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# Optimal carbon taxation and horizontal equity: A welfare-theoretic approach with application to German household data

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A B S T R A C T

We develop a model of optimal taxation and redistribution under an ambitious climate target. We take into account vertical income differences, but also explicitly capture horizontal equity concerns by considering heterogeneous energy efficiencies. By deriving first- and second-best rules for policy instruments including carbon and labor taxes, transfers and energy subsidies, we investigate analytically how vertical and horizontal inequality is considered in the welfare maximizing tax structure. We calibrate the model to German household data and a 30 percent emission reduction goal and show that redistribution of carbon tax revenues via household-specific transfers is the first-best policy. Under plausible assumptions on inequality aversion, transfers to energy-intensive households should be about five times higher than transfers to energy-efficient households. Equal per-capita transfers do not require to observe households' efficiency type, but increase equity-weighted mitigation costs by around 5 percent compared to the first-best. Mitigation costs increase by less, if the government can implement a uniform clean energy subsidy or household-specific tax-subsidy schemes on energy consumption and labor income that target heterogeneous energy efficiencies. Horizontal equity concerns may therefore constitute a new second-best rationale for clean energy policies or differentiated energy taxes.

## 1. Introduction

The trade-off between equity and efficiency is one of the central topics in economics and economic policy. Analyzing this central trade-off with respect to the implementation of climate policy poses an urgent challenge: On the one hand economic theory clearly suggests that Pigouvian carbon pricing should be at the heart of economically efficient and environmentally effective climate policy (Pigou, 1920; Nordhaus, 2019), which is broadly supported by economists worldwide (Financial Times, 2019; The Economist, 2021) as well as by recent empirical findings (Andersson, 2019; Gugler et al., 2021). On the other hand, there are many factors that impede the timely required implementation and political feasibility of carbon pricing (Levi, 2021; Edenhofer et al., 2021). Among these factors, distributional consequences for low-income households are a key concern (Shammin and Bullard, 2009; Parry, 2015; Pizer and Sexton, 2019) resulting in a political debate, which is often charged with emotions and thus provides breeding ground for conflicts, turmoil and deadlock. The yellow vest movement, for example, rose in France in November 2018 to protest against

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fuel price increases due to $CO_2$ taxation. It is one example illustrating that climate policy analysis needs to consider appropriate redistribution measures that address equity concerns.

Due to the regressive first-order effect of carbon pricing in middle and high-income countries (Wang et al., 2016; Dorband et al., 2019; Ohlendorf et al., 2021), the existing literature has mostly focused on vertical equity between different income deciles. Empirical studies, however, have highlighted the importance of horizontal equity since distributional effects show an even larger variation within income groups (Poterba, 1991; Rausch et al., 2011; Cronin et al., 2019; Pizer and Sexton, 2019). The impact of carbon pricing on carbon-intensive energy consumption can vary across households with similar incomes due to household characteristics and behavior such as the climate surrounding the household, commuting distance of its members or the energy efficiency standard of a building (Rausch et al., 2011). Indeed, in the public debate negative distributional outcomes for households that are hardship cases due to their high carbon footprints, have frequently been used to argue against carbon pricing, even when the overall distributional effects are progressive, for example, due to per-capita transfers.

Because household characteristics that determine horizontal inequality are mostly outside the scope of governmental regulation (Kaplow, 1989, 1992), only very few studies have included horizontal equity concerns in a welfare-theoretic framework. While some studies like Auerbach and Hassett (2002) suggest the introduction of a separate parameter for horizontal inequality aversion, others view horizontal inequality as being related to loss aversion (Slesnick, 1989; Fischer and Pizer, 2019). In this paper we put forward an alternative perspective on social welfare and horizontal equity in the context of ambitious climate policy. Specifically, we define horizontal inequality as heterogeneous endowments with energy-efficient capital of household of the same vertical income decile, that is, of same labor productivity. To achieve a certain level of utility, a long-distance commuter living in a badly insulated house, for example, is likely to require a higher amount of energy than a city dweller living in a modern apartment building. As heterogeneous capital endowments result in heterogeneous marginal abatement costs for households with same income, carbon pricing increases horizontal inequality. Because this inequality is orthogonal to income inequality (that is, vertical inequality), conventional distributional policies like progressive income taxes or transfers cannot reduce horizontal inequality. Our welfare-theoretic approach explicitly considers both types of inequality. This allows us to derive optimal policies that consider both dimensions. In particular, our model can explicitly capture the trade-off of policy mixes that reduce horizontal equity at the expense of larger aggregate mitigation costs.

While we focus on the aspect of energy efficient capital, there are many other dimensions of horizontal inequality. Moreover, our approach does not introduce an explicit welfare (or utility) based horizontal inequality aversion as in Slesnick (1989) and Fischer and Pizer (2019) which would create an additional rationale to take horizontal inequality into account. While future work could combine these two approaches, we aim to isolate horizontal inequality in a conventional welfare economics framework.[2]

Methodologically our approach builds on Cremer et al. (2003) and Kaplow (2008), who suggest non-linear (energy) tax rules to take into account households' heterogeneity. In our case, however, the heterogeneity is not modeled as a 'taste' as in Cremer et al. (2003) and Kaplow (2008) but, at least in the short-run, as exogenous household-specific capital endowment capturing how efficient households can convert energy into individual well-being. This enables us to consider the empirical evidence on horizontal heterogeneity in energy expenditure shares, which we model as the main driver of horizontal inequality due to ambitious climate policy. Our paper further borrows from previous works on optimal environmental policy and vertical inequality that focused, for example, on non-linear Engel-curves in energy use (Klenert et al., 2018; Jacobs and van den Ploeg, 2019, for example). In this context a recent study by van der Ploeg et al. (2022) investigates which policies affect public support for the green tax reform by estimating an EASI energy demand system for the German economy. Our paper mainly differs by considering a less detailed expenditure structure but by explicitly incorporating horizontal and vertical inequality in a normative social welfare framework.

