

Three papers come closest to our setup. Beetsma and Debrun (2007) develop a theoretical model assessing the revised EU Stability and Growth Pact's impact on fiscal discipline and economic reforms, and, thereby, they analyze many relevant aspects of the deficit–investment trade-off. We extend their analysis in three ways. First, we analytically derive the optimal deficit rule and provide comparative statics on key parameters. Second, our simplified model allows for an explicit characterization of three policy alternatives, complementing their numerical approach with greater generality. Third, our model's simplicity highlights the core mechanism at play: the deficit–investment trade-off. Boyer et al. (2024) present a model, focusing on a political economy framework where parties compete for electoral support by directing resources to voter subgroups. While deficit caps constrain policymakers' ability to redistribute future gains, making reforms less appealing, we extend their work by investigating the optimal balance of the deficit–investment trade-off in this context. In concurrent research, Janeba (2025) also models the deficit–investment trade-off in a two-period framework, but differs from our approach in two key ways. First, public investment enters utility directly rather than increasing second period resources. As a result, present bias alone is sufficient to distort public good provision, and imposing a deficit cap worsens this inefficiency. Second, we address the question of how to optimally balance the competing pressures of overspending and underinvestment while Janeba (2025) uses the model as a point of departure for an empirical investigation.

In another related contribution, Bouton et al. (2020) study the effect of fiscal rules in a model with public debt and entitlements, capturing pensions and social security. By doing this, the authors formally capture the argument that focusing on government debt obfuscates the role that other government obligations like pensions play in matters of intergenerational distribution (see e.g, Kotlikoff, 1988; Kotlikoff and Burns, 2012). On a more abstract level, the authors make a similar point as we do: By targeting public debt or deficits, one might overlook substitution relationships with other important variables, e.g., entitlements or, as in our case, public investment. Other important theoretical contributions on the effects of fiscal rules were made by Huber and Runkel (2008) and Dovis and Kirpalani (2018), who study fiscal rules in the context of a federal state, Hatchondo et al. (2022), who are concerned with sovereign defaults, as well as Piguillem and Riboni (2021) who analyze the value of fiscal rules that can be overridden with a simple majority, yet take effect as "bargaining chip" in the political decision-making process. Kellner (2023) studies stock-pollutants in a political-competition setting in which governments are present-biased à la Tabellini and Alesina (1990). Strategic interactions between parties with heterogeneous preferences can increase low-emission public good provision, an effect absent in our reduced-form approach where political turnover does not generate such incentives.

3. The deficit–investment trade-off

We analyze a two-period small open economy in which a government maximizes an intertemporal welfare function through spending and borrowing decisions. The resource constraints for each period are defined as

$$c_1 + i = y_1 + b, \tag{1}$$

$$c_2 = y_2 + F(i, A) - R\,b. \tag{2}$$

Initial endowment in both periods is given by y_1 and y_2. The government allocates funds to consumption c_t and public investment i which we think of as an intertemporal public good. The latter is transformed into second-period resources via the production function $F(i, A)$.[2] This function is continuous, with first derivatives with respect to i $F_i > 0$, second derivative $F_{ii} < 0$, and third derivative $F_{iii} > 0$ (e.g., an isoelastic production function), non-negative on its domain $[0, \infty)$, and fulfills the Inada conditions. Additionally, the productivity of public investment, A, is modeled as a multiplicative factor in the production function: $F(i, A) = A \cdot \tilde{F}(i)$. The budget in the first period can be extended by issuing bonds, b, at the cost of reducing the second-period budget, with R representing the exogenous interest rate on bonds.[3] As the model concludes after the second period, all government debt must be redeemed.[4] We assume perfect commitment. The government maximizes intertemporal welfare given by

$$W_\beta = u(c_1) + \beta\,\delta\,u(c_2). \tag{3}$$

We denote the discount factor by $\delta > 0$. Furthermore, we assume $\beta \in (0, 1]$ and interpret $1/\beta$ as the government's degree of present bias.[5] This present bias constitutes the difference between the principal and the agent in the subsequent sections. It is a reduced form parameter that captures the essence of different political economy distortions like uncertain re-election, aggregation of heterogeneous preferences or common pool problems within governments. All these mechanisms lead to a government acting *as if* it had present-biased preferences.[6] The utility function $u(\cdot)$ is continuous and thrice differentiable, with $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'''(\cdot) > 0$ (e.g., a CRRA or a CARA utility function), and fulfills the Inada conditions.

The government chooses c_1, i, and b and is subject to a deficit cap $\bar{b}$, so that

$$b \leq \bar{b}. \tag{4}$$

We assume perfect enforcement of the deficit rule. The government maximizes the objective function (3) subject to the budget constraints (1) and (2) and the deficit rule (4), where we denote the Lagrange-multiplier from this last constraint with μ. The first-order conditions for this maximization problem can be rearranged to give the following two optimality conditions[7]:

$$u'(c_1) = \beta\,\delta\,R u'(c_2) + \mu, \tag{5}$$

$$F_i = R + \frac{\mu}{u'(c_2)\,\beta\,\delta}. \tag{6}$$

First-best. To analyze the welfare effects of imposing a deficit cap on the government versus allowing it to act freely, we require a benchmark scenario. For this purpose, we consider a social planner that solves the agent's problem but neither succumbs to a present bias, nor is bound by a deficit cap. The first-order conditions of this benchmark scenario correspond to Eqs. (5) and (6) with $\beta = 1$ and $\mu = 0$. In other words, the social planner chooses consumption according to a standard Euler Equation of consumption and determines investment such that the marginal product of the public investment equals the exogenously given interest rate. We refer to the resulting allocation as the social planner solution $(c_1^{SP}, c_2^{SP}, i^{SP}, b^{SP})$.

The following proposition establishes that the deficit incurred by a present-biased government is excessive in the absence of a deficit rule and that a binding deficit cap effectively harnesses the deficit and current consumption at the expense of public investment.

Proposition 1.

(i) In the absence of a binding deficit cap ($\mu = 0$) a present-biased agent ($\beta < 1$) incurs higher deficits and prioritizes current consumption at the expense of future consumption compared to their respective first-best levels. Public investment is equal to its efficient level, satisfying $F_i = R$.

[2] An alternative modeling approach would be to include the benefits of the public investment as argument of second-period utility, as it is done in Boyer et al. (2024), Janeba (2025) and discussed in Edenhofer et al. (2025). Our results are robust to this modeling choice. Minor changes are highlighted in the text, whenever relevant.

[3] We discuss the consequences of modeling an endogenous risk premium in the conclusion.

[4] The assumption of debt redemption is the analog of a transversality condition in an infinite horizon model. For the qualitative results of this model to hold, it is sufficient that the bonds given out in the first period impose some cost in the second period, e.g., by increasing the risk of a sovereign debt crisis or expected inflation.

[5] Other authors refer to β as the government's present bias (see Halac and Yared, 2014). We deviate from this convention because we find it more convincing to say that an increase in present bias leads to a government putting less weight on the second period.

[6] See Yared (2019) for an overview of the political economy mechanisms and Halac and Yared (2014, 2022) for analyses using this approach.

[7] The strict concavity of the agent's objective function (3) ensures that the conditions are sufficient and that the maximum is unique.

(ii) The introduction of a binding deficit cap ($\mu > 0$) reduces the ratio of first-period to second-period consumption c_1/c_2. Public investment falls below its efficient level, so that $F_i > R$.

Proof. (i) Follows directly from Eqs. (5) and (6) and the concavity of $u(\cdot)$ and $F(\cdot)$ for $\beta < 1$. (ii) Follows directly from Eqs. (5) and (6) and $\mu > 0$. □

Proposition 1 captures the deficit–investment trade-off for present-biased governments. First, in the absence of a deficit rule, a present-biased agent would increase deficits to enable higher present consumption at the expense of future consumption, while investing efficiently into the public capital stock in order to maximize intertemporal resources.[8] This aligns with the explanation by Yared (2019) for the rise in debt-to-GDP ratios between the mid-1970s and 2000s, as well as the narrative that debt rules are essential for harnessing debt-to-GDP ratios and safeguarding the interests of future generations against the preferences of the present. Second, a deficit rule can counteract the effects of the present bias by limiting the skew toward first-period consumption. However, this correction comes at a cost. A binding deficit cap introduces a wedge between the marginal product of investment F_i and the interest rate R, leading to a reduction in public investment below its efficient level. The interests of future generations are thus compromised by a reduction in public investment that might affect infrastructure, education, or climate mitigation. We refer to these two effects as the deficit–investment trade-off.

