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The cost of events in temporally-resolved energy system models: a stochastic programming approach

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Abstract
To deal with random exogenous events, energy systems require the ability to flexibly adjust their operational schedule. Here we explore how such events can be incorporated into temporally resolved energy system models as forced deviations from planned operation at random times. This leads to a formulation of the investment and dispatch problem as a stochastic program. We apply our approach to a conceptual model of an industry park with renewable generation, subject to randomly timed, homogeneously distributed, events requiring it to lower or increase power consumption. The cost of events occurring can exceed the cost of maintaining the capacity to adjust to them. We find that incorporating a probabilistic sample of such events during system planning can reduce the cost of adjusting to them during operation by up to 50%. Furthermore, investment decisions seem to have a minor impact on the cost of adjusting to the events - the main benefits lie in an optimized operation of the industry park. Thus, in this setting, existing assets can be leveraged to provide a cost effective response.

1 Introduction

Energy systems are subject to a multitude of exogeneous events. When optimizing operations of energy systems for a short time horizon (e.g., the dispatch for the coming hours and days), a standard approach to incorporate uncertain events is stochastic programming. Decisions that have to be taken "here-and-now" are optimized with respect to the expected cost of future events. In a two stage stochastic program, this

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means there are variables representing decisions taken at the time of planning, and variables for decisions taken later, in a second "recourse" stage, after the random event occurred [14]. In long horizon planning (e.g. whole year operation which is commonly considered for investment decisions), where we can expect many events to occur one after the other, multistage stochastic programming with one set of decision variables per possible event would be required. This would lead to a combinatorial explosion of states, and is not directly feasible [5]. Consequently, the standard approach here is to incorporate representative scenarios instead.

In this paper we explore an alternative approach to leveraging stochastic programming in medium and long horizon, multiple event energy system tasks. We treat the decisions on investment as well as operation for the entire time horizon as a first stage, then draw a sequence of events that occur and treat the resulting changes to decisions after these events as the second stage decisions. To make this approach consistent we require that these second stage decisions return to planned operations in a relatively short time window. Uncharacteristically, our second stage decisions are not after the first stage decision in a temporal sense, and our second stage decisions are associated to random times.

This provides us with a little explored type of stochastic program for long and medium horizon tasks subject to relatively infrequent events. We then explore the methodological properties of this approach by studying a conceptual model of an industry park with renewables electricity generation and heat sector, subject to a very simple distribution of requests to increase or decrease energy consumption by a random amount.

In this setting, we find that the cost of events can be considerably larger than merely ensuring the capacity to respond to them. We see that our approach can successfully optimize investment and operational decisions, and the cost of events can be reduced by up to 50%. To implement the optimization we discretize the stochastic program using a sample of events. In such a sampled stochastic program, over-fitting to the concrete sample can occur, and we show that this can lead to underestimating the cost of events by up to 30%. Numerical evidence indicates that the sampling approximation converges, and the cost of events can be reduced by up to 50%.

From a mathematical point of view, the problem of approximating multi-stage problems using fewer stages was explored in [12]. In the context of energy systems, probable deviations from scheduled operation have been considered in the context of energy system adequacy assessments [1, 8]. A setting that is similar to ours has been considered for the short-horizon task of planning operations in the context of a larger study for participating in energy flexibility markets in [16]. In [2] the problem of optimizing maintenance schedules given generator failures was cast into a two-stage problem in which the event of failure is a random variable.

The paper is structured in the following way: in Sect. 2 we introduce the random events and the appropriate notion of flexibility in our setting: the ability to react to exogeneous requests which require to increase or decrease energy consumption. In Sect. 3 we describe how these give rise to a stochastic program modelling the presence of uncertain flexibility requirements in the energy system, and Sect. 4 describes the various notions cost of flexibility and events that arise in such a model. Section 5 describe the energy system model of the industry park we consider as a case study.

Section 6 explicitly describes how we introduce the flexibility models in this context. The results of studying this model in detail are then given Sect. 7.

2 Capacity for flexibility and events

In the following we consider Energy System Models (ESMs) that find the investment decisions I and operational schedules O for a given energy system that minimize the total system costs, while ensuring that a predefined load profile can be met. The period over which the model is defined is a year, with hourly resolution. We will consider events that require the system to increase or decrease its consumption, thus the appropriate notion of flexibility in our setting is the ability of the system to deviate from the anticipated load and production profiles. A flexible reaction of the system can involve adjusting generation levels, using various forms of storage, or shifting demand. Both of the latter can involve non-electric energy sectors, such as heat.

In order to ensure the system has the capacity to deviate from the schedule, we need to extend our ESM. We assume that the energy system can coordinate the use of flexibility arising from all components of our system. Therefore, the positive and negative flexibility capacity of the system $F^{\pm}(t)$ at a given time is the sum of the capacity to deviate of all components. This capacity to deviate will depend on the concrete state of the latter, such as state of charge, planned curtailment of production, unused capacity of energy converters or shiftable loads currently being served. We extend the ESM by adding a capacity constraint, that ensures that the flexibility capacity of the system is maintained at all times:

$$F^{+}(t) \geq F_c, \; -F^{-}(t) \geq F_c \;\; \forall t \in T \tag{1}$$

However, this capacity constraint does not allow us to actually consider the cost of individual events. Nor does it ensure that the system can succesfully operate in the time span following an event, e.g. if storage that was intended for latter use is prematurely depleted, and cannot be recharged in time.

Our events are flexibility requests ξ, a forced deviation of the amount of energy our industry park exchanges with the grid. These requests are characterized by three variables: The time at which they occur t_ξ, the amount of requested energy F_ξ and the sign of the request s_ξ. Here $s_\xi = 1$ denotes a positive flexibility request which means that the system must provide additional energy to the grid, while $s_\xi = -1$ denotes negative flexibility. In this case, the system must act as a sink.