We show that the government's first-best solution is to set the carbon tax equal to the Pigouvian level and recycle the carbon tax revenue through household-specific transfer payments that account for both vertical and horizontal household heterogeneity. Subsequently, we explain the government's equity–efficiency trade-off when the carbon tax deviates from the Pigouvian level and show the implications of restricting climate policy to uniform transfers and a subsidy on clean energy production. We find that the interplay between household-specific endowments of energy efficient capital, the curvature of the utility function, normative welfare weights and the scarcity of public funds determines the optimal policy. For example, the more the underlying normative standpoint leans towards the utilitarian principle, the more likely it will be welfare-enhancing to redistribute tax revenue to energy efficient households as they can better increase social welfare due to their higher endowments with energy efficient capital. If in turn distributional motives dominate the equity–efficiency trade-off, then less energy efficient households receive larger transfers.

We apply our findings to empirical data on energy consumption in Germany to quantify optimal policies. The numerical analysis considers a 30 percent emission reduction target and disregards the vertical inequality dimension (different labor productivities) to obtain a clear understanding about the trade-off between horizontal equity and efficiency in particular. We show that irrespective of conventional ranges of inequality aversion the first-best policy requires higher lump-sum transfers for less energy efficient households and zero marginal income taxation for all households. The first-best policy and also a second-best policy package that has a fixed lump-sum transfer but household-specific marginal income tax rates require that governments can perfectly observe households' energy efficiency types. We therefore consider a set of second-best policy instruments with lower informational requirements, like subsidies for clean energy consumption, in combination with uniform carbon prices and transfers. We show that it is socially optimal

[2] To clearly conceptualize the effects of horizontal heterogeneity on social welfare, we further disregard an explicit consideration of different types of energy efficient capital, but rather measure it at an aggregate level related to its carbon-intensity only. Micro-econometric studies, like for example Jacobsen and van Benthem (2015), offer additional insights into how policies should consider heterogeneity within one particular type of capital (for example, old versus new cars).

to subsidize the production of clean energy by using up to 15% of the tax revenue generated by climate policy. Subsequently, we discuss the labor market effects of the different policy packages and calculate the increase in mitigation welfare costs, which range from 1% to 12% compared to the first-best optimum.

The paper is structured as follows. Section 2 discusses related literature on the economics of horizontal equity in detail. In Section 3, we set up the theoretical model and provide some basic insight on first- and second-best policies to target vertical and horizontal inequality. In Section 4, we introduce functional forms and the calibration approach to German household data. Section 5 presents the numerical results for a richer set of first- and second-best policy packages. Finally, Section 6 summarizes our results and discusses limitations of the approach taken in this paper.

## 2. The economics of horizontal equity

The literature on the economics of horizontal equity can be divided into (i) applied studies that quantify the magnitude of horizontal inequality due to some (environmental) policy reform, (ii) studies suggesting a welfare measure that disentangles vertical from horizontal equity and (iii) theoretical studies that consider horizontal equity within an optimal taxation framework. Here, we summarize each of these literature strands.

The first strand reports considerable within-decile variation in energy expenditure shares. Poterba (1991) analyzes gasoline expenditures in the United States and finds considerable within-decile variability especially among low-income households. Pizer and Sexton (2019) confirm this high variation also for other countries like Mexico and the UK. Rausch et al. (2011) show that the impacts of carbon taxation in the United States puts indeed the highest burden on low-income deciles, while Cronin et al. (2019) also consider the capacity of existing transfer payments to address horizontal equity. They show that a uniform increase in all existing transfer payments increases horizontal inequality even further, which thus calls for a more targeted redistribution approach. Also emphasizing the substantial within-group variation of impacts of climate policy, Landis et al. (2019) quantify policy costs in a comparison of a tax-based approach with a command-and-control approach, the former outperforming the latter in terms of cost-effectiveness by a factor of five.

The second strand is on welfare indices that incorporate horizontal equity. Slesnick (1989) proposes a welfare measure for horizontal equity that is consistent with social choice axioms and is calculated as the difference between welfare under a horizontally egalitarian distribution and the existing distribution of individual welfare. Using U.S. data from 1947–1985 the study finds increasing horizontal inequality due to the heterogeneous effects of taxation on households' welfare. Auerbach and Hassett (2002) argue that horizontal equity should be justified within the context of the Atkinson inequality aversion index (Atkinson, 1970). They differentiate between aversion to vertical and aversion to horizontal inequality by using a two parameter specification similar to the one that has been suggested by Epstein and Zin (1989). Based on U.S. data, they find horizontal inequality to be less severe the higher the standard Atkinson inequality aversion index is. In Fischer and Pizer (2019) horizontal equity concerns are seen as being related to loss aversion. They suggest a welfare measure that can incorporate both vertical and horizontal equity and is based on the concept of equal sacrifice relative to a status-quo (Slesnick, 1989; Kahneman and Tversky, 1979). Findings show that non-Pigouvian policies like tradable performance standards lead to more horizontal inequality as compared to Pigouvian policies like a cap and trade system with equal per household rebates.

Compared to the literature on welfare measures, our paper specifically introduces income heterogeneity in households' energy expenditure shares in the modeling structure, but otherwise applies a standard utilitarian social welfare approach to evaluate policies. Thus, our model is capable of capturing horizontal inequality with the standard Atkinson inequality aversion index as suggested by Auerbach and Hassett (2002). While in Fischer and Pizer (2019) more horizontal equity is always welfare-increasing, we do not only consider the positive effect on welfare due to a more egalitarian within-decile income distribution, but also take into account that society sacrifices efficiency gains when compensating hardship cases with higher transfer payments. Our approach can thus be considered more general as we seek to understand under which conditions (that is, social preferences) a benevolent government would care about horizontal equity.