4. The optimal deficit rule

In the previous section, we described the deficit–investment trade-off, demonstrating that imposing a binding deficit cap on a present-biased agent reduces excessive present consumption at the cost of distorting the investment decision. In this section, we examine how to optimally balance these conflicting effects. Following Amador et al. (2006); Halac and Yared (2014, 2018, 2022), we formulate a principal-agent-like problem where the principal sets a deficit cap to influence the decisions of the present-biased agent to maximize welfare.

We can interpret the principal and his objective function normatively, as in Halac and Yared (2014). Alternatively, the principal could be an incumbent government with a constitutional majority and the agent a future government with only a simple majority as in Piguillem and Riboni (2021). The 'Principal-Agent' terminology we employ derives from this interpretation, where an incumbent government does not know its successor's present bias. While we assume perfect knowledge about the present bias by the principal in this section, asymmetric information motivates the analysis in Section 5.

In this section, we obtain three results. First, we show that the principal always imposes a binding deficit cap on the agent. Second, we characterize the optimal deficit cap as a second-best instrument. Third, we perform comparative statics and analytically show how the optimal deficit cap increases in reaction to a decrease in the present bias $1/\beta$ and an increase in the productivity of public investment A.

4.1. The principal-agent problem

In our setting, the principal sets a deficit cap $\bar{b}$, anticipating that the present-biased agent will choose both the deficit b and public investment i in response. Formally, we solve two maximization problems. The agent's maximization problem is given by

$$\begin{aligned} \max_{\{b\in\mathbb{R},i\geq 0\}} \quad W_\beta &= u(c_1) + \beta\,\delta\,u(c_2) \\ \text{s.t.} \quad c_1 &= y_1 + b - i, \\ c_2 &= y_2 - R\,b + F(i, A), \\ b &\leq \bar{b}. \end{aligned} \tag{7}$$

Denoting by $b^*(\bar{b})$ and $i^*(\bar{b})$ the solutions to this problem, the principal's maximization problem is given by

$$\begin{aligned} \max_{\{\bar{b}\in\mathbb{R}\}} \quad W_1 &= u(c_1^*) + \delta\,u(c_2^*) \\ \text{s.t.} \quad c_1^* &= y_1 + b^*(\bar{b}) - i^*(\bar{b}), \\ c_2^* &= y_2 - R\,b^*(\bar{b}) + F(i^*(\bar{b}), A). \end{aligned} \tag{8}$$

The principal acts as the leader and the agent as the follower. The two objective functions only differ in the present bias $1/\beta$ and the different control variables.

The following proposition establishes that, whenever the agent is present-biased, the principal always optimally chooses a deficit cap that is binding for the agent, so that $b^*(\bar{b}) = \bar{b}$. Let $(c^*_{1,nc}, c^*_{2,nc}, i^*_{nc}, b^*_{nc})$ denote the unconstrained allocation chosen by the agent in the absence of a deficit cap.

Proposition 2. *If the agent is present-biased ($\beta < 1$), there exists a binding deficit cap $\bar{b} < b^*_{nc}$ under which the principal's welfare W_1 strictly exceeds welfare evaluated at the unconstrained agent's allocation $(c^*_{1,nc}, c^*_{2,nc}, i^*_{nc}, b^*_{nc})$.*

[8] In a model where public investment enters second-period utility directly, public investment by a present-biased government is inefficiently low, even if the latter is non-constrained (Janeba, 2025). The result that introducing a binding deficit cap (further) reduces public investment pertains.

Proof. See Appendix A.2 □

Proposition 2 establishes that the deficit cap binds in equilibrium, $b^*(\bar{b}) = \bar{b}$. Therefore, the agent's problem (7) simplifies to the choice of i only:

$$\begin{aligned} \max_{\{i \geq 0\}} \; W_\beta &= u(c_1) + \beta\,\delta\,u(c_2) \\ \text{s.t.} \quad c_1 &= y_1 + \bar{b} - i, \\ c_2 &= y_2 - R\,\bar{b} + F(i, A). \end{aligned} \tag{9}$$

We use this simplified formulation throughout the remainder of the analysis.

By solving (8) and (9), we obtain two optimality conditions pinning down our two variables of interest: investment, i^*, and the deficit cap, $\bar{b}^*$. The optimality condition of the agent, which implicitly determines a unique i^* is

$$u'(c_1) = F_i u'(c_2)\beta\delta, \tag{10}$$

where both c_1 and c_2 are functions of i, too.[9] The optimality condition of the principal implicitly defines the optimal deficit cap $\bar{b}^*$ and is given by:

$$(1 - i^*_{\bar{b}})u'(c_1^*) = \delta\,u'(c_2^*)\left(R - F_i(i^*, A)i^*_{\bar{b}}\right), \tag{11}$$

where c_1^*, c_2^*, i^*, and $i^*_{\bar{b}} := \partial i^*/\partial \bar{b}$ are functions of $\bar{b}$. Eq. (11) determines a unique solution of the principal's maximization problem if $\partial^2 i^*/\partial \bar{b}^2 < 0$ (i.e., if i^* is a concave function of $\bar{b}$), which we will assume for the rest of the analysis.[10]

The optimal deficit cap maximizes the principal's welfare, given its influence on the present biased agent's investment decision. Since we know that investment is inefficiently low for any binding deficit cap from Eq. (6), we immediately obtain the following corollary:

Corollary 1. *The optimal deficit cap $\bar{b}^*$ is a second-best instrument. It implies that investment is inefficiently low, i.e., $F_i(i^*(\bar{b}^*), A) > R$.*

Corollary 1 highlights two central insights. First, the optimal deficit rule constitutes a second-best instrument: it cannot induce the social optimum. This limitation arises because the deficit cap addresses the problem of excessive spending in the present, but in doing so it simultaneously depresses investment below its efficient level. Because the principal cannot correct both distortions with a single instrument, the resulting allocation remains second-best. Second, the corollary implies that observing a social rate of return F_i higher than the interest rate R is a characteristic of the second-best solution rather than an indication of a poorly designed deficit rule.

4.2. Comparative statics

We now examine how the optimal deficit cap $\bar{b}^*$ responds to changes in key parameters. More specifically, we analyze its comparative statics with respect to the present bias $1/\beta$ and the productivity of public investment A. The response to the present bias $1/\beta$ is particularly relevant, as the historical argument for introducing fiscal rules is based on the premise that, starting in the mid-1970s, governments in advanced economies exhibited an increasing degree of present bias (Yared, 2019). The impact of shocks to the productivity of public investment A is interesting, as it affects the first-best deficit. Examples of such shocks include the emergence of new productivity-enhancing technologies dependent on public infrastructure (e.g., the internet) or new scientific evidence on the severity of future climate damages.

The results of this section require assumptions about the relative curvature of the utility function $u(\cdot)$ and the production function $F(\cdot)$. We simplify the expressions by assuming logarithmic utility, that is $u(\cdot) = log(\cdot)$ in this section. In this case, we must make the following assumption—reminiscent of assumption (c) in (Tabellini and Alesina, 1990, footnote 7)—that is necessary for the results with respect to β and sufficient with respect to A

$$(1 + \delta)F_i F_{iii} > (1 + 2\delta)\left(F_{ii}\right)^2. \tag{A}$$

Examples of functional forms that satisfy this assumption are an isoelastic production function with output elasticity between 0 and 1 and the natural logarithm.

Furthermore, we assume that when it comes to the effect of the productivity of investment A on the agent's choice of investment i^*, the substitution effect dominates the income effect and therefore $\partial i^*/\partial A > 0$. We are now able to derive the following comparative-static results:

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[9] Existence and uniqueness follow from the concavity of (3).

[10] The second derivative of the principal's objective function W_1 with respect to $\bar{b}$ is given by

$$\frac{\partial^2 W_1}{\partial \bar{b}^2} = (1 - i_{\bar{b}^*})^2 u''(c_1^*) + \delta\left(i_{\bar{b}^*} F_i(i^*, A) - R\right)^2 u''(c_2) + \delta F_{ii}(i^*, A)(i_{\bar{b}^*})^2 u'(c_2^*) + \delta(1 - \beta)F_i(i^*, A)u'(c_2^*)\frac{\partial^2 i^*}{\partial \bar{b}^2},$$

which is negative for all $\bar{b}$, if $\partial^2 i^*/\partial \bar{b}^2 < 0$. Note that this condition is sufficient but not necessary.