To absorb or supply this additional energy, the system needs to deviate from the schedule planned in absence of flexibility requests. This changed schedule will also change the operational costs. We denote the cost of servicing the demand $s_\xi F_\xi$ and then returning to the planned schedule using a recovery schedule R, as $\mathrm{CR}(\xi, R)$. Following [15], we define the cost of servicing the request ξ as the minimum cost of all feasible recovery schedules. Thus

$$\mathrm{CR}\left(\xi\right)=\min_{R}\,\mathrm{CR}\left(\xi,R\right) \tag{2}$$

CR may be interpreted as the operational cost of the event, i.e., the cost of servicing requests once the system's investment decision and planned operation are fixed. Note that for individual events this cost can be negative, for example, if a shiftable load is served earlier than expected due to a negative flexibility request, the system needs to buy less energy in the future. Also note that this optimization might be infeasible. In that case, there is no possible schedule that recovers the system after the event to the previously planned operation.

During the time period considered, several events $\xi_1 \ldots \xi_N$ will be present. We model response to each of them and its cost in the same way. As our focus is on infrequent events, we assume no two events occur closer than t_r, where t_r is the length of the recovery schedule. The cost of a set of flexibility request $\{\xi_n\}$ with $1 \leq n \leq N$ is simply the sum of the costs of the components.

$$\mathrm{CR}\left(\{\xi_n\}\right)=\sum_{n}\min_{R_n}\,\mathrm{CR}\left(\xi_n,R_n\right) \tag{3}$$

The fact that our costs factorize this way is a direct consequence of the approach to impose the condition that the events recover to the original planned schedule after some time t_r. If events occur closer in time than t_r, the above is only an approximation.

3 Stochastic programming for energy systems with events

The occurrence times, magnitudes and signs of flexibility requests are fundamentally unknown during planning. Our approach is to model the ξ_n as random variables. That is, we assume t_{ξ_n}, F_{ξ_n}, and s_{ξ_n} are randomly distributed with a probability density ρ. For simplicity, we assume that the probability of a request occurring is the same at all times, and thus t_{ξ_n} is evenly distributed. The rate with which events occur is parametrized by the expected interval between events Δt, inter event time is bounded from below by t_r and Poisson distributed. We draw $s_{\xi_n} = \pm$ with equal probability, and we take $F_{\xi_n} \in [0.7F_c, F_c]$ uniformly. As ρ is fully parametrized by Δt and F_c, the expected cost of flexibility will depend on these two quantities:

$$\mathrm{CR}\left(\Delta t,F_c\right)=\mathbb{E}_{\Delta t,F_c}\left[\,\mathrm{CR}\left(\{\xi_n\}\right)\right] \tag{4}$$

We are now in a position to extend our ESM to also consider the expected cost of events by adding $\mathrm{CR}\left(\Delta t,F_c\right)$ to the system costs. The resulting optimization problem is a Stochastic Program.

Stochastic Programs are optimization problems which include random variables and in which the cost function depends on the expected value of an inner optimization that depends on the random variable. Decision variables that occur solely inside the expectation are called second stage, all others first stage. The intuition is that there are variables that need to be chosen without knowing what values of the random variable are actually realized (first stage), and there are those that can be adjusted once the value is known (second stage) [4]. In our case, the first stage includes the investment decisions I and the planned operational schedule O, and the second stages are the recovery schedules R_n. As noted above this leads to an unusual structure where second stage decisions are not temporally after the first stage decisions in a strict sense.

The full cost function of our stochastic program for our energy system is of the form

$$C(I, O, \Delta t, F_c) = \left(\mathrm{CI}\,(I) + \mathrm{CO}\,(O) + \mathbb{E}_{\Delta t, F_c} \left[\sum_{n=1}^{N} \min_{R_n} \mathrm{CR}\,(\xi_n, R_n, O) \right] \right) \tag{5}$$

where $\mathrm{CI}\,(I)$ and $\mathrm{CO}\,(O)$ are the direct costs of investment and operations, and we have made the dependence of the cost of flexibility on the planned schedule explicit in $\mathrm{CR}\,(\xi_n, R_n, O)$.

The system cost, given a distribution of requests Δt, F_c, and a minimum flexibility capacity F_c, is then given by

$$C(\Delta t, F_c) = \min_{I,O} \left(\mathrm{CI}\,(I) + \mathrm{CO}\,(O) + \mathbb{E}_{\Delta t, F_c} \left[\sum_{n=1}^{N} \min_{R_n} \mathrm{CR}\,(\xi_n, R_n, O) \right] \right) \tag{6}$$

$$\text{subject to} \tag{7}$$

$$g_{\mathrm{ESM}}(I, O) \geq 0 \tag{8}$$

$$g^r_{\mathrm{ESM}}(I, O, R_n) \geq 0 \; \forall n \tag{9}$$

$$\pm F^{\pm}(I, O, t) \geq F_c \; \forall t \; , \tag{10}$$

where $g_{\mathrm{ESM}}(I, O) \geq 0$ refers to all the constraints that define the standard ESM, and we have now written the dependence of $F^{\pm}$ on I and O explicitly. The recovery schedules R_n also have to satisfy the appropriate ESM constraints g^r_{ESM}. In the following, we will omit the constraints when writing minimizations. All minima are taken over the set of investment, operational and recovery decisions that satisfy the ESM constraints.

As such a model cannot typically be evaluated directly, we approximate it by discretization. That is, we replace the expectation value $\mathbb{E}_{\Delta t, F_c}$ by a sum over a sample drawn from ρ. A sample of size n_s consists of a set $\{\xi_n\}$ of $n_s \cdot N$ individual events. To shorten notation we denote as S the full information characterizing the event sample, that is, the set $\{\xi_n\}$, as well as Δt, n_s, N and F_c. Writing $\xi_s \in S$ for the individual events in the sample, we obtain:

$$C(S,I,O) = \left(\mathrm{CI}\,(I) + \mathrm{CO}\,(O) + \frac{1}{n_s}\left[\sum_{\xi_s \in S} \min_{R_s} \mathrm{CR}\,(\xi_s, R_s, O) \right] \right) \quad (11)$$

The error induced by this approximation goes to zero as n_s is increased. A further consequence of discretization is the risk of overfitting to the given sample during the optimization, that is, it is possible that the optimizer will optimize the first stage variables, I and O for the supplied sample ξ_s, effectively rendering the stochastic aspect of the approach moot. To see whether this overfitting is occurring, we can evaluate the cost *C* on a different sample S' while fixing I and O at the result of optimization with the sample *S*:

$$I(S), O(S) = \arg\min_{I,O} C(S,I,O) \quad (12)$$

With this we define the discretized cost of an energy system with events *S*, *C*(*S*), as

$$C(S) = C(S', I(S), O(S)) \quad (13)$$

Finally we will also define the partially optimized costs. As a means of comparing costs of different systems we will fix either the investment decisions or both investment and operation decisions to some given values I and O:

$$C(S|I) = C(S', I, O(S)) \quad (14)$$

$$C(S|I,O) = C(S,I,O) \quad (15)$$

It is instructive to compare our approach to a purely scenario based one [9, 10]. In the latter we could include several representative sets of events $\{\xi_n\}$ as scenarios. This would lead to an objective function of the form $\mathrm{CI}\,(I) + \frac{1}{n_s}\sum_{\text{scenarios}} \mathrm{CO}\,(O_s)$. While investment decisions are taken with respect to the expectation over scenarios, each operational decision is taken with perfect foresight of the events that will occur.