The third strand of the literature deals with optimal taxation (Ramsey, 1928; Diamond and Mirrlees, 1971) with the aim of implementing a tax system that maximizes a social welfare function subject to economic constraints (Mankiw et al., 2009). Traditional utilitarian welfare theory (Bentham, 1789) is based on the principle of diminishing marginal utility of income, which motivates the dominating interest in vertical equity in optimal taxation models. Horizontal equity, in turn, is in this context typically interpreted as treating tax payers at equal positions equally, which is the more fundamental and widely accepted principle of fairness as an acceptable pattern of differentiation between income groups must be chosen (Musgrave, 1990). Nevertheless, the literature on optimal taxation and horizontal equity is relatively scarce (Atkinson and Stiglitz, 1976; Fischer and Pizer, 2019).

Stiglitz (1982) shows that horizontal equity cannot be derived from a utilitarian social welfare function and can be inconsistent with Pareto efficiency. Jordahl and Micheletto (2005) incorporate a horizontal equity constraint in the problem of finding an optimal utilitarian tax structure, which has already been suggested by Atkinson and Stiglitz (1976) in order to circumvent the equity–efficiency trade-off when horizontal equity is built into the measurement of social welfare itself. The horizontal equity constraint in Jordahl and Micheletto (2005) is based on the interpretation of Bossert (1995) in terms of 'equal transfers for equal circumstances' and requires that heterogeneous households with the same abilities should pay the same taxes. Based on a utility function that can embody different types of heterogeneity, Kaplow (2008) shows that preference heterogeneity can lead to both higher and lower levels of income taxation depending on the type of heterogeneity, it's strengths, and the concavity of private utility and the social welfare function. However, both Jordahl and Micheletto (2005) and Kaplow (2008) do not make any explicit connection to environmental policy and carbon taxation specifically.

Within the optimal taxation literature there is an established sub-field on optimal taxation and environmental externalities[3] that goes back to Pigou (1920). Later Sandmo (1975) contributed the seminal paper based on a model of heterogeneous households and optimal linear taxation of a commodity that generates a negative atmospheric externality. The optimal commodity tax rule that results from this modeling setup includes one additive term that corrects for the externality thereby fulfilling the so-called 'additivity property'.

This literature has been extended to consider a broader set of different tax system (Cremer et al., 2003), which could in principle also include horizontal inequality as a source of heterogeneity. Cremer et al. (2003) analyze how taste heterogeneity in households' preferences is captured in different systems of optimal environmental taxation and applies the model to energy consumption in France. The authors argue for non-linear environmental taxes when consumption levels are observable as in the case of electricity.

Compared to the previous literature, our paper considers horizontal inequality explicitly by modeling heterogeneous endowments of energy efficient capital, like well-insulated buildings or fuel-efficient cars, *within* income deciles. Thus, welfare-optimal redistributive policies, such as targeted transfers, taxes or subsidies, explicitly take into account the effect of climate policy on horizontal inequality. In contrast to previous literature, our study thus exhibits the trade-off between equity and efficiency in the context of climate policy and horizontal inequality from a normative social welfare perspective in a transparent way.

## 3. Theoretical model

In the following, we introduce a parsimonious model in order to convey a few basic intuitions about optimal policies. In Section 3.2, we characterize the socially optimal allocation that a social planner would implement. Then, we compare the social optimum with the outcomes that a government can achieve by using different sets of first-best (Section 3.3) and second-best policy instruments (Section 3.4). Finally, in Section 3.5, we discuss an extension of the static model to a dynamic two-periods model, which allows us to include the possibility of households investing in energy efficiency over time.

### *3.1. Basic model*

We assume that a benevolent government seeks to maximize the welfare of $j \in \{1, \dots, n\}$ heterogeneous households, which derive utility $u^j$ from a numeraire consumption good $c^j$, carbon-intensive energy services $E^j$ and leisure $z^j$. Leisure $z^j$ is defined as a time endowment normalized to one minus the fraction of time spent on work $l^j$, that is, $z^j = 1 - l^j$. Households differ in labor productivity captured by the before tax hourly wage rate $w^j$, but also in their energy efficiency. Specifically we assume that they demand raw energy $\widetilde{E}^j$ and use a technology $f$ to convert it to energy services $E^j$. Thus, we assume that $E^j = f(x_0^j)\tilde{E}^j$ where we define $f(x_0^j) =: \alpha_j$. All households use the same technology, but are heterogeneous with respect to their capital endowments $x_0^j$. For example, energy services constitute an optimal room temperature which can require more or less raw energy depending on the type of heating and building insulation or the surrounding climate conditions; mobility – reaching certain places for working, shopping etc. – constitutes another energy service, which requires different amounts of raw energy depending on distance, availability of public transport infrastructure or fuel efficiency of the household's car.

In our theoretical model household heterogeneity is thus two-dimensional. The first dimension is standard in the literature on optimal taxation and reflects vertical income heterogeneity through differences in labor productivity. The second dimension is novel and reflects horizontal heterogeneity within each income decile by household-specific capital endowments $x_0^j$. For now, we will assume that households cannot make additional investments in efficiency enhancing capital. This is reasonable for time periods of five to ten years over which housing location decisions or energy investments of buildings hardly change. Later, we will discuss how relaxing this assumption affects our results. Thus, we have

$$\begin{aligned} u^j &= u(c^j, E^j, z^j) \\ E^j &= f(x_0^j)\tilde{E}^j = \alpha_j \tilde{E}^j \end{aligned} \tag{1}$$

where $x_0^j > 0$ and $f' > 0 > f''$. The households' budget equation is

$$\underbrace{w^j l^j - T(w^j l^j)}_{b^j} = c^j + \underbrace{(p_E + t_E^j)}_{=:q^j} \tilde{E}^j \tag{2}$$

Households spend their labor income $I^j = w^j l^j$ net of labor income taxes $T(I^j)$ on numeraire consumption and raw energy. Note that the tax function $T(I^j)$ can be non-linear in gross income $I^j$. The producer price of energy $p_E$ is assumed to be fixed and given. Possible policy instruments that the government could implement include carbon taxes $t_E^j$ on $CO_2$-intensive energy consumption and labor income taxes $T^j$, which may be uniform or household-specific and can include both lump-sum transfers and marginal income taxes.