Proposition 3. *(i) Assume logarithmic utility, i.e.,* $u(\cdot) = log(\cdot)$ *and Assumption* (A) *with respect to the curvature of* $F(\cdot)$*. Then:*

$$\frac{\partial \bar{b}^*}{\partial \beta} > 0$$

(ii) Additionally assume that $\frac{\partial i^*}{\partial A} > 0$*. Then:*

$$\frac{\partial \bar{b}^*}{\partial A} > 0$$

Note that Assumption (A) *is necessary for* (i) *and sufficient for both results.*

Proof. See Appendix A.3 □

Proposition 3 tells us two things. First, the higher the present bias (the lower β), the stricter the optimal deficit cap the principal imposes. In other words, the stronger the agent's preference for present consumption due to the present bias, the more the principal will act to counterbalance this tendency by imposing a tighter cap. If the present bias of governments has increased over the past decades, this could justify both the introduction and the tightening of deficit rules during this period. Second, the higher the productivity of public investment, the higher the optimal deficit cap. An increase in investment productivity raises the returns to investment. If the agent reacts to this with an increase in investment, the principal finds it advantageous to relax the binding deficit cap. This result is an interesting point of departure for discussions about fiscal rule reforms in times of new scientific evidence about the severity of future climate damages. It means that if the estimated climate damages caused by greenhouse gas emissions today increase and if, by the same token, investing to reduce these emissions today avoids more climate damages and is thus more productive, the optimal deficit cap implies a higher deficit for the present-biased government today.

5. Policy

In the previous section, we assumed that the principal observes the agent's present bias and designs an optimal deficit cap based on it. However, if we interpret the principal as an incumbent government with a constitutional majority that seeks to bind a successor, knowledge about the present bias is implausible. Moreover, a government's present bias likely varies over the course of time, for example when approaching the end of a legislative period, further complicating the design of an optimal deficit rule.

Thus, in this section we study the case where the principal does not know the agent's degree of present bias and compare the welfare effects of three specific policy instruments with no informational requirements about the level of present bias:

1. **Balanced budget rule**: The agent is not allowed to take on any debt, $\bar{b} = 0$.
2. **Absence of a deficit rule**: There is no rule in place and the present-biased agent is free to choose the deficit.
3. **Benchmark deficit rule**: The agent is prevented from incurring a deficit higher than the social planner would, $\bar{b} = b^{SP}$.

We do not study rules that condition the admissible deficit on observable behavior by the agent (in our case first period consumption or investment) as in Huber and Runkel (2008); Halac and Yared (2014, 2018). Such a rule would be subject to the same "creative accounting" critique golden-rule type fiscal rules are subject to (see Section 2). We take no stance on whether this critique is ultimately insurmountable; carefully designed conditional rules may well mitigate it in the future.[11] However, given the prominence of this critique in shaping actual fiscal-rule design, we base our analysis on the assumption that the principal cannot design a rule conditional on observed public investment.

We now analyze the three simple deficit rules with respect to their impact on the principal's welfare. The results are general and part of Proposition 4 below. For illustrative purposes, we base our discussion on Fig. 1, a graphical illustration of a calibrated version of the model. For the latter, we use log-utility and Cobb-Douglas production and calibrate the simple 2-period model roughly to the US economy and a 5-year time period. The calibration is described in Appendix B. Importantly, we assume that the time discount factor is given by $\delta = (1 + g)/R$, where g is the growth rate of the exogenous endowment y_t. This assumption implies that the only reason for the social planner to incur a positive deficit is to achieve the desired allocation of net benefits of public investment over time. In the absence of public investment, that is for $i = 0$, the optimal deficit would thus be zero. In other words, we abstract from consumption smoothing motives for debt.

Before diving into the welfare comparison of the three deficit rules, we note that Fig. 1 illustrates the analytical results from the previous sections. First, in absence of a binding deficit cap, the agent chooses higher deficit levels (squares) than in first best (vertical dashed line).[12] Second, the optimal deficit cap (solid vertical lines) is a second-best instrument since welfare is always below its first best level (normalized to 1). This is the content of Corollary 1. Third, the optimal deficit cap decreases in the level of present bias (increases in β), as stated in Proposition 3.

We can now compare the BBR, the absence of a deficit rule, and the benchmark deficit rule. Fig. 1 shows that for low levels of present bias (high β, blue line), the principal is better-off by not constraining the agent compared to imposing a balanced budget

[11] See Edenhofer et al. (2025) for a recent proposal of a conditional rule tailored to climate policy that addresses past criticisms of the classical golden rule.

[12] Note that for deficit caps higher than the unconstrained agent's choice (deficit caps to the right of the squares), the welfare level as evaluated by the principal does not change. The reason is, that deficit caps higher than the unconstrained agent's choice are non-binding. They do not affect the agent's deficit, consumption and investment choices and consequently do not affect welfare.

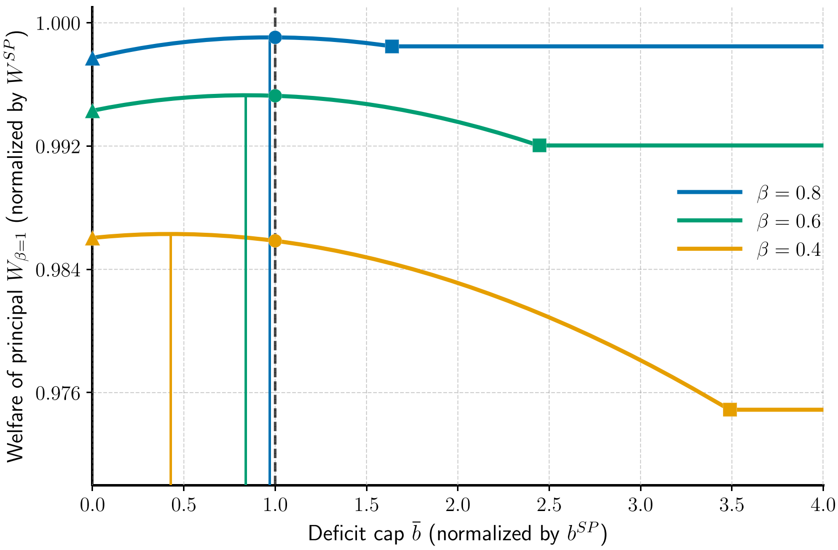

Fig. 1. Welfare evaluated by the principal as a function of the imposed deficit cap $\bar{b}$ for different levels of present bias β. Both variables are normalized by their counterparts from the social planner solution, W^{SP} and b^{SP} (see Section 3). Solid vertical lines indicate the optimal deficit cap imposed by the principal. Triangles indicate the welfare level for the balanced budget rule, circles for the benchmark deficit rule and squares for the absence of a deficit rule.

rule (the blue square achieves higher welfare than the blue triangle). The opposite is true for higher levels of present bias (green and yellow triangles achieve higher welfare than the green and yellow squares). Furthermore, the graphical illustration shows that the benchmark deficit rule achieves higher welfare than the absence of any rule (circles achieve higher welfare than squares) and that for moderate levels of present bias the benchmark rule achieves higher welfare than a balanced budget rule (blue and green circles achieve higher welfare than the blue and green triangles). This observation is particularly interesting because, as discussed previously, the optimal deficit cap is a function of the agent's present bias, which plausibly changes over time and is difficult to assess. Ignoring the present bias and requiring the deficit not to exceed its first best level might be a good rule of thumb to design a deficit rule. The following proposition generalizes our illustrative results formally.

Proposition 4.

(i) *The principal always prefers the benchmark rule to the absence of a rule.*
(ii) *There exists a level of present bias $1/\beta'$ for which the principal is indifferent between the balanced budget rule and the absence of a rule. If β' is unique, the principal prefers the absence of a rule for all $\beta > \beta'$ and the balanced budget rule for all $\beta < \beta'$.*
(iii) *There exists a level of present bias $1/\beta$" for which the principal is indifferent between the balanced budget rule and the benchmark rule. If β" is unique the principal prefers the benchmark rule for all $\beta > \beta$" and the balanced budget rule for all $\beta < \beta$". If both β' and β" are unique, then $\beta'' < \beta'$.*

Proof. See Appendix A.4 □

Proposition 4 provides conceptual guidance for situations in which implementing the optimal deficit cap is not feasible. It characterizes how the desirability of a BBR depends on the degree of present bias. For low levels of present bias, the welfare loss from reducing public investment outweighs the welfare gain from curbing excessive consumption of a BBR, making the rule undesirable. For higher levels of present bias, these effects reverse and the principal prefers a BBR over the absence of a deficit rule. This is also true when comparing the BBR to the benchmark deficit level, which seems to be a good approximation of the optimal deficit cap for a certain range of values of present bias $1/\beta$. The level of present bias $1/\beta$" for which the BBR becomes preferable over the benchmark deficit level is higher than $1/\beta'$. How much higher? While we are not able to answer this question analytically, a rough calibration of our conceptual model (see Appendix B) suggests that the second threshold for $1/\beta$ is indeed much higher than the first, i.e., the BBR is welfare superior to the benchmark deficit rule only for high degrees of present bias.