4 The cost of maintaining capacity and reacting to events

We want to understand how well our approach can improve the cost of random events in the system. To this end, we need to compare the cost of the full system against costs that do not take events into account. We introduce two baselines as special case of our full model, one with neither events or the capacity to flexibly react, and one with the capacity for flexibility, but without accounting for actual events.

To describe the model without events, and no flexibility capacity, the appropriate sample is the empty set $\varnothing$, $N = 0$, and $F_c = 0$. We will denote this sample as *B*. Then we recover the original ESM whose objective value naturally serves as a baseline of system cost:

$$B : \varnothing, n_s = 1, N = 0, F_c = 0 \tag{16}$$

$$I_B = I(B) \tag{17}$$

$$O_B = O(B) \tag{18}$$

$$C_B = C(B) \tag{19}$$

We denote the investment and operation decisions of the original baseline ESM as I_B and O_B. The objective value C_B of the baseline model with these decisions is the optimal cost of the energy system without consideration of events, and no capacity to react to them.

The second baseline model maintains capacity to react to events up to $F_c = F_C$, but does not account for the cost of actual events occurring. We denote this sample as *F*, and call this the "capacity only" model:

$$F : \varnothing, n_s = 1, N = 0, F_c = F_C \tag{20}$$

$$I_F = I(F) \tag{21}$$

$$O_F = O(F) \tag{22}$$

$$C_F = C(F) \tag{23}$$

Following the same approach as with the baseline model, we denote "capacity only" investment and operation decisions as I_F and O_F. The objective value of the capacity only model C_F represents the cost of the linear optimization with the constraint guaranteeing a level of flexibility F_C.

Note that the objective values C_B and C_F do not include any events actually occurring. The difference $C_F - C_B$ can be interpreted as the cost of maintaining a flexibility capacity at level F_C at all times. The difference between the objective value of full system and the baseline, $C(S) - C_B$, is the full cost of maintaining capacity and dealing with events as described by *S*.

A key question we will consider is whether it is necessary to consider the stochastic events that will occur when deciding the investments and operational schedule of the energy system. To evaluate this, we will consider the cost of the system when decisions are made with the capacity only model. $C(S|I_F, O_F)$ denotes the objective value of the optimized system experiencing events *S*, with investments and operations planned using the capacity only model, and is obtained by only optimizing the recourse schedules. Similarly, $C(S|I_F)$ is the objective value of the system where operations have been planned taking *S* into account, but investments were made using the capacity only model.

Table 1 Overview: Stochastic modeling of capacity to react and events

Model	Objective value	Flexibility Capacity	Investment	Operation	Recovery after event	Stoch
Baseline	C_B	0	optimal (I_B)	optimal (O_B)	not considered	no
Capacity only	C_F	F_C	optimal (I_F)	optimal (O_F)	not considered	no
Recourse only	$C(S\|I_F, O_F)$	F_C	fixed at I_F	fixed at O_F	evaluated	no
Operations and Recourse	$C(S\|I_F)$	F_C	fixed at I_F	optimal	optimal	yes
Full	$C(S)$	F_C	optimal	optimal	optimal	yes

B refers to the baseline model in which no capacity for events and no events are accounted for. F refers to the model where a capacity to react to events up to F_C is maintained, but the reaction to events is not taken into account. The Recourse only model evaluates the cost of events, but does not optimize either investments or operations for them, "Operations and Recourse" takes investment decisions without taking events into account. Throughout S is the sample of events on which the cost is evaluated. For $C(S|I_F)$ and $C(S)$ the sample on which the investment and operation are optimized are different from the one on which the cost is evaluated

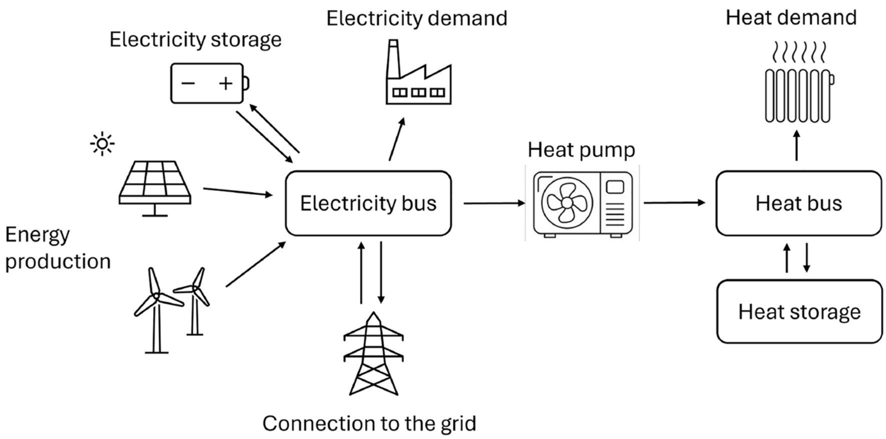

Fig. 1 Simplified energy system model based on an industrial park

The standard value of stochastic solution is not directly well defined in our case but, but we can define an equivalent value of the stochastic solution by comparing the full cost to the cost of maintaining capacity $C(S|I_F, O_F) - C(S)$. Similarly, we can consider the value of stochastic operation as $C(S|I_F, O_F) - C(S|I_F)$ and the value of stochastic investment $C(S|I_F) - C(S)$. An overview of all costs and values introduced in this section is provided in Table 1.