Households maximize utility by choosing numeraire consumption, energy service consumption and leisure time subject to their budget constraint. The Lagrangian of households is given by

$$L^H = u(c^j, E^j, z^j) + \lambda^j (b^j - c^j - \frac{q^j E^j}{\alpha_j}), \tag{3}$$

[3] See Aronsson and Sjögren (2018) for a very good overview of this literature.

where we have used (1) to eliminate raw energy $\tilde{E}^j$. The first-order conditions are as follows:

$$u_c^j = \lambda^j \tag{4}$$

$$\frac{u_E^j\,\alpha_j}{q^j} = \lambda^j \tag{5}$$

$$\frac{u_z^j}{w^j(1-T_I)} = \lambda^j. \tag{6}$$

Equations (5) and (6) reveal that the marginal utility of the individual household's income $\lambda^j$ depends on both vertical income heterogeneity captured by $w^j$ and horizontal heterogeneity in energy efficiency captured by $\alpha_j$. Marginal utility of income decreases in labor productivity, that is, $\lambda^j(w^j)$ with $\lambda^{j\prime}(w^j) = \frac{-u_z^j}{w^j(1-T_I^j)} < 0$ (standard decreasing marginal utility of income) and increases in energy efficiency, that is, $\lambda^j(\alpha_j)$ with $\lambda^{j\prime}(\alpha_j) = \frac{u_E^j}{q^j} > 0$. Combining Equations (4) and (5) as well as (4) and (6) yields the following two conditions:

$$\frac{u_E^j}{u_c^j} = \frac{p_E + t_E^j}{\alpha^j} \tag{7}$$

$$\frac{u_z^j}{u_c^j} = w^j(1-T_I^j) \tag{8}$$

Eq. (7) shows that in the optimum the marginal rate of substitution between consuming an additional unit of energy services and consuming an additional unit of the numeraire consumption good must be equal to the market price of energy scaled by the energy efficiency parameter $\alpha_j$. Condition (8) reveals that the household chooses optimal labor supply such that the marginal rate of substitution between leisure and numeraire consumption is equal to the after-tax net wage rate. Here $T_I^j = T_I(w^j l^j)$ is the marginal income tax rate that household $j$ faces.

An individual household's maximization results in the conditional demand functions $c^j = c(b^j, q^j, z^j)$ and $E^j = E(b^j, q^j, z^j)$, which together determine the household's indirect utility function $v^j = v(b^j, z^j, q^j)$.

### 3.2. Social planner optimum

A social planner chooses numeraire consumption, energy service consumption and leisure time to maximize a Bergson–Samuelson social welfare function $W(u^1, \dots, u^n)$ with $\frac{\partial W}{\partial u^j} \geq 0$ and $\frac{\partial^2 W}{\partial u^{j2}} \leq 0$ for all $j$, subject to an exogenous aggregate environmental target $E^* = \sum_j \tilde{E}^j$ and a resource constraint $\sum_j I^j - c^j - p_E \tilde{E}^j = 0$. We abstract from an explicit representation of environmental damages to keep the analysis as simple as possible. In the context of climate change, this is also reasonable as damages occur in the very-long run and are globally distributed.

From the social planner's first-order conditions[4] it follows that

$$\frac{W_u^i}{W_u^j} = \frac{u_c^j}{u_c^i} = \frac{u_E^j}{u_E^i}\frac{\alpha_j}{\alpha_i} = \frac{u_z^j}{u_z^i}\frac{w^i}{w^j} \quad \forall i, j.$$

Thus, the social planner chooses an allocation that balances households' welfare weights $W_u^j$, their marginal utilities $u_E^j$, $u_c^j$ and $u_z^j$ and their energy efficiencies $\alpha_j$. Normative distributional social preferences are balanced with efficiency in consumption.

To further interpret these equations, assume that $i$ has a higher normative welfare weight than $j$ ($W_u^i > W_u^j$). Then, $i$ must also have a higher level of numeraire consumption (corresponding to a lower marginal utility of consumption). Moreover, if $\alpha_i = \alpha_j$, household $i$ must also have higher level of energy service consumption. However, if $j$ is more energy efficient than $i$ ($\alpha_i < \alpha_j$), then the difference between normative welfare weights could be offset by energy efficiency considerations. If efficiency considerations (normative welfare weights) dominate, social optimality requires the social planner to allocate relatively more (less) energy service consumption to households that are more (less) efficient in transforming energy services to utility and hence social welfare. Similarly, if $w^i = w^j$, household $i$ must have more leisure time than $j$, but if $w^i > w^j$, then the difference between normative weights could be offset by labor productivity considerations.

### 3.3. First-best optimal governmental policy for the decentralized economy

The government takes into account each individual household's optimization behavior when it chooses numeraire consumption, energy service consumption and leisure time to maximize the same social welfare function $W(u^1, \dots, u^n)$ as the social planner does. The government is constrained by the same aggregate environmental target $E^* = \sum_j \tilde{E}^j$ and optimizes, in addition, subject to its budget constraint $\sum_j (t_E^j \tilde{E}^j - T^j) = 0$. We denote by $\mu$ and $\gamma$ the respective shadow prices. Then, $\frac{\mu}{\gamma}$ expresses the social value of relaxing the environmental constraint in income units. Moreover, we use $\widehat{\tilde{E}_z^j} = \tilde{E}_z^j - \tilde{E}_b^j MRS_{z,b}^j$ to denote how increased leisure

[4] See Appendix A.1

time affects the household's conditional demand for energy. We abstract from problems of self-selection to clarify the fundamental mechanisms that determine the optimal tax system when vertical and horizontal heterogeneity is observable.

The first-best case serves as an important policy benchmark. However, we first state a more general result in Lemma 1, which we subsequently use to characterize both the first-best optimal tax system (Proposition 1), but also, in the next subsection, a generic second-best optimal tax system (Proposition 2).