6. Conclusion

This paper develops a simple two-period model to examine the trade-off between deficits and investment faced by present-biased governments under deficit rules. We focus on deficit rules that restrict the permissible deficit without exemptions for specific spending categories as opposed to investment rules because of the difficulty in implementing the latter in practice. By framing the problem as a principal–agent-like interaction, we derive the optimal deficit cap that balances two opposing effects: limiting overspending and avoiding underinvestment. We show that the optimal deficit cap is a second-best instrument. Any deficit rule that curbs excessive current consumption also leads to inefficiently low levels of public investment.

We further show analytically that the optimal deficit cap depends negatively on the degree of present bias and positively on the productivity of public investment for plausible functional forms. If there is truth to the narrative that an increase in the present bias of governments can explain the increasing debt-to-GDP ratios observed in advanced economies since the 1970s, then a tightening of deficit rules—as observed in the past decades—can be rationalized, bearing in mind that deficit caps can only be second-best instruments. It also implies that new scientific evidence pointing to more severe future climate damages from today's greenhouse gas emissions—as documented by the Intergovernmental Panel on Climate Change over the past decades—provides a rationale for relaxing deficit rules. Such evidence increases the expected returns to public investment in mitigation, thereby strengthening the case for higher permissible deficits.

Because the optimal deficit rule depends on information that is difficult to observe in practice—notably the degree of present bias—we compare three simpler policy alternatives: a balanced budget rule, the absence of a deficit rule, and a benchmark deficit rule that limits deficits to the level chosen by the social planner. We show that the benchmark rule always dominates the absence of a rule, and that a balanced budget rule is welfare-inferior to the absence of a rule when present bias is small. Only for higher degrees of present bias does the balanced budget rule outperform the benchmark deficit rule.

With our model, we highlight the role of public investment in the design of fiscal rules and contribute not only to the theoretical literature but also to the policy debate by providing a transparent formalization of a recurring criticism of fiscal rules. For the sake of clarity and tractability, the model is deliberately kept simple, but it could be extended along several dimensions. First, a natural extension would be to introduce an endogenous risk premium on government debt that increases with the deficit. This would harness the agent's appetite for deficits but decrease public investment in the absence of a deficit cap. The principal would take this additional cost into account and, since it accrues in the second period, give relatively more importance to it. We anticipate no qualitative changes to our results as it merely adds additional cost to taking on deficits.

Second, one could reformulate the model as a closed economy. In that case, the crowding out of private capital and the subsequent increase in the real interest rate constitute an additional cost, similar to the endogenous risk premium case. Accordingly, we do not expect qualitative changes to our results.

Third, an extension of the model could endogenize the agent's present bias. Some of the sources of present bias described in Yared (2019), such as the demographic change in ageing societies, are outside of a government's immediate control. Others, like the re-election probability important for present bias stemming from political turnover are directly influenced by an incumbent government. In our model, this could be done by re-interpreting the present bias parameter β as the re-election probability of the latter and assuming that it positively depends on first period consumption, $\partial\beta(c_1)/\partial c_1 > 0$ (see e.g., Raveh and Tsur (2020) for a recent contribution using such an approach in the tradition of the political agency literature (Barro, 1973; Maskin and Tirole, 2004)). This would add another twist to the model: A deficit cap, designed to curb a government's present bias effectively *increases* it by reducing first period resources, first period consumption expenditures and thus the government's re-election probability. From a principal's perspective, this speaks in favor of choosing a laxer cap, ceteris paribus. However, any binding cap worsens the government's present bias, making a tighter cap more attractive for the principal. We anticipate the qualitative results to be the same and leave the quantitative analysis for future numerical work.

Fourth, extending the time horizon and analyzing both the steady-state and the transition dynamics would be worthwhile. This is more demanding, since the agent's time inconsistent preferences entail strategic behavior that is best analyzed as a dynamic game solved for Markov-Perfect equilibria. Because such equilibria are likely indeterminate (Krusell and Smith, 2003; Cao and Werning, 2018), one must specify an equilibrium selection process. We expect steady state welfare to be higher, the tighter the deficit cap, since lowering the deficit cap forces the agent to save. While this reduces public investment in the short run, we expect this effect to be temporary. However—analogous to the welfare–deficit relationship depicted in Fig. 1—very tight deficit caps reduce period utility in the initial periods, whereas moderate caps can raise utility in all subsequent periods (for the principal, not the agent). Hence, from the principal's perspective, the absence of a deficit cap likely is dynamically inefficient. These expectations rest on intuition. Given the prominence of the deficit–investment trade-off in the policy debate, putting them on firmer ground seems worthwhile.

CRediT authorship contribution statement

Tobias Bergmann: Writing–review & editing, Writing–original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Nikolaj Moretti:** Writing–review & editing, Writing–original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used GPT-4, GPT-5, Opus 4.6 and 4.7 in order to improve the readability and refine the writing style of the manuscript. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Nikolaj Moretti reports that financial support was provided by the German Federal Ministry of Research, Technology and Space (BMFTR) as part of the ARIADNE project (grant reference: 03SFK5J0). Nikolaj Moretti also reports that financial support was provided by the German Federal Environmental Foundation (DBU). Tobias Bergmann reports that financial support was provided by the Heinrich Boll Foundation. Tobias Bergmann also reports that financial support was provided by the German Federal Ministry of Research, Technology and Space (BMFTR) as part of the ARIADNE project (grant reference: 03SFK5J0). If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proofs

A.1. Proof Lemma 1

For the remainder of this section it will be useful to define some implicit derivatives in a Lemma.

Lemma 1. *The partial derivatives of the investment function of the agent, $i^*(\beta, \bar{b}, A)$ with respect to $\bar{b}$ and β are positive and given by*

$$i^*_{\bar{b}} = \frac{\partial i^*}{\partial \bar{b}} = \frac{u''(c_1) + F_i u''(c_2) R\beta\delta}{u''(c_1) + F_i^2 u''(c_2)\beta\delta + F_{ii} u'(c_2)\beta\delta} \in (0,1), \tag{12}$$

$$i^*_{\beta} = \frac{\partial i^*}{\partial \beta} = -\frac{F_i u'(c_2)\delta}{u''(c_1) + F_i^2 u''(c_2)\beta\delta + F_{ii} u'(c_2)\beta\delta} > 0. \tag{13}$$

The derivative with respect to A is given by

$$i^*_A = \frac{\partial i^*}{\partial A} = -\frac{\left(F_A F_i u''(c_2) + F_{iA} u'(c_2)\right)\beta\delta}{u''(c_1) + \left(F_i^2 u''(c_2) + F_{ii} u'(c_2)\right)\beta\delta}, \tag{14}$$

which is positive if $F_A F_i u''(c_2) + F_{iA} u'(c_2) > 0$, that is if the income effect is dominated by the substitution effect, which we assume to be the case. Inserting the optimality condition for the principal and the agent as well as log-utility, this condition can be rewritten only in terms of the production function and is given by

$$F_{ii} F \left(R - F_i \beta\right) + F_i^2 \left(F_i - R\right)\beta\left(1 + \delta\right) > 0. \tag{15}$$