In principle, the stochastic design of the total cost approach described here can be applied to any energy system. To concretely test whether it can be useful in practice, we will consider a system with on-site energy production and consumption and a

Table 2 Investment and operational schedule variables$t$

Notation and value range	Meaning and unit
$u_{pv} \in [0, pv_{cap}]$	Investment in PV energy production, kWp
$u_w \in [0, w_{cap}]$	Investment in wind energy production, kWp
$u_{st} \geq 0$	Investment in electricity storage, kWh
$u_{hs} \geq 0$	Investment in heat storage, kWh
$u_{hp} \geq 0$	Investment in heat pumps, kW of heat
$soc(t) \in [0, u_{st}]$	Electricity storage state of charge, kWh
$st^{in,out}(t) \in [0, u_{st}]$	Flow into electricity storage from the bus and out of storage into the bus, kWh
$soc_{hs}(t) \in [0, u_{hs}]$	Heat storage state of charge, kWh
$hs^{in,out}(t) \in [0, u_{st}]$	Flow into heat storage from the heat bus and from heat storage into the heat bus, kWh
$hp(t) \in [0, u_{hp}/\mathrm{COP}]$	Flow of energy through the heat pump
$cur_{pv}(t) \in [0, u_{pv}pv(t)]$	Curtailment of PV energy production, kW
$cur_w(t) \in [0, u_w w(t)]$	Curtailment of wind energy production, kW
$gci(t) \geq 0$	Flow of energy from the grid, kW
$gco(t) \geq 0$	Flow of energy into the grid, kW

The flow and state of charge variables are given for the hour

single connection to the rest of the grid inspired by an industry park. We will first give the standard model explicitly, and then give the new additions $F^{\pm}$ and CR.

5 ESM of an industry park

We will study the flexibility potential of a simplified energy system consisting of electricity and heat demand, wind and photo-voltaic (PV) components with potential production $w(t) \geq 0$ and $pv(t) \geq 0$, electricity and heat storage, heat pumps and a

grid connection point (see Fig. 1). The model is close to the model study of the Max Bögl industrial park used in the open_plan validation study [6]. We do not consider shiftable demand and thus the electricity and heat demand time-series $d_{el}(t) \geq 0$ and $d_h(t) \geq 0$ are fixed. The variables defining the system's components are presented in Sect. 2.

The system is studied in discrete time: $t \in T, T = 1 \ldots t_f$ with a time resolution of 1 h and $t_f = 365 \cdot 24$, i.e., one full year of operation.

Wind and solar potentials *w*(*t*) and *pv*(*t*) are given for a unit component with $1kWp$. They are scaled by u_{pv} and u_w that represent investment. We allow curtailment of energy production $0 \leq cur_{pv}(t) \leq u_{pv} * pv(t)$, $0 \leq cur_w(t) \leq u_w * w(t)$.

Component costs are given by $u_k c_k$, c_k is the cost of $k \in pv, w, sto, sto_h, hp$ component per kW peak nominal production capacity. To account for the longer expected lifetime of components, their costs are scaled down to the hourly timescale with a factor $\frac{1}{20*365*24}$, where 20 is the component lifetime in years. Costs of components and their operation are given in Table 3. We also include storage losses and heat

Table 3 Costs and parameters of the system's components

Notation	Meaning and unit	Value
c_{pv}	Cost of PV components per kWp, Euro	800
c_w	Cost of wind turbines per kWp, Euro	1150
c_{st}	Cost of electricity storage per kWh, Euro	300
c_{hs}	Cost of heat storage per kWh, Euro	3.1
c_{hp}	Cost of heat pumps per kW of energy transformed, Euro	1006
c_{gci}	Cost of buying a kW of energy from the grid, Euro	0.165
c_{gco}	Revenue for selling a kW of energy to the grid, Euro	0.02
η_{st}	Efficiency of charge and discharge	0.97
η_{hs}	Efficiency of charge and discharge of thermal storage	1
l_h	Rate of discharge of thermal storage	0.0001
COP	Heat pump coefficient of performance	3.5

pumps coefficient of performance (COP). The values of parameters corresponding to PV, wind and the electric system are taken from the pilot study of the open_plan project [6]. The heat sector is not parametrized to a concrete industry park, but uses the model developed in [13] for a conceptual heat sector, with parameters taken from the PyPSA-Europe project [11].

We denote energy flow in and out from the grid connection point as $gci(t) \geq 0$ and $gco(t) \geq 0$. The energy is bought at the price c_{gci} and sold at c_{gco}, where $c_{gci} > c_{gco}$

. The grid connection point serves as the connection to the energy market, where the system may trade energy to balance its own energy demand and production.

The flow of energy into the bus from the battery and into the battery is $st^{in}(t) \geq 0$

and $st^{out}(t) \geq 0$. The battery state of charge is given by the variable *soc* and the following constraints:

$$soc(1) = \frac{u_{st}}{2}$$
$$soc(t+1) = soc(t) - st^{out}(t)/\eta_{st} + st^{in}(t)\eta_{st} \, \forall t \in T \tag{24}$$

In this paper the state of charge is defined in absolute value, i.e. $0 \leq soc \leq u_{st}$.

Following conventions of energy system modelling, we set a cyclic constraint equating storage state of charge at the first and last time steps:

$$soc(t_f) = soc(1) \tag{25}$$

The finite battery capacity is reflected in the following constraint:

$$\forall t: \; 0 \leq soc(t) \leq u_{st}$$

The system contains heat storage and heat pumps. The usage of heat storage is defined by $hs^{in}(t) \geq 0$ and $hs^{out}(t) \geq 0$. The heat storage state of charge is given by the variable soc_{hs} and the following constraints:

$$\begin{aligned} soc_{hs}(1) &= \frac{u_{hs}}{2} \\ soc_{hs}(t+1) &= soc_{hs}(t) - hs^{out}(t)/\eta_{hs} + hs^{in}(t)\eta_{hs}\ \forall t \in T \\ soc_{hs}(N) &= soc_{hs}(1) \end{aligned} \tag{26}$$

The use of heat components is defined by the heat demand and the balance of heat in- and out-flows at each hour. The transformation of electricity to heat through the heat pump is defined through the flow of energy to heat denoted *hp* and coefficient of performance COP. These coefficients are likewise defined in Table 3.

$$\forall t \in T:\ hs^{out}(t) - hs^{in}(t) + \mathrm{COP}hp(t) - l_h soc_h(t) - d_h(t) = 0 \tag{27}$$

$$0 \leq hp(t) \leq u_{hp}/\mathrm{COP} \tag{28}$$

Flows of electricity in the system must be balanced at every hour, which is reflected in the energy balance constraint:

$$\begin{aligned} \forall t \in T:\ & u_{pv}(t)pv(t) - cur_{pv}(t) \\ & + u_w(t)w(t) - cur_w(t) \\ & + gci(t) - gco(t) + st^{out}(t) - st^{in}(t) \\ & - d_{el}(t) - hp(t) = 0 \end{aligned} \tag{29}$$

6 Flexibility extensions of the ESM

We now show concretely how the additional flexibility capacity constraint and events look like in this ESM.