**Lemma 1.**

*1. For a given set of carbon taxes* $\{t_E^j\}_j$*, if the government can target each household with individualized transfers, the optimal marginal income tax for household* $j$ *is*

$$T_I^{j*} = \frac{\widehat{\tilde{E}_z^j}}{w^j}\left(t_E^j - \frac{\mu}{\gamma}\right). \tag{9}$$

*2. For a given set of income taxes and transfers, the optimal carbon tax can be expressed as*

$$t_E^{j\,*} = \frac{\mu}{\gamma} + \underbrace{\left(\frac{W_v^j \lambda^j}{\gamma} - 1\right)\frac{\tilde{E}^j}{\tilde{E}_q^j}}_{\Phi_j(w^j,\alpha_j)}. \tag{10}$$

*The optimal individual carbon tax is the sum of two components:*

*(i) a Pigouvian component* $\mu/\gamma$*, which is the same for each household.*[5]

*(ii) a distributional, household-specific component* $\Phi_j(w^j, \alpha_j)$*, which is determined by an equity–efficiency trade-off that takes into account both vertical inequality resulting from heterogeneous labor productivity* $w^j$ *and horizontal inequality due heterogeneous energy efficiency* $\alpha_j$*.*

**Proof.** See Appendix A.2. □

We can now state the implications of Lemma 1 when the carbon tax is set to the Pigouvian level. We find that in combination with individual lump-sum transfers the Pigouvian carbon tax is sufficient to achieve the first-best. Thus, even with horizontal inequality due to heterogeneity in energy efficiency, Sandmo's principle of targeting (Sandmo, 1975) still holds.

**Proposition 1.** *If the government uses a uniform carbon tax* $t_E^j = t_E\ \forall j$ *and household-specific lump-sum transfers* $T^j$*, it can achieve the first-best optimum by setting*

$$t_E^* = \frac{\mu}{\gamma}\ \forall j, \tag{11}$$

*Then, the optimal marginal income tax is zero*

$$T_I^{j*} = \frac{\widehat{\tilde{E}_z^j}}{w^j}\left(t_E^* - \frac{\mu}{\gamma}\right) = 0. \tag{9'}$$

**Proof.** See Appendix A.3. □

Proposition 1 indicates that in a first-best setting with perfect information about labor productivity and energy efficiency the government has no redistributive reason to distort the labor market. Distributional concerns can be addressed by lump-sum transfers $T^j$ directly targeted at each household $j$. The distributional component $\Phi_j$ in expression (10) is still helpful to illustrate the equity–efficiency trade-offs that the government faces, which we will refer to in the subsequent discussion of second-best policies.

### *3.4. Second-best policies*

In this section, we first discuss the second-best in general, that is, we explain the government's equity–efficiency trade-off when type-specific lump-sum transfers are not possible (Section 3.4.1). Then, we show the implications of restricting the optimal policy instrument mix to a combination of a uniform carbon tax, a uniform lump-sum transfer and a uniform subsidy for clean energy production (Section 3.4.2).

[5] We are aware of the fact that the Pigouvian component is typically associated with capturing the sum of marginal environmental damages. We abstract from environmental damages here and instead introduce a fixed upper bound on total energy use. However, our shadow price $\mu/\gamma$ plays a similar role as the corresponding shadow price in the literature on optimal taxation and environmental externalities.

*3.4.1. Equity–efficiency trade-off*

Lemma 1, and in particular the distributional term $\Phi_j(w^j, \alpha_j)$ in expression (10) allow us to precisely characterize the determinants of the government's choices in the equity–efficiency trade-off that arises from heterogeneity in labor productivity $w^j$ and energy efficiency $\alpha_j$. In the first-best, $\Phi_j = 0$, and all households are taxed at the Pigouvian level. In general, however, $\Phi_j$ could be greater or less than zero, implying that a second-best optimal household-specific carbon tax lies above or below the Pigouvian level in order to take both environmental and distributional considerations into account. Accordingly, the second-best optimal marginal income tax is then proportional to $\Phi_j$, that is, $T^*_{I^j} = \frac{\widehat{\tilde{E}^j_z}}{w^j}\Phi_j$.

Without loss of generality we focus in the following on the case in which $\Phi_j < 0$ and, hence, $t^{j\,*}_E < \frac{\mu}{\gamma}$. The discussion of the case $\Phi_j > 0$ would be analogous. Assuming that energy is a normal good in the sense that $\tilde{E}^j_q < 0$, we have

$$t^{j\,*}_E < \frac{\mu}{\gamma} \quad \Longleftrightarrow \quad \Phi_j < 0 \quad \Longleftrightarrow \quad \frac{\gamma}{\lambda^j} < W^j_v. \tag{12}$$

Above inequality (12) allows us to infer four reasons for the government to tax a household below the Pigouvian rate. We now discuss each of these four reasons assuming all else is equal. First, the optimal individual carbon tax is more likely to be set below the Pigouvian level if the government puts a relatively high marginal welfare weight on household $j$, that is, $W^j_v$ is relatively large. Households whose utility contributes more to social welfare thus have to bear less of the tax burden. The choice of the welfare function is normative in nature. It has, however, no direct vertical or horizontal equity motive but is influenced by both dimensions to the extent that indirect utility is affected along both dimensions: Households with low labor productivity $w_j$ or with low energy efficiency $\alpha_j$ both tend to have, *ceteris paribus*, lower utility levels. Under a prioritarian welfare measure (that is, where $W$ is concave in $v^j$), households with low $w_j$ or low $\alpha_j$ should therefore receive preferential treatment due to lower taxes or higher transfers.

Second, (12) is more likely to hold, the larger the household's marginal utility of income $\lambda^j$ is. Since marginal utility is decreasing in income and labor productivity ($\frac{\partial \lambda^j}{\partial w_j} < 0$), this channel is directly related to vertical inequality: The government should reduce the tax burden on poorer households (with low labor productivity), as they are more efficient in transforming an additional unit of income to utility and, hence, social welfare.

Third, for efficiency reasons, the government may want to shift energy consumption towards the household that generates most utility from a given quantity of energy, that is, the household with the highest $\alpha_j$. It holds that $\frac{\partial \lambda^j}{\partial \alpha_j} > 0$ and thus, the higher $\alpha_j$, the more likely it is that (12) holds and household $j$ is taxed below the Pigouvian level. This channel is therefore related to the horizontal heterogeneity of households.