The second-order derivatives are given by

$$\begin{aligned} i^*_{\beta\beta} = \frac{\partial^2 i^*}{\partial\beta\partial\beta} = \frac{F_i u'(c_2)\delta^2}{D^3} \Big(& F_i^4 \beta\delta\big(2\big(u''(c_2)\big)^2 - u'''(c_2)u'(c_2)\big) + 2F_{ii}u'(c_2)\big(u''(c_1) + F_{ii}u'(c_2)\beta\delta\big) \\ & + F_i^2 u''(c_2)\big(2u''(c_1) + F_{ii}u'(c_2)\beta\delta\big) + F_i u'(c_2)\big(u'''(c_1) - F_{iii}u'(c_2)\beta\delta\big)\Big), \end{aligned} \tag{16}$$

$$\begin{aligned} i^*_{\bar{b}\bar{b}} = \frac{\partial^2 i^*}{\partial\bar{b}\partial\bar{b}} = \frac{\beta\delta}{D^3}\Big(& -\big(u''(c_1)\big)^2\big(F_{iii}u'(c_2) + F_{ii}u''(c_2)\big(3F_i - 2R\big) + F_i u'''(c_2)\big(F_i - R\big)^2\big) + \Big(u'''(c_1)\big(F_i^2 u''(c_2) + F_{ii}u'(c_2)\big)^2 \\ & - 2R\Big(F_i^2\big(u''(c_2)\big)^2\big(2F_{ii}u''(c_1) + F_i u'''(c_1)\big) + u'(c_2)\Big(F_{iii}F_i u''(c_1)u''(c_2) - F_{ii}^2 u''(c_1)u''(c_2) \\ & + F_{ii}F_i\big(u''(c_2)u'''(c_1) - F_i u''(c_1)u'''(c_2)\big)\Big)\Big) + F_i\beta\delta\Big(F_i u''(c_2)^2 u'''(c_1) + 2F_{ii}u''(c_1)\big(u''(c_2)^2 - u'''(c_2)u'(c_2)\big)\Big)R^2\Big) \\ & - F_i R^2\beta^2\delta^2\Big(F_{ii}F_i^2 u''(c_2)^3 + F_{iii}F_i u''(c_2)^2 u'(c_2) + F_{ii}^2 u'(c_2)\big(u'''(c_2)u'(c_2) - 2u''(c_2)^2\big)\Big)\Big), \end{aligned} \tag{17}$$

$$\begin{aligned} i^*_{\bar{b}A} = \frac{\partial^2 i^*}{\partial\bar{b}\partial A} = \frac{\beta\delta}{D^2}\Bigg(& -\frac{1}{u''(c_1) + F_i u''(c_2)R\beta\delta}\Big(F_{ii}F_A u''(c_1)u''(c_2) + 2F_i F_{iA}u''(c_1)u''(c_2) + F_A F_i u''(c_2)u'''(c_1) \\ & + F_A F_i^2 u''(c_1)u'''(c_2) + F_{iiA}u''(c_1)u'(c_2) + F_{iA}u'''(c_1)u'(c_2) + \beta\delta\Big(2F_i^2\big(-F_{ii}F_A + F_i F_{iA}\big)\big(u''(c_2)\big)^2 \\ & + u''(c_2)u'(c_2)\big(F_{ii}^2 F_A + F_i\big(-F_{iii}F_A + F_{iiA}F_i\big) - F_{ii}F_i F_{iA}\big) + F_i^2 u'''(c_2)u'(c_2)\big(F_{ii}F_A - F_i F_{iA}\big) \\ & + \big(F_{ii}F_{iiA} - F_{iii}F_{iA}\big)\big(u'(c_2)\big)^2\Big)\Big) + F_{iA}\Big(u'''(c_1)u'(c_2) + u''(c_1)u''(c_2)R + F_i^2\big(\big(u''(c_2)\big)^2 - u'''(c_2)u'(c_2)\big)R\beta\delta\Big) \\ & + F_A F_i\Big(u''(c_2)u'''(c_1) + u''(c_1)u'''(c_2)R + F_{ii}\big(-\big(u''(c_2)\big)^2 + u'''(c_2)u'(c_2)\big)R\beta\delta\Big)\Bigg), \end{aligned} \tag{18}$$

$$\begin{aligned} i^*_{\bar{b}\beta} = \frac{\partial^2 i^*}{\partial\bar{b}\partial\beta} = \frac{\delta}{D^3}\Bigg(& F_i^4 u''(c_1)\beta\delta\big(-\big(u''(c_2)\big)^2 + u'''(c_2)u'(c_2)\big) - F_{ii}u''(c_1)u'(c_2)\big(u''(c_1) + F_{ii}u'(c_2)\beta\delta\big) + F_i^3\beta\delta\Big(u''(c_2)u'''(c_1)u'(c_2) \\ & - u'''(c_2)u'(c_2)R\big(u''(c_1) + F_{ii}u'(c_2)\beta\delta\big) + \big(u''(c_2)\big)^2 R\big(u''(c_1) + 2F_{ii}u'(c_2)\beta\delta\big)\Big) + F_i^2 u''(c_2)\Big(-\big(u''(c_1)\big)^2 \\ & + F_{ii}u''(c_1)u'(c_2)\beta\delta + u'(c_2)R\beta\delta\big(-u'''(c_1) + F_{iii}u'(c_2)\beta\delta\big)\Big) + F_i\Big(\big(u''(c_1)\big)^2 u''(c_2)R + F_{iii}u''(c_1)\big(u'(c_2)\big)^2\beta\delta \\ & + F_{ii}\big(u'(c_2)\big)^2\beta\delta\big(u'''(c_1) - F_{ii}u''(c_2)R\beta\delta\big)\Big)\Bigg), \end{aligned} \tag{19}$$

where $D = u''(c_1) + \left(F_i^2 u''(c_2) + F_{ii}u'(c_2)\right)\beta\delta$.

Proof. First, define the function $H(i, \beta, \bar{b}, A)$ by rearranging the optimality condition of the agent Eq. (10)

$$H(i, \beta, \bar{b}, A) := -u'(y_1 + \bar{b} - i) + F_i(i, A)u'(y_2 + F(i, A) - R\,\bar{b})\beta\delta. \tag{20}$$

Note that since $u(\cdot)$ and $F(\cdot)$ are thrice continuously differentiable, H is continuous in $i, \beta, \bar{b}$, and A (and twice continuously differentiable in these variables). Furthermore, for all $\beta_0, \bar{b}_0$, and A_0, there is an $i_0 \,\epsilon\, (0, y_1 + \bar{b}_0)$ such that $H(i_0, \beta_0, \bar{b}_0, A_0) = 0$. This must hold, since for all $\beta_0, \bar{b}_0$, and A_0, $W_\beta(i)$ has a maximum at a value i_0, where necessarily $H(i_0, \beta_0, \bar{b}_0, A_0) = 0$. This follows from $u(c_1) \to -\infty$ for $i \to y_1 + \bar{b}_0$ and $F(i) \to \infty$ for $i \to 0$ and the strict concavity of the agent's objective function W_β in i. The derivative of H with respect to i is given by

$$H_i(i, \beta, \bar{b}, A) = u''(c_1) + \beta\delta\left(F_{ii}(i, A)u'(c_2) + F_i(i, A)^2 u''(c_2)\right) < 0. \tag{21}$$

It is strictly negative and in particular non-zero, which guarantees the existence and uniqueness of the implicit function $i^*(\beta, \bar{b}, A)$ defined by Eq. (10). The partial derivatives then follow from applying the implicit function theorem. From Eq. (12) we also conclude that $i^*_{\bar{b}} < 1$, since $1 < R < F_i$. □

A.2. Proof Proposition 2

Proof. The principal sets a deficit cap $\bar{b}$, and the agent chooses $b \le \bar{b}$ and i. We prove two claims: (i) For $\bar{b} < b^*_{nc}$, the agent's best response is $b^*(\bar{b}) = \bar{b}$, and (ii) the principal optimally sets $\bar{b} < b^*_{nc}$.

Step 1: Agent's best response when $\bar{b} < b^*_{nc}$: $b^*(\bar{b}) = \bar{b}$.

The unconstrained objective $W_\beta(b, i)$ is strictly concave in (b, i), as u is strictly concave, F is strictly concave, and the budget map is linear. Hence the reduced-form value function $\tilde{W}(b) := \max_i W_\beta(b, i)$ is strictly concave in b, with a unique maximizer b^*_{nc}. Therefore $\tilde{W}$ is strictly increasing on $(-\infty, b^*_{nc}]$, and for any $\bar{b} < b^*_{nc}$ the agent's best response is the corner solution $b^*(\bar{b}) = \bar{b}$.

Step 2: The principal optimally sets $\bar{b} < b^*_{nc}$.