6.1 Capacity constraint

The flexibility capacity of renewable energy production is given in terms of the potential to increase or decrease curtailment:

$$F^+_{pv}(t) = cur_{pv}(t) \tag{30}$$

$$F^-_{pv}(t) = u_{pv} \cdot pv(t) - cur_{pv}(t) \tag{31}$$

$$F^+_w(t) = cur_w(t) \tag{32}$$

$$F_w^-(t) = u_w \cdot w(t) - cur_w(t) \tag{33}$$

For the electricity sector, the flexibility capacity is limited by the state of charge of the electricity storage, and the maximum flow in and out of storage in one hour, which we assume to be 0.8 of the battery capacity u_{st} here. Denote the total flow from storage as $st(t) = st^{out} - st^{in}$, then

$$F_{el}^+(t) = \min(soc(t+1) \cdot \eta_{st}, 0.8u_{st} - st(t)) \tag{34}$$

$$F_{el}^-(t) = \max((soc(t+1) - u_{sto})/\eta_{st}, st(t) - 0.8u_{st}) \tag{35}$$

The flexibility capacity of the heat sector differs in so far as no energy can be taken from the heat storage to meet electricity demand. The only possibility is to reduce the amount of energy flowing into the heatpump and supplement it by drawing from heat storage. The flexibility capacity is given in terms of the electricity equivalent of the storage capacity and the operational limits of the heat pump:

$$F_{hs}^+(t) = \min(soc(t+1) \cdot \eta_{hs}/COP, hp(t)) \tag{36}$$

$$F_{hs}^-(t) = \max((soc(t+1) - u_{hs}) \cdot \eta_{hs} \cdot COP, hp(t) - u_{hp}/COP) \tag{37}$$

The total positive and negative flexibility capacity is then given by the sum of all these individual capacities:

$$F^{\pm} = F_{pv}^{\pm} + F_w^{\pm} + F_{el}^{\pm} + F_{hs}^{\pm} \tag{38}$$

Table 4 Second-stage variables and additional constraints defining the event time, and the return to the planned operational schedule

Second-stage variable	Constraints
$\widetilde{soc}$	$\widetilde{soc}(t_\xi) = soc(t_\xi)$
	$\widetilde{soc}(t_\xi + t_r) = soc(t_\xi + t_r)$
$\widetilde{st}^{in,out}$	$\widetilde{st}^{in,out}(t_\xi + t_r) = st^{in,out}(t_\xi + t_r)$
$\widetilde{soc}_h$	$\widetilde{soc}_h(t_\xi) = soc(t_\xi)$
$\widetilde{hs}^{in,out}$	$\widetilde{hs}^{in,out}(t_\xi + t_r) = hs^{in,out}(t_\xi + t_r)$
$\widetilde{hp}$	$\widetilde{hp}(t_\xi + t_r) = hp(t_\xi + t_r)$
$\widetilde{cur}_{pv}$	$\widetilde{cur}_{pv}(t_\xi + t_r) = cur_{pv}(t_\xi + t_r)$
$\widetilde{cur}_w$	$\widetilde{cur}_w(t_\xi + t_r) = cur_w(t_\xi + t_r)$
$\widetilde{gci}$	$\widetilde{gci}(t_\xi) = gci(t_\xi)$ and
	$\widetilde{gci}(t_\xi + t_r) = gci(t_\xi + t_r)$

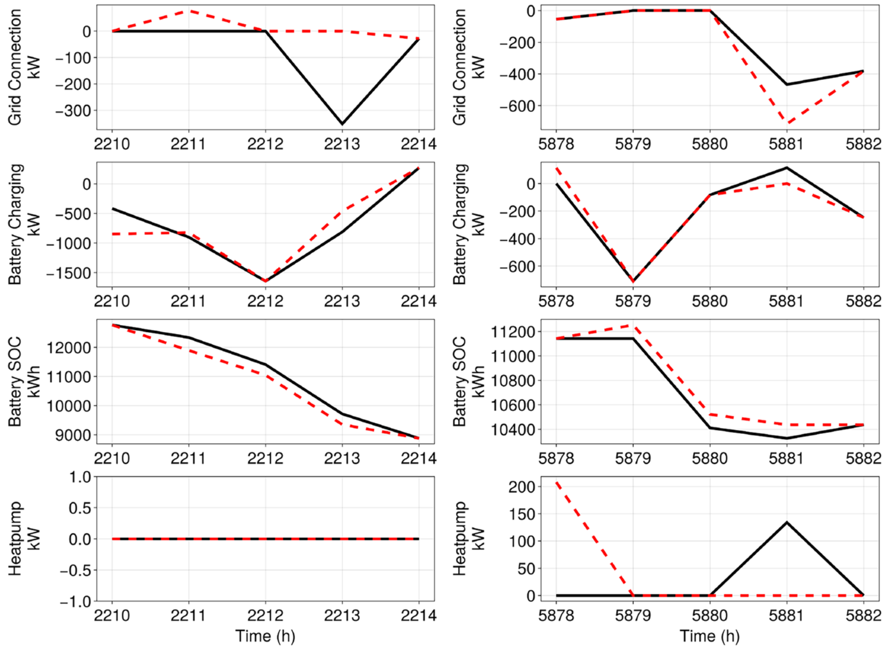

Fig. 2 Recovery schedules, with originally planned schedule in black, and behavior during the event and the recovery given by the dashed red lines. At the time of the event, the energy balance deviates from the planned schedule, subsequently it returns to the planned operational schedule over the course of several hours. The flexibility event on the left is served by the electric battery, the one on the right, by the heat sector