Fourth, the lower the social marginal value of public funds $\gamma$, the more likely (12) will hold. This channel has again no direct link to vertical or horizontal equity but only indirectly: When there exist strong redistributional motives due to the above mentioned channels, redistributive tax volumes will be high. Because redistributive taxes also cause deadweight losses due to distorted incentives to work, $\gamma$ will also be high. Hence, additional public funds would contribute relatively strongly to social welfare, and the optimal tax is less likely to be below the Pigouvian level.

In Appendix B we show that the discussion on how the optimal tax system is affected when the government is constrained by implementing a uniform carbon tax on all households, that is, $t^j_E = t_E$ for all $j$ is similar to the above reasoning.

The following corollary summarizes the implications of the equity–efficiency trade-off for the second-best optimal system of taxation:

**Proposition 2.** *In setting the optimal carbon tax, the government faces an equity–efficiency trade-off. The equity motive is determined by the welfare function, while the efficiency motive is determined by the curvature of households' indirect utility function with respect to disposable income (measured by $\lambda^j$), their energy-efficiency (measured by $\alpha_j$) and the marginal social value of public funds $\gamma$.*

*If the optimal carbon tax deviates from the Pigouvian level, the government also has to correct the marginal income tax that is then given by*

$$T^*_{I^j} = \frac{\widehat{\tilde{E}^j_z}}{w^j}\Phi_j(w^j, \alpha_j). \tag{13}$$

*Thus, the second-best optimal marginal income tax is sensitive to distributional concerns due to heterogeneous labor productivity $w^j$ (vertical inequality) and due to heterogeneous energy efficiency $\alpha_j$ (horizontal inequality).*

*3.4.2. Subsidies for clean energy production in a uniform system of optimal linear taxation*

In the preceding analysis we have abstracted from the possibility to decarbonize energy production. The only mitigation option to achieve the environmental target $E^*$ was to reduce energy consumption, for which the government used the carbon tax on energy consumption $t_E$. The model's production side, however, can be extended to include a production function for energy that takes clean energy $X$ and fossil-based energy $Z$ as input factors where factor prices $p_X$ and $p_Z$ are fixed. Then, the environmental target consists of keeping the use of fossil resources below a certain threshold, $Z \leq Z^*$.

Now, the government can implement a uniform carbon tax $\tau_Z$ on the use of $Z$ in production, a uniform lump-sum tax $T$ and a uniform subsidy $s_X$ for clean energy to incentivize the decarbonization of energy production.[6] The only additional equations that are

[6] Household-specific carbon taxes as considered in the preceding analysis cannot be modeled under this setup. Instead, we can consider household-specific subsidies $s^j_E$ for energy expenditures that take over the role of energy taxes $t^j_E$. We follow this strategy in our numerical application in Section 4.

added by this extension are the production function $E(X, Z)$, which we assume to satisfy the Inada conditions and to have constant returns to scale, and the competitive energy producer's first order conditions associated with its profit maximization:

$$p_E \tilde{E}_X = p_X - s_X \quad (14)$$

$$p_E \tilde{E}_Z = p_Z + \tau_Z \quad (15)$$

We use the extended analytical model to characterize the role of subsidies for the production of clean energy within a system of uniform taxation:

**Proposition 3.** *Consider the case where clean energy can substitute for fossil inputs in energy production and the government uses only linear and uniform tax policies. In this case, it is, in general, welfare-improving to implement a clean energy subsidy alongside the carbon tax and the uniform lump-sum transfer. The second-best optimal subsidy is*

$$s_X = \frac{1}{\Omega}\left[-\frac{\mu}{\gamma} + \left(\frac{\sum_j W_v^j \lambda^j}{\gamma} - n\right)\frac{1}{Z_b} + \frac{\tilde{E}_Z}{\tilde{E}_X} p_X - p_Z\right] \quad (16)$$

*where we have defined* $\Omega := \frac{X_b}{Z_b} + \frac{\tilde{E}_Z}{\tilde{E}_X}$.

**Proof.** See Appendix A.4. □

The key intuition of this proposition is that a government should also exploit the possibility to change relative prices between clean and fossil energy, to influence overall energy prices when household-specific transfers are not possible. This, in turn, reduces the welfare losses of energy-intensive households which benefit from lower energy prices.

**Corollary 1.** *When the government uses only the linear uniform instruments $\tau_Z, s_X, T$ as described above, and sets the carbon tax to the Pigouvian level, that is, $\tau_Z = \frac{\mu}{\gamma}$, then the second best clean energy subsidy is*

$$s_X = \left(\frac{\sum_j W_v^j \lambda^j}{\gamma} - n\right)\frac{1}{X_b}. \quad (17)$$

**Proof.** See Appendix A.4. □

Corollary 1 shows that the level of the optimal subsidy is determined by the equity–efficiency trade-off discussed in Section 3.4.1. In particular, the clean energy subsidy decreases in the marginal cost of public funds $\gamma$, while it increases in the normative welfare weight $W_v^j$ and in the marginal utility of income $\lambda^j$. In the first-best, individual transfers ensure that the expression in parentheses in (17) vanishes (c.f. the social planner's first order conditions in Appendix A.1). Hence, the subsidy would be zero. Under a uniform lump-sum transfer $T$, however, the expression in parentheses does not vanish in general. The more the second-best deviates from the first-best, the more $\left(\frac{\sum_j W_v^j \lambda^j}{\gamma} - n\right)$ deviates from zero, and hence, also the larger the subsidy becomes in absolute terms. Thus, implementing a subsidy can be a second-best optimal policy option, which may be motivated by equity and/or efficiency reasons.

### 3.5. Extension to two periods with efficiency enhancing investments

Assuming that households' energy efficiency $\alpha_j$ is fixed, limits the model's applicability to the short run. Our conceptual framework can be extended to include the possibility of households to increase their energy efficiency by saving some of their income with the purpose of investing it in a second time period in efficiency enhancing capital.