We prove Step 2 by showing that the derivative of the principal's welfare W_1 with respect to $\bar{b}$, evaluated at the unconstrained agent's allocation $(c^*_{1,nc}, i^*_{nc}, b^*_{nc})$, is strictly negative.[13] Computing this derivative and applying the agent's FOCs (with $\mu = 0$ at b^*_{nc}), we obtain

$$\begin{aligned}
\left.\frac{\partial W_1}{\partial \bar{b}}\right|_{(c^*_{1,nc}, i^*_{nc}, b^*_{nc})} &= u'(c^*_{1,nc})(1 - i^*_{\bar{b}}) + \delta u'(c^*_{2,nc})(-R + F_i\, i^*_{\bar{b}}) \\
&= u'(c^*_{1,nc})(1 - i^*_{\bar{b}}) - \delta R\, u'(c^*_{2,nc})(1 - i^*_{\bar{b}}) \\
&= u'(c^*_{1,nc})(1 - i^*_{\bar{b}})(1 - 1/\beta) < 0,
\end{aligned}$$

where the second line uses (6) and the third uses (5). The sign follows from $\beta < 1$ and $i^*_{\bar{b}} \in (0, 1)$ (Lemma 1). □

A.3. Proof Proposition 3

Proof. First, we define the function $G(\bar{b}^*, \beta, A)$ by rearranging the optimality condition of the principal (11) to obtain

$$G(\bar{b}^*, \beta, A) = (1 - i^*_{\bar{b}})u'(c^*_1) - \delta\, u'(c^*_2)\left(R - F_i(i^*, A)i^*_{\bar{b}}\right) = 0. \tag{22}$$

(i): Using the implicit function theorem we receive the partial derivative of $\bar{b}^*$ with respect to β given by

$$\frac{\partial \bar{b}^*}{\partial \beta} = -\frac{\frac{\partial G(\bar{b}^*, \beta, A)}{\partial \beta}}{\frac{\partial G(\bar{b}^*, \beta, A)}{\partial \bar{b}}}. \tag{23}$$

The denominator of Eq. (23) is equal to the second-order condition of the principals maximization problem and has to be negative for a maximum, $\frac{\partial G(\bar{b}^*, \beta, A)}{\partial \bar{b}^*} = \frac{\partial^2 W_1}{\partial \bar{b}^2} < 0$. To prove that $\frac{\partial \bar{b}^*}{\partial \beta} > 0$, the numerator of Eq. (23) has to be positive. We are thus left to prove that

$$\frac{\partial G(\bar{b}^*, \beta, A)}{\partial \beta} = i^*_\beta\left(i^*_{\bar{b}}\delta\left(F_{ii}u'(c_2) + F_i^2 u''(c_2)\right) - \left(1 - i^*_{\bar{b}}\right)u''(c_1) - R\delta F_i u''(c_2)\right) + \left(\delta F_i u'(c_2) - u'(c_1)\right)i^*_{b\beta} > 0. \tag{24}$$

We can simplify this expression by using Lemma 1 and the optimality conditions of the agent and the principal given by Eqs. (10) and (11). Furthermore, if we apply log-utility and by this the following relations $u''(c_t) = u'(c_t)^2$ and $u'''(c_t) = 2u'(c_t)^3$ we can simplify Eq. (24) to

$$\frac{\partial G(\bar{b}^*, \beta, A)}{\partial \beta} = \frac{\delta(R - F_i\beta)^2}{(1 - \beta)^2 F_i^2 (R + F_i\beta\delta)^2}\frac{(R - F_i)}{F_{ii}}\mathcal{A}. \tag{25}$$

Since, we know that $F_i > R$ from Proposition 1 and the sign of the derivatives of the production function, this expression is positive if (and only if) $\mathcal{A} := (1 + \delta)F_i F_{iii} - (1 + 2\delta)F_{ii}^2 > 0$ which is Eq. (A) from the main text.

[13] We do not need to check whether the sign changes for higher levels of $\bar{b}$, since the principal —using a deficit *cap*— can only lower the agent's chosen deficit but not increase it.

(ii): Analogous to (i), we use the implicit function theorem to derive the partial derivative of $\bar{b}^*$ with respect to A given by

$$\frac{\partial \bar{b}^*}{\partial A} = -\frac{\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A}}{\frac{\partial G(\bar{b}^*,\beta,A)}{\partial \bar{b}^*}}. \tag{26}$$

The denominator of Eq. (26) is again equal to the second-order condition of the principals maximization problem and negative. To prove that $\frac{\partial \bar{b}^*}{\partial A} > 0$, the numerator of Eq. (26) has to be positive. We are thus left to prove that

$$\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} = i^*_{\bar{b}}\delta\Big(F_A F_i u''(c_2) + F_{iA}u'(c_2)\Big) - F_A R u''(c_2)\delta + i^*_A\Big(\big(u''(c_1) + F_i^2 u''(c_2)\delta + F_{ii}u'(c_2)\delta\big)i^*_{\bar{b}} - u''(c_1) - F_i R u''(c_2)\delta\Big) + i^*_{bA}\big(F_i u'(c_2)\delta - u'(c_1)\big) > 0. \tag{27}$$

We can simplify this expression by using Lemma 1 and by applying log-utility and by this the following relations $u''(c_t) = u'(c_t)^2$ and $u'''(c_t) = 2u'(c_t)^3$ as well as linearity of A in $F(i,A)$, so that $F_A = \frac{F(i,A)}{A}$, $F_{iA} = \frac{F_i}{A}$ and $F_{iiA} = \frac{F_{ii}}{A}$. Furthermore, we insert the optimality conditions of the agent and the principal given by Eqs. (10) and (11). Thus, we can rewrite Eq. (27) to

$$\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} = -\frac{\big(R - F_i\beta\big)^2\delta}{A F_{ii} F_i^4\big(F_i - R\big)\big(\beta-1\big)^2\big(1+\delta\big)\big(R + F_i\beta\delta\big)^2}\Big[-F_{ii}F_{iii}F_iF\big(F_i - R\big)\big(F_i\beta - R\big)\big(1+\delta\big) + F_{iii}F_i^3\big(F_i - R\big)^2\beta\big(1+\delta\big)^2 + F_{ii}^3 F\Big(R^2\big(1+2\delta\big) - 2F_iR\big(1+\delta+\beta\delta\big) + F_i^2\beta\big(1+\delta+\beta\delta\big)\Big) - F_{ii}^2F_i^2\big(1+\delta\big)\Big(R^2\beta\big(1+2\delta\big) + F_i^2\beta\big(1+\delta+\beta\delta\big) - F_iR\big(1+\beta+4\beta\delta\big)\Big)\Big] \tag{28}$$

Since, we know that the term in front of the parentheses is positive, we have to show that the expression in parentheses is positive as well. Rearranging this we receive

$$\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} = -\frac{\big(R - F_i\beta\big)^2\delta}{A F_{ii} F_i^4\big(F_i - R\big)\big(\beta-1\big)^2\big(1+\delta\big)\big(R + F_i\beta\delta\big)^2}\Big[\Big(\big(F_i - R\big)\Big(F_{ii}F\big(F_i\beta - R\big) - F_i^2\big(F_i - R\big)\beta\big(1+\delta\big)\Big)\Big) \times \Big(F_{ii}^2\big(1+2\delta\big) - F_iF_{iii}\big(1+\delta\big)\Big) + F_{ii}^2F_i\big(1-\beta\big)\big(R + F_i\beta\delta\big)\Big(F_i^2\big(1+\delta\big) - F_{ii}F\Big)\Big], \tag{29}$$

where the last line $F_{ii}^2F_i\big(1-\beta\big)\big(R+F_i\beta\delta\big)\Big(F_i^2\big(1+\delta\big) - F_{ii}F\Big) > 0$. The second line is Eq. (A), which states $F_iF_{iii}\big(1+\delta\big) > F_{ii}^2\big(1+2\delta\big)$. To show that the first line is negative we know from Proposition 1 that $F_i > R$ and from Eq. (15) in Lemma 1 that $F_{ii}F\big(F_i\beta - R\big) - F_i^2\big(F_i - R\big)\beta\big(1+\delta\big) < 0$. Thus, for $\frac{\partial i^*}{\partial A} > 0$ (that is, the substitution effect dominating the income effect) the first line is negative, resulting in an overall positive expression. Note that both conditions (A) and (15) are sufficient and not necessary to prove that $\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} > 0$. □

A.4. Proof Proposition 4

Proof. In this proof, we formalize the graphical intuition that can be derived from Fig. 1. Interpret the objective function of the principal (8) as a function of both the deficit cap $\bar{b}$ and the present bias $1/\beta$, i.e., $W_1(\bar{b},\beta)$.

Furthermore, let $\bar{b}^*(\beta)$ denote the optimal deficit cap for present bias $1/\beta$, $b^*(\beta)$ the choice of an unconstrained agent with present bias $1/\beta$ in the absence of a deficit rule, and $b^*(1)$ the benchmark deficit. We prove the three parts of the proposition by using the fact that, when facing an agent with present bias, the principal prefers policy option A associated with deficit level b_A to option B associated with deficit level b_B, if and only if $W_1(b_A,\beta) \geq W_1(b_B,\beta)$.