6.2 Events

The model for the recovery schedules R largely follows the model of the operational schedule of the ESM. The recovery window has length $t_r + 1$, and runs from $t_\xi \ldots t_\xi + t_r$. Every time-dependent variable $x(t)$ of the ESM has a corresponding second stage variable $\tilde{x}(t)$. At t_ξ the flows in the system can deviate from the first

stage operational schedule, the state of charge of heat and electricity storage will deviate from $t_\xi + 1$ onwards. By $t_\xi + t_r$ the system needs to have recovered to its optimal schedule with the exception of the heat storage, which is allowed to stay out of balance. This is necessary as the heat storage can only be charged, but not additionally discharged during the recovery window. The mismatch is accounted for in the cost of the scenario, with a surplus being rewarded with c_{gco} and a deficit penalized with c_{gci}.

Of the flows, only *gci* and *gco* are required to stay the same as the first stage variables at the request time. The deviation cannot simply be met by drawing from the grid, as we want to model the ability of the system to use its own sources of flexibility. The additional constraints satisfied by the second stage versions of the operational variables during the recovery window are summarized in Table 4.

Examples of recovery schedules are given in Fig. 2

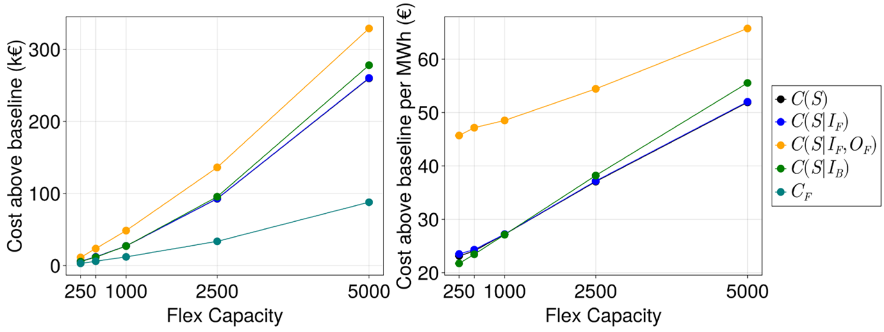

Fig. 3 Difference of total system costs (left) and costs per MWh of flexibility provided (right) of different model configurations compared to the no capacity, no event baseline. C_F is the cost of only maintaining the capacity for flexibility, $C(S|I_F, O_F)$ is the cost of dealing with events without optimizing investment or operations for them, $C(S|I_X)$ is the same cost when optimizing operations but taking investments from the capacity only model (I_F) or the baseline model (I_B), and $C(S)$ is the cost when

7 Results

We implemented the above model first using `StochasticPrograms.jl` [3] and then in JuMP [7] directly. All code and data is available at https://github.com/FHell/StoFlex3. We then considered several different parameters for the flexibility events, with events occurring once, twice or four times per day, and the system having the capacity to deal with events ranging from 250kW to 5000kW. This should be compared to an average hourly electricity demand of 2400kW and a heat demand of 1300kW. We present results for discretization with 12 samples which implies the overall set of events contains 6 to 24 events per day.

7.1 Cost of events and capacity, and value of stochastic modelling

We begin with the results for the full cost of dealing with four events per day. The results are plotted in Fig. 3. All costs are given above the baseline C_B. As expected, we see that cost of maintaining flexibility capacity (C_F) rises with the amount of flexibility required. For the full costs of flexibility we consider four different models. The fully optimized system $C(S)$, the system with investments as determined by the capacity only optimization $C(S|I_F)$, the system with investments as determined by the background model $C(S|I_B)$, and the system with investment and operations determined by the capacity only optimization $C(S|I_F, O_F)$.

We see that the cost of events is considerable, and for larger flexibility requests exceeds the cost of maintaining the capacity for flexibility.

We also see that the cost of events per MWh is smaller for smaller flexibility capacities. This shows that the system can use its existing resources to provide cheaper flexibility, up to a point.

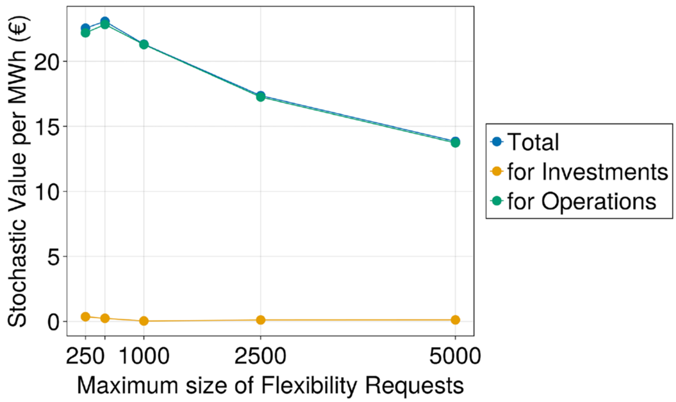

Fig. 4 Added value of stochastically optimizing investments, operations and the full system for handling events

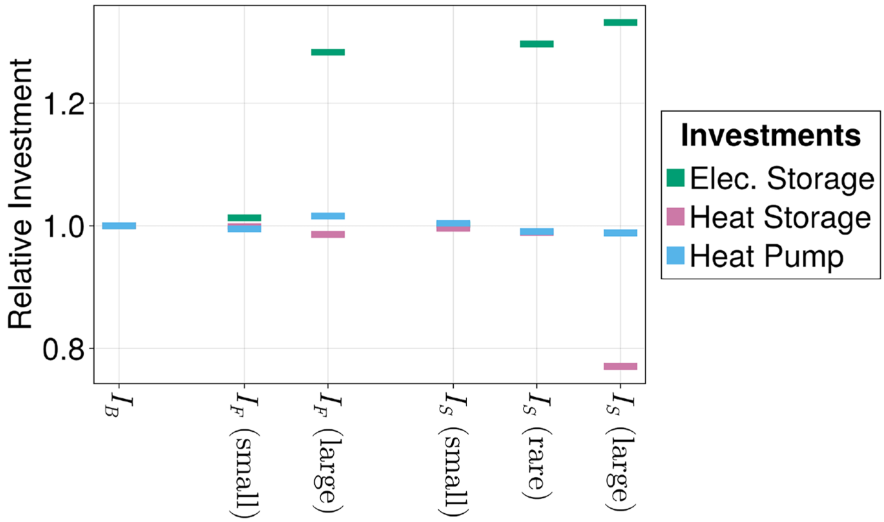

Fig. 5 Capacity only investments I_F and full model investments I_S given: rare, (5000kW, once per day), large (5000kW, four times per day) and small (250kW, four times per day) events