Consider households who live for two periods $t = 1, 2$ and who are able to save part of their income in order to invest in improving their energy efficiency $\alpha_j$. Examples include investments into building insulation or moving house to locations closer to the work-place or with better access to public transport infrastructure. Final energy services consumed by household $j$ in periods $t = 1$ and $t = 2$ would then be $E_1^j = \alpha_{j,1}\tilde{E}_1^j = f(x_0^j)\tilde{E}_1^j$ and $E_2^j = \alpha_{j,2}\tilde{E}_2^j = f(x_0^j + x^j)\tilde{E}_2^j$, where $x_0^j, x^j > 0$ and $f' > 0 > f''$. In addition to the carbon tax and the lump-sum transfer, the government could implement a subsidy on efficiency-enhancing investments.

However, extending the model to allow for household investments in efficiency-enhancing capital yields no further qualitative insights as compared to the ones obtained from the basic model: For example, Proposition 1 still remains valid, that is, the first-best can be implemented with a uniform Pigouvian carbon tax and household-specific lump-sum transfers. Importantly, horizontal heterogeneity is not removed in a dynamic context since differences in initial endowments $x_0^j$ continue to constitute heterogeneous (discounted) marginal abatement costs within income groups. If the government is restricted to a uniform linear system of optimal taxation, it can be welfare enhancing to allow for a subsidy on efficiency-enhancing investments (for details, see Appendix C).

## 4. Numerical application

To quantify the implications of our analysis for optimal climate policy in the face of achieving a 30% reduction target of carbon-intensive production, we develop a numerical model and calibrate it to German household data. We can run the model in three different ways: a baseline version that best fits the data, a version with optimized policy but without climate target (business-as-usual, BAU), and finally with optimized policies under a climate target.

Even in the absence of a climate target, there is significant room for improvement over the calibrated baseline by optimizing the system of taxation and redistribution. The reason is that even without a climate target, differences in energy efficiency constitute horizontal inequality, which influences social welfare. More importantly, we calculate the socially optimal response of the tax system when effective climate policy is introduced. To characterize this response, we compare the optimal tax and transfer system with and without climate target.

We allow for substitution of fossil energy by clean energy production as described in Section 3.4.2. Otherwise, the climate target can only be achieved by energy demand reduction, implying significant welfare losses and requiring relatively high carbon prices. In order to purely focus on the role of horizontal inequality for optimal policy instruments, we disregard the vertical inequality dimension and optimize the numerical model for median labor productivity.[7] This enables us to quantify the relevant implications of horizontal inequality due to heterogeneous energy efficiencies in a transparent way. Socially optimal policy instruments are then calculated for ten different efficiency deciles $\alpha_j$ within that median income household group. Efficiency decile 1 includes the most energy efficient households ($\alpha_1 = 5.01$) whereas efficiency decile 10 captures the least energy efficient households ($\alpha_{10} = 0.26$).[8]

We further abstract from including household investments in energy efficiency (as described in Section 3.5), because it is less relevant for the considered time period (5–10 years) and because household-specific data on energy efficiency capital and its energy demand effect is lacking. In the following, we will describe the functional forms of the numerical model, the empirical calibration procedure and our results.

### 4.1. Functional forms

#### 4.1.1. Utility and social welfare

We assume a linear expenditure system (LES)[9] with Stone–Geary type utility functions for the households of the form

$$u^j(c^j, 1-l^j, \tilde{E}^j) = \frac{\left( c^{j\,\beta} \left( \alpha_j \tilde{E}^j - \bar{E} \right)^{1-\beta} - \psi \frac{l^{j\,1+\varphi}}{1+\varphi} \right)^{1-\eta}}{1-\eta}. \tag{18}$$

The sub-utility function $\phi(l^j) = \psi \frac{l^{j\,1+\varphi}}{1+\varphi}$ determines households' preferences about labor supply, where $\varphi$ measures the inverse of the Frisch elasticity of labor supply and $\psi$ is the dis-utility of labor cost parameter. Further, $\bar{E}$ denotes a subsistence level of utility-relevant energy consumption and $\alpha_j$ the conversion efficiency for raw energy $\tilde{E}$ to utility-enhancing energy-intensive services such that $\alpha_j \tilde{E}^j = E^j$. The subsistence requirement enables us to model non-constant energy-expenditure shares over different income deciles; moreover, non-homothetic utility functions allow to consider horizontal inequality due to different energy efficiency technologies. Finally, $\eta$ is the elasticity of the marginal utility of comprehensive consumption determining the curvature of the individual household utility function $u$ or private inequality aversion.

We assume a Bergson–Samuelson social welfare function of the form

$$W(u) = \sum_j \frac{u^{j\,1-\zeta}}{1-\zeta}, \tag{19}$$

where $u = (u^1, \ldots, u^n)$ and $\zeta$ is the Atkinson measure for inequality aversion (Atkinson, 1970). While $\zeta$ is a normative welfare parameter capturing governmental inequality aversion in our case, $\eta$ is a descriptive parameter that can in principle be calibrated according to data about households' risk aversion. Kaplow (2010) showed that the combined private and governmental inequality aversion can be expressed as $\epsilon = \eta + (1-\eta)\zeta$. Thus, in the numerical optimization we vary the households' elasticity of the marginal utility of comprehensive consumption $\eta$ and thereby also the combined concavity parameter $\epsilon$ within a range from 0.5 (low inequality aversion) to 2 (high inequality aversion). This is in line with typical ranges for this parameter applied in the literature (Del Campo et al., 2021) and allows us to capture different normative views on social preferences for horizontal equity.[10]

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[7] The household at the 5th income decile has an exogenous wage of 23.80 € per hour corresponding to a median net income (adult-equivalent expenditure) of 18392 € per year.

[8] See Table E.3 for all values.