(i): First, from Proposition 3 we know that the optimal deficit cap is increasing in the level of β, i.e., $\partial\bar{b}^*/\partial\beta > 0$. Hence, for all $\beta < 1$, we know that the benchmark deficit (which is the optimal deficit for $\beta = 1$, that is $b^*(1)$) is bigger than the optimal deficit cap, i.e., $b^*(1) \equiv \bar{b}^*(1) > \bar{b}^*(\beta)$. Second, we know that for a non-constrained agent, the incurred deficit is increasing in the level of present bias (decreasing in β), i.e., $\partial b^*/\partial\beta < 0$.,[14] Hence, for all $\beta < 1$, the deficit chosen by the agent in absence of a deficit rule is higher than the benchmark deficit, $b^*(\beta) > b^*(1)$. The result follows since W_1 is decreasing for $\bar{b} \in [\bar{b}^*(\beta), b^*(\beta)]$ given our assumption that i^* is concave in $\bar{b}$ (see footnote 10).

(ii): Remember that we defined W_1 as a function of both the deficit cap $\bar{b}$ (first argument) and the inverse present bias β (second argument), for the purpose of this proof. We prove that there exists a $\beta' \in (0,1]$ for which $W_1(0,\beta') = W_1(b^*(\beta'),\beta')$. Define

$$\Delta W(\beta) := W_1(0,\beta) - W_1(b^*(\beta),\beta), \qquad \beta \in (0,1]. \tag{30}$$

We show

a) $\Delta W(1) = W_1(0,1) - W_1(b^*(1),1) < 0$,

[14] To see this, consider the first-order Equations for a non-constrained agent's decision problem which are given by Eqs. (5) and (6) for $\mu = 0$. Since investment is pinned down by R and does not change with present bias β, the only possibility to adjust the intertemporal consumption path to the benefit of c_2 with increasing β, as implied by the Euler Eq. (5) is by reducing b.

b) $\Delta W(\beta) = W_1(0,\beta) - W_1(b^*(\beta),\beta) > 0$, for sufficiently small β,
c) ΔW is continuous on $(0,1]$.

The existence of β' then follows from the intermediate value theorem.

Proof of a): If $\beta = 1$, the agent's choice is optimal from the principal's perspective and $b^*(1)$ is the unconstrained optimal deficit from both perspectives. Furthermore, we know that $0 < b^*(1)$ because of the Inada conditions on $F(i)$ and our choice of δ that eliminates consumption smoothing motives. Hence $W_1(0,1) < W_1(b^*(1),1)$, because of the strict concavity of W_1 in $\bar{b}$. Therefore, $\Delta W(1) < 0$.

Proof of b): Consider the agent in the absence of a deficit rule. As noted in the footnote above, for $\mu = 0$ the agent's investment is pinned down by $F_i(i,A) = R$ and hence equals a constant $\bar{i}$, independent of β. The agent's Euler equation then reads

$$u'(c_1(\beta)) = \beta\,\delta R\,u'(c_2(\beta)), \tag{31}$$

where $c_1(\beta) = y_1 + b^*(\beta) - \bar{i}$ and $c_2(\beta) = y_2 + F(\bar{i},A) - R\,b^*(\beta)$. By feasibility, $b^*(\beta) \le \bar{b}_{\max} := \big(y_2 + F(\bar{i},A)\big)/R$, so $c_2(\beta) \ge 0$.

We claim that $c_2(\beta) \to 0$ as $\beta \to 0$. Assume, towards a contradiction, that there exist $\varepsilon > 0$ and a sequence $\beta_n \to 0$ such that $c_2(\beta_n) \ge \varepsilon$ for all n. Then $u'(c_2(\beta_n))$ is bounded above by $u'(\varepsilon) < \infty$. Moreover, $c_1(\beta_n)$ is bounded above (since $b^*(\beta_n) \le \bar{b}_{\max}$) so $u'(c_1(\beta_n))$ is bounded away from zero. Now, let $\beta_n \to 0$. Then, the right-hand side of (31) converges to 0, while the left-hand side stays strictly positive. Since Eq. (31) must hold for all $\beta \in (0,1]$, this is a contradiction. Hence $c_2(\beta) \to 0$ as $\beta \to 0$.

Since $u(c) \to -\infty$ as $c \to 0$, it follows that along the unconstrained allocation

$$W_1(b^*(\beta),\beta) = u(c_1(\beta)) + \delta\,u(c_2(\beta)) \to -\infty \qquad \text{as } \beta \to 0. \tag{32}$$

In contrast, under a balanced budget rule ($\bar{b} = 0$) we have $c_2 = y_2 + F(i^*(\beta,0),A) > 0$ for all $\beta \in (0,1]$. This holds, since $F(i) \ge 0$ on its domain. Hence, $W_1(0,\beta)$ remains finite as $\beta \to 0$. Therefore,

$$\lim_{\beta\to 0} \Delta W(\beta) = +\infty, \tag{33}$$

and thus there exists $\underline{\beta} \in (0,1]$ such that $\Delta W(\underline{\beta}) > 0$.

Proof of c): We already know that $i^*(\beta,\bar{b},A)$ is continuously differentiable in β by Lemma 1, hence $W_1(0,\beta)$ is continuous. It remains to argue the continuity of $\beta \mapsto W_1(b^*(\beta),\beta)$. For $\mu = 0$, $b^*(\beta)$ is pinned down by the Euler Eq. (31) (with $\bar{i}$ fixed) and can be written as the unique solution to the equation of the form

$$\Phi(b,\beta) \equiv u'(y_1 + b - \bar{i}) - \beta\,\delta\,R\,u'(y_2 + F(\bar{i},A) - Rb) = 0 \tag{34}$$

Since Φ is continuously differentiable and $\partial\Phi/\partial b \neq 0$ by the strict concavity of u, the implicit function theorem implies that $b^*(\beta)$ is continuous on $(0,1]$. By the continuity of u and F, the composition $\beta \mapsto W_1(b^*(\beta),\beta)$ is continuous, hence ΔW is continuous on $(0,1]$.
Conclusion. We have $\Delta W(\underline{\beta}) > 0$ for some $\underline{\beta} \in (0,1]$ and $\Delta W(1) < 0$. By the intermediate value theorem there exists $\beta' \in [\underline{\beta},1]$ such that $\Delta W(\beta') = 0$, that is, $W_1(0,\beta') = W_1(b^*(\beta'),\beta')$. If the root is unique, then $\Delta W(\beta) > 0$ for all $\beta < \beta'$ and $\Delta W(\beta) < 0$ for all $\beta > \beta'$, which yields the stated preference ordering.

(iii): Define

$$\widetilde{\Delta W}(\beta) := W_1(0,\beta) - W_1(b^*(1),\beta), \qquad \beta \in (0,1], \tag{35}$$

where under the benchmark rule the principal directly sets $b = b^*(1)$ and the agent chooses only investment.

We follow the same steps as in part (ii) of the proof and show:

a) $\widetilde{\Delta W}(1) < 0$,
b) $\widetilde{\Delta W}(\beta) > 0$ for sufficiently small $\beta > 0$,
c) $\widetilde{\Delta W}$ is continuous on $(0,1]$.

Existence of β" then follows from the intermediate value theorem.

Proof of a): Analogous to part (ii) of the proof, at $\beta = 1$, the agent's choice is optimal from the principal's perspective and $b^*(1)$ is optimal from both perspectives. Hence $W_1(0,1) < W_1(b^*(1),1)$ and $\widetilde{\Delta W}(1) < 0$.

Proof of b): Fix any debt level $b \ge 0$. The agent chooses investment by solving

$$u'(y_1 + b - i) = \beta\delta\,F_i(i,A)\,u'(y_2 - Rb + F(i,A)). \tag{36}$$

We claim that $i^*(\beta,b) \to 0$ as $\beta \to 0$. Towards a contradiction, assume that there exists $\varepsilon > 0$ and a sequence $\beta_n \to 0$ such that $i^*(\beta_n,b) \ge \varepsilon$. Then the left-hand side is bounded away from zero, while the right-hand side converges to zero for $\beta_n \to 0$. This happens because $F_i(i,A) \le F_i(\varepsilon,A)$ is finite. Thus, the right-hand side is proportional to β_n and tends to zero. Since Eq. (36) must hold for all $\beta \in (0,1]$, this is a contradiction. Hence $i^*(\beta,b) \to 0$ as $\beta \to 0$.

Therefore,

$$\lim_{\beta\to 0} W_1(0,\beta) = u(y_1) + \delta u(y_2), \tag{37}$$

$$\lim_{\beta\to 0} W_1(b^*(1),\beta) = u(y_1 + b^*(1)) + \delta u(y_2 - Rb^*(1)). \tag{38}$$

As discussed in Section 5, we choose the discount factor δ to abstract from consumption smoothing motives for deficits. Therefore, in the absence of public investment ($i = 0$) the principal optimally chooses zero debt. Hence, for $b^*(1) > 0$,

$$u(y_1) + \delta u(y_2) > u(y_1 + b^*(1)) + \delta u(y_2 - Rb^*(1)),$$

implying $\widetilde{\Delta W}(\beta) > 0$ for sufficiently small $\beta > 0$.