Finally, we observe that the difference between the cost of flexibility between the different investment decisions is very small if the operational schedule is optimized for flexibility requests. Only for the largest request sizes is there a substantive difference between the background investment and the capacity only investment. Optimizing the investment for flexibility requests brings no further benefits, $C(S)$ and $C(S|I_F)$ are virtually identical. On the other hand, optimizing the operational schedule to not just provide the capacity for flexibility, but taking the cost of the recovery required after the event into account, has a dramatic impact on system costs. For the smallest event sizes we see a reduction in the cost per MWh by up to half. This can be further illustrated by considering the value of stochastic operations as defined above as shown in 4.

7.2 Investment decisions

Given that investments seem to play a minor role in improving system costs, we investigate next how investment decisions actually change in the different models. We will consider three event parameters, infrequent large requests (5000kW, once per day), frequent large requests (5000kW, four times per day) and frequent small requests (250kW, four times per day).

The investments found by the various optimizations in these situations are plotted in Fig. 5. We see that for small requests the investment decisions don't differ significantly from the baseline. This agrees with the observation above that for small amounts of flexibility, the system can make use of existing investments. However, for large requests we see that the optimizations taking flexibility into account invest significantly more into storage and heat pump. In both, the situation with rare large and frequent large requests the capacity only investments I_F for large requests differ significantly from the full system investments I_S.

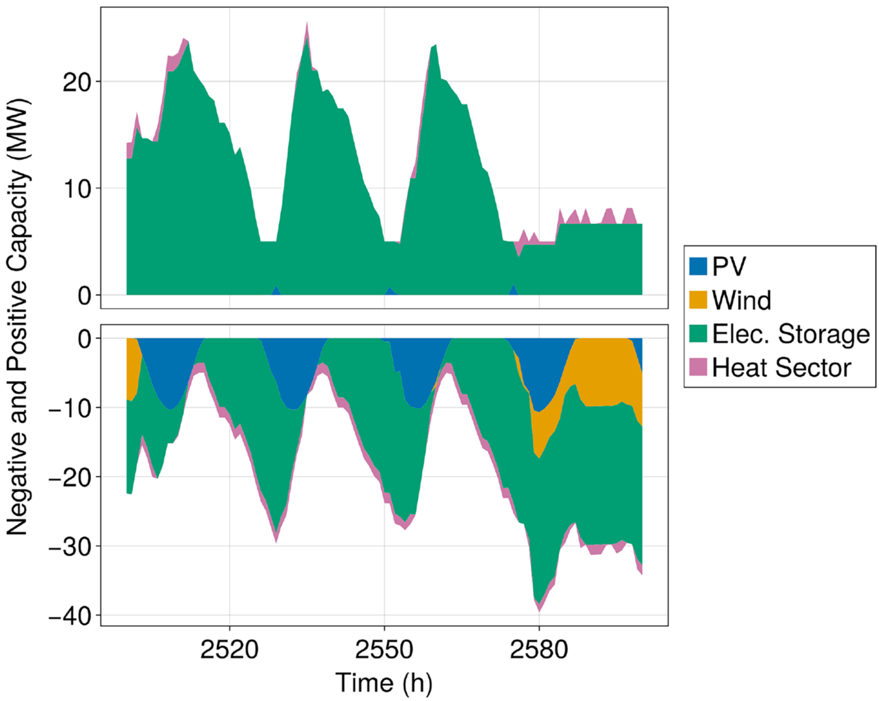

Fig. 6 Source of negative and positive flexibility capacity

7.3 Sources of flexibility

Using the fact that the capacity constraint Equation (1) is the sum of components, we can also see which parts of the system contribute to the flexibility capacity Fig. 6. We see that in the system considered, electrical storage plays the main role in providing both, efficient use of solar energy, and flexible operations. For some hours in the day, curtailment and the heat sector also contribute to reach the required flexibility capacity.

Further, we see that the flexibility capacity is often much larger than the minimum required.

7.4 Necessity of resampling

We define *C*(*S*) with respect to a different sample of events (S') than the one used to optimize the system (*S*), i.e., $C(S) := C(S', I(S), O(S))$. When the system is optimized for a given sample *S*, the optimization has perfect foresight of the events - namely, the timing and magnitude of flexibility request in the sample *S*. Hence, the optimized scheudle may take advantage of that information, e.g., by increasing storage levels just before the event. By choosing a different sample to evaluate the cost, we eliminate that information from the cost calculation and arrive at a more robust estimate of the costs of adjusting to unforeseen events. The mismatch between the costs illustrated in Fig. 7, relative to 3 can be interpreted as the mistake caused by perfect foresight of the events in the sample. In our model this error is above 30% for the smaller flexibility requests and still around 20% for the 5 MWh requests. This error is closely related to how good the chosen sample covers the space of all possible events and we expect it to decrease if a larger sample is chosen.