Proof of c): Since $b^*(1)$ is constant and $i^*(\beta, b)$ is continuous in β (by the implicit function theorem applied to the agent's FOC), $\widetilde{\Delta W}$ is continuous on $(0, 1]$.
Since $\widetilde{\Delta W}(1) \leq 0$ and $\widetilde{\Delta W}(\beta) > 0$ for small β, continuity implies the existence of $\beta'' \in (0, 1]$ such that $\widetilde{\Delta W}(\beta'') = 0$. If the root is unique, the stated preference ordering follows.

Finally, assume both β' and β" are unique. From part *(i)* we know that for any $\beta < 1$ the benchmark rule is preferred to the absence of a rule, that is, $W_1(b^*(1), \beta) > W_1(b^*(\beta), \beta)$. Evaluating this at $\beta = \beta'$ and using $W_1(0, \beta') = W_1(b^*(\beta'), \beta')$ from part *(ii)* yields

$$W_1(0,\beta') = W_1(b^*(\beta'),\beta') < W_1(b^*(1),\beta'), \tag{39}$$

hence $\widetilde{\Delta W}(\beta') < 0$. Since $\lim_{\beta\to 0} \widetilde{\Delta W}(\beta) = +\infty$ and $\widetilde{\Delta W}$ is continuous with a unique root, it follows that the unique root β" satisfies $\beta'' < \beta'$. □

Appendix B. Calibration

This section describes our functional forms and parameter values used for the numerical analysis in Section 5.

B.1. Functional forms

To facilitate the calibration, we assume that public investment contributes to the public capital stock K_t for the numerical analysis. The dynamics of the stock are given by

$$K_2 = (1 - \delta_K)K_1 + i, \tag{40}$$

where δ_K is the depreciation of public capital. We assume that the production function from public capital is Cobb-Douglas:

$$F(K_t) = \frac{A}{\alpha} \cdot K_t^{\alpha}, \tag{41}$$

where α is the output elasticity of public capital, and A is an exogoneous TFP parameter. The utility function is assumed to be logarithmic

$$u(c_t) = \log(c_t). \tag{42}$$

B.2. Parameter values

For the numerical illustration, we calibrate the two-period model to broadly resemble the structure of the US economy in 2024. One model period corresponds to five years, reflecting the average legislative term length in many democracies, so that $T = 5$.

GDP of the US economy in 2024 corresponds to output produced using the public capital stock, $F(K_t)$, and output produced using all other resources like labor, private capital or natural resources, which we summarize under the exogenous term, $\hat{y}_1$. For simplicity, we impose an additive structure and write

$$T \cdot GDP_{US,2024} = \hat{y}_1 + F(K_1). \tag{43}$$

Government resources in the first period correspond to the goods and services produced using the public capital stock and a fraction τ of all other goods and services, so that

$$G_1 = \tau\hat{y}_1 + F(K_1). \tag{44}$$

We do so to stick to the structure from the main text with $y_1 := \tau\hat{y}_1$, where $\tau = 0.25$ corresponds to the tax share in GDP in the United States in 2023 (OECD, 2024).

The initial public capital stock in period one is defined using a share of GDP from Ramey (2020) of 63%, so that $K_1 = 0.63 \cdot GDP_{US,2024}$. The productivity parameter of the production function is set to $A = T \cdot A_y = 5$, which reflects a normalization of yearly productivity to $A_y = 1$. To calibrate α, we use the empirical estimate of the output elasticity of core public capital in the long-run, $\varepsilon_{Y,K} = 0.17$, from Bom and Ligthart (2014). This output elasticity is in our model defined as

$$\frac{\partial(T \cdot GDP_{US,2024})}{\partial K_t} \frac{K_t}{T \cdot GDP_{US,2024}} = \frac{A \cdot K_t^{\alpha}}{T \cdot GDP_{US,2024}} \overset{!}{=} \varepsilon_{Y,K}. \tag{45}$$

Rearranging and taking logs gives us an expression for the curvature of our production function

$$\alpha = \frac{\log \varepsilon_{Y,K} + \log GDP_{US,2024}}{\log K_1}. \tag{46}$$

The exogenous endowment in period 1 is defined as the difference between GDP taken from data, $T \cdot GDP_{US,2024}$ and the produced output in model period one using the capital stock in period one, $F(K_1)$, so that

$$\begin{aligned}\hat{y}_1 &= T \cdot GDP_{US,2024} - F(K_1)\\ &= T \cdot GDP_{US,2024} - \frac{A}{\alpha} \cdot K_1^{\alpha}\\ &= \left(1 - \frac{\varepsilon_{Y,K}}{\alpha}\right) T \cdot GDP_{US,2024},\end{aligned} \tag{47}$$

where we use Eq. (45) for the last equality. Thus, the exogenous government resources are $y_t = \tau \hat{y}_t$. $\hat{y}_t$ is assumed to grow with yearly growth rate g_y. The latter is set equal to 2%, roughly corresponding to the 20-year average annual growth rate of real GDP of the US economy (U.S.BEA, 2025). Exogenous endowment in period two is then given by

$$y_2 = (1+g)y_1, \tag{48}$$

where the per period growth rate is given by $g = (1+g_y)^T - 1$. The annual real interest rate on US government bonds is set to $r = 0.01$. Again, this value represents roughly the 20-year average of the 10-Year real interest rate (Cleveland FED, 2025). The per period risk-free gross return is thus

$$R = (1+r)^T. \tag{49}$$

To isolate the effect of discounting, we assume the discount factor δ to be equal to $(1+g)/R$. This specification ensures that the only reason to take on debt is to allocate the cost and benefit of public investment between today and tomorrow. In absence of public investment, the optimal deficit is zero. In our calibration, this assumption leads to a discount factor $\delta > 1$. This means, that given our values of the real interest rate r and growth rate g, the principal must give a higher weight to the second period, if we want to eliminate the consumption smoothing motive for debt. A discount factor of $\delta = 1$ or lower would imply a positive optimal deficit level for the purpose of consumption smoothing. As we already stated, we abstract from the latter.

Finally, the 5-year depreciation rate of public capital δ_K is computed from the quarterly depreciation rate provided in Ramey (2020) of $\delta_{K,quart} = 0.01$, so that $\delta_K = 1 - (1 - \delta_{K,quart})^{4T}$. Table B.1 summarizes the calibration.

Table B.1
Calibration of parameters for the numerical analysis.

Parameter	Description	Value	Source/Definition
(a) Directly taken from data or literature			
τ	Average tax share	0.25	OECD (2024)
g_y	20y avg. annual growth rate	0.02	U.S.BEA (2025)
r	20y avg. annual real interest rate	0.01	Cleveland FED (2025)
$\varepsilon_{Y,K}$	Output elasticity of public capital	0.17	Bom and Ligthart (2014)
$\delta_{K,quart}$	Quarterly depreciation rate	0.01	Ramey (2020)
$GDP_{US,2024}$	US GDP in trln USD in 2024	23.3	U.S.BEA (2025)
$K_1/GDP_{US,2024}$	Public capital–output ratio	0.63	Ramey (2020)
(b) Assumptions and normalizations			
T	Model period (years)	5	Legislative term length
A	Productivity parameter	5	$A = T \cdot A_y$, with $A_y = 1$
(c) Parameters calculated or implied from data			
R	5y gross return	1.051	$R = (1+r)^T$
g	5y growth rate	0.104	$g = (1+g_y)^T - 1$
δ	Discount factor	1.05	$\delta = (1+g)/R$
δ_K	5y depreciation rate	0.182	$\delta_K = 1 - (1-\delta_{K,quart})^{4T}$
α	Curvature production function	0.51	Calibrated from $\varepsilon_{Y,K}$
$\hat{y}_1$	Residual GDP	77.85	$\hat{y}_1 = (1 - \varepsilon_{Y,K}/\alpha)(T \cdot GDP_{US,2024})$
y_1	First-period resources	19.5	$y_1 = \tau \cdot \hat{y}_1$
y_2	Second-period resources	21.5	$y_2 = \tau \cdot \hat{y}_2 = \tau \cdot (1+g_y)^T \hat{y}_1$
K_1	First-period capital stock	14.68	$K_1 = 0.63 \cdot GDP_{US,2024}$

Data availability

The data used for the calibration is referenced and freely available.

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