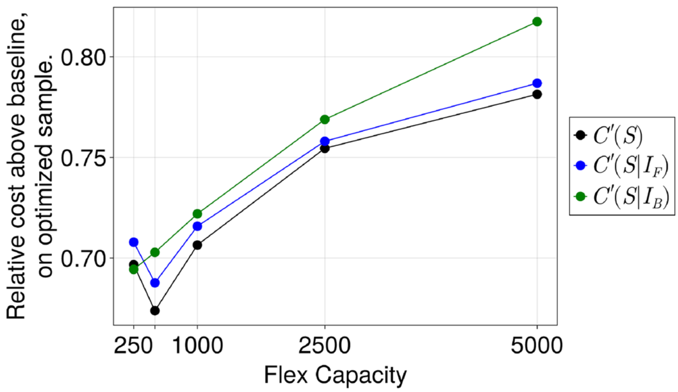

Fig. 7 Costs of flexibility without resampling $(C'(S) = C(S|I(S), O(S)))$ relative to cost of flexibility

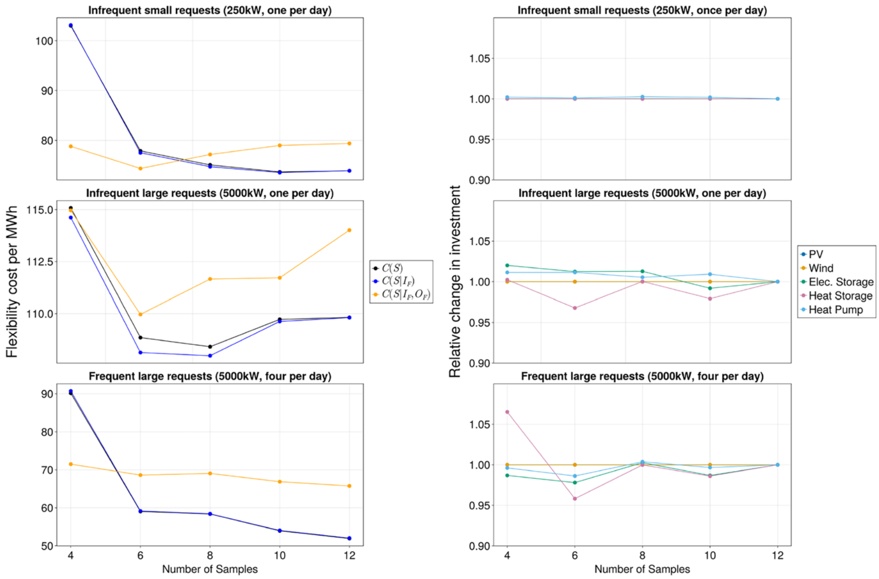

Fig. 8 Cost of flexibility and investment decisions versus number of samples used to discretize the stochastic program, in three different flexibility request models

7.5 Convergence of the stochastic program

Finally, we check whether the results depend on the number of samples n_s used to discretize the stochastic program. We consider three scenarios, infrequent large request, infrequent small request and frequent large requests. We see in Fig. 8 that the investment decisions do not depend strongly on the number of samples. On the other hand, the cost of events is possibly not fully converged at $n_s = 6$ for all cases. We see that additional samples can reduce the cost further. Recall that we evaluate the costs on a different sample than we use to optimize the decisions. As we have seen, optimizing the operation for the actual cost of events has a large impact on the latter. We suspect that we see a similar situation here, as the number of samples increases, more individual hours are directly optimized for the cost of dealing with the events.

We also note here that having the capacity constraint is not sufficient to actually ensure that a permissible recovery schedule exists. In situations in which the background schedule plans to operate the system at the edge of its capacities in the coming hours, it can be impossible to actually return to the planned schedule exactly in the recovery window. Such events were very rare for the parameters chosen here. Finally, we also found that in our system it is not reliably possible to investigate the cost of events using a purely stochastic approach, foregoing the capacity constraint. In this case, we observed significant overfitting to the sample used to optimize the system.

8 Discussion

In this paper, we extended a standard temporally-resolved ESM for medium horizon optimization tasks, to deal with random events. We introduced a deterministic constraint that ensures the system has the capacity to deviate from the intended operational schedule, and a stochastic model of the actual deviations being forced and their associated cost.

In a case study of a renewable based industry park, we found that the cost of deviating from the schedule after an event can be larger than the cost of merely ensuring the capacity to deviate. Taking deviations into account when optimizing the operating schedule considerably reduces the overall cost of flexibility by up to 50%.

On the other hand, investment decisions had only a minor impact on the final system costs when operations were optimized to provide flexibility with the given resources.

The costs of deviation events per MWh we obtained with this model are somewhat larger than what is typically seen on flexibility markets today. However, it should be noted that the price we obtain should not be interpreted as the price with which an operator could bid on flexibility markets. For one, we saw that the flexibility capacity is often significantly larger than the minimum required by the capacity constraint. This additional flexibility could be offered, significantly lowering the cost per MWh. Secondly, the recovery conditions we impose are very strict and are not needed in practice. Finally, when participating in the market, the operator can intelligently choose when to offer how much flexibility, according to the expected state of the system. This contrasts to our model, where we assume the same minimum flexibility capacity at every hour of the year.

9 Conclusion

Events that force a deviation from planned operation are a fact of life in energy systems. There is a large literature on optimizing the cost of maintaining the capacity to react flexibly, in comparison the cost of the actual deviations is rarely considered. We have seen that by incorporating random events in an appropriate sense into an ESM, such that they are considered during the operation and investment planning, the cost of such deviations can be reduced. Furthermore we found that by evaluating the model decisions on events for which they were not optimized, it is possible to estimate the overfitting error that an approach with perfect foresight of the sampled events would have produced.

The natural setting for our model of exogenous events is in the context of optimizing systems for cost efficient resilience and adequacy. For concrete applications, this poses the challenge to model appropriate events, recourse actions and capacity constraints though. Another context where a similar approach might be relevant, is for extending systems for the participation in flexibility markets. However, in the case study we explored we did not find that our approach helped with investment decisions. As already noted in the discussion, properly evaluating the approach for this use-case would also imply extending the model substantially. To actually plan investments for participation in markets, we would need to extend the model in the direction of [15], which, while in principle feasible, is beyond the scope of this work.

It would also be interesting to explore whether our stochastic model of intermittent events could be used in combination with scenario based approaches [9, 10]. For example, by treating forecasting errors as events, and taking into account unanticipated load and renewable fluctuations, and by optimizing the system to make forecasting errors cheap. This would complement established approaches that make sure a wide range of possible conditions are covered.

Author Contributions All authors reviewed the manuscript. E.Z. wrote the first version of the manuscript and implemented the model. P.P. contributed the heat sector. M.L. and F.H. developed and ran the final version of the model and finalized the manuscript.

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Data Availability All code and data to reproduce the results is available at https://github.com/FHell/StoFlex3

Declarations

Acknowledgements and Ethics Not applicable.

Conflict of interest The authors declare no conflict of interest.

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