[9] In principle, more flexible demand systems (like the Almost Ideal Demand System (AIDS)) could also be applied here. The advantages over our LES specification would be that the more flexible functional forms could accommodate the non-linearity of environmental Engel curves better (Jacobs and van den Ploeg, 2019). However, in our numerical analysis, we disregard income differences to focus only on the horizontal dimension. Therefore, we expect an LES formulation to yield only very small errors as income heterogeneity is only a second order effect, resulting from (slightly) different labor responses due to heterogeneous energy efficiency. Moreover, to our knowledge, there is no straightforward way to model horizontal heterogeneity (which is not correlated to income and might even not be fully explained by other observable factors) within the existing demand system approaches. An advantage of the LES approach taken here is that it is consistent with the analytical model presented in Section 3 and makes the analysis more tractable.

[10] Note that every $\epsilon$ can be obtained by varying only $\eta$ while holding $\zeta$ constant equal to zero. We follow this strategy to solve the numerical optimization model efficiently and obtain the results in Section 5. Details on the combined concavity parameter $\epsilon$ for different values of $\eta$ and $\zeta$ can be found in in Table D.1 in Appendix D.

We use equivalent variation (EV) to monetize households' policy costs. Given household j's net budget in the case with climate target $b^j$ and in the business-as-usual case (BAU) without climate target $b^j_{BAU}$, as well as given the gross and net energy prices, $p_E$ and $q^j$ respectively, $EV^j$ is indirectly obtained by the following indifference condition:

$$v^j(b^j_{BAU} + EV^j, z^j_{BAU}, p_E) = v^j(b^j, z^j, q^j). \tag{20}$$

The aggregate social welfare loss is then given by the sum of the individual household's $EV^j$ weighted by the marginal utility of income $v^j_{b^j}$ and the household's welfare weight $W_{v^j}$ (see for example Fankhauser et al., 1997). In order to calculate the monetized social welfare loss $M_W$ we then divide the aggregate social welfare loss by the product of the welfare weight and the marginal utility of income of the average household for the case without climate target $W_{\bar{v}}\bar{v}_{\bar{b}}$, that is,

$$M_W = \frac{\sum_j W_{v^j} v^j_{b^j} EV^j}{W_{\bar{v}}\bar{v}_{\bar{b}}}. \tag{21}$$

*4.1.2. Energy demand*

Let $b^j = c^j + q^j \tilde{E}^j$ be total expenditures and $m^j_E = \frac{q^j \tilde{E}^j}{b^j}$ the energy expenditure share. With $\frac{\partial u^j}{\partial E^j}\alpha_j = q^j \frac{\partial u^j}{\partial c^j}$ from the first-order conditions of the household optimization problem (Equation (4)–(5)), we obtain for the energy expenditure share:

$$m^j_E = 1 - \beta + \frac{\beta q^j \bar{E}}{\alpha_j b^j}. \tag{22}$$

Hence, for a homothetic utility function (with $\bar{E} = 0$), energy expenditure shares are constant and equal to $1 - \beta$. Because of subsistence energy consumption $\bar{E}$, energy expenditure shares decrease with rising total expenditure. Further, if energy conversion efficiency $\alpha_j$ is high, energy expenditure share is lower. Importantly, there is no horizontal heterogeneity in energy expenditure shares if preferences are homothetic and $\bar{E} = 0$.

*4.1.3. Energy production*

For the energy production sector, we consider a CES function with constant elasticity of substitution $\sigma \in \mathbb{R}^+_0$, share parameter $a \in (0, 1)$ and scaling parameter $A$:

$$E(X, Z) = A\left(aX^\rho + (1-a)Z^\rho\right)^{\frac{1}{\rho}}, \text{ where } \rho = \frac{\sigma - 1}{\sigma}.$$

The firms in the energy production sector sell raw energy $\tilde{E}$ at price $p_E$. They choose clean and fossil energy inputs $X$ and $Z$ at prices $p_X$ and $p_Z$, which the government may subsidize or tax. Firms maximize profits

$$\pi_E = p_E \tilde{E}(X, Z) - (p_X - s_X)X - (p_Z + \tau_Z)Z,$$

which yields the following first order conditions:

$$p_X - s_X = p_E A\left(aX^\rho + (1-a)Z^\rho\right)^{\frac{1-\rho}{\rho}} aX^{\rho-1}$$
$$p_Z + \tau_Z = p_E A\left(aX^\rho + (1-a)Z^\rho\right)^{\frac{1-\rho}{\rho}} (1-a)Z^{\rho-1}.$$

*4.1.4. Labor supply and labor income tax function*

Using Equation (8) we can derive optimal labor supply, which increases in the level of labor productivity $w^j$ and energy efficiency $\alpha_j$ while it decreases in the marginal labor income tax $T^j_I$.

$$l^{j*} = {}^1\!/\!_\psi\left(w^j(1 - T^j_I)\beta c^{j\,\beta-1}(\alpha_j \tilde{E}^j - \bar{E})^{1-\beta}\right)^{1/\varphi} \tag{23}$$

We assume a simplified linear labor income tax function of the form

$$T(I^j) = \max(0, T^j_I I^j - R^j), \tag{24}$$

where $T^j_I$ denotes the marginal income tax and $R^j$ is a lump-sum transfer (positive or negative) for household $j$. In our case this simple linear functional form reflects the difference between households' gross incomes and expenditures for the case without climate policy and optimized labor income tax (see Section 4.2) and thus not only includes income taxes but also social security contributions, transfers and other deductibles.

*4.1.5. Budget of the government*

The governmental budget constraint reads

$$\tau_Z Z + \sum_j T(I^j) = s_X X + \sum_j s^j_E \tilde{E}^j + G. \tag{25}$$

The revenue that the government raises through optimal carbon taxation $\tau_Z Z$ and labor income taxation $\sum_j T(I^j)$ is redistributed back to households via a combination of policy instruments. In the first-best the government uses only lump-sum transfers $R^j$, while possible second-best revenue recycling options include clean energy subsidies $s_X X$ and energy consumption subsidies $\sum_j s^j_E \tilde{E}^j$. $G$ is an exogenous revenue requirement determined by the labor income tax revenue for the case without climate policy and tax optimization. Thus, the budget equation makes sure that first- and second-best policies are revenue-neutral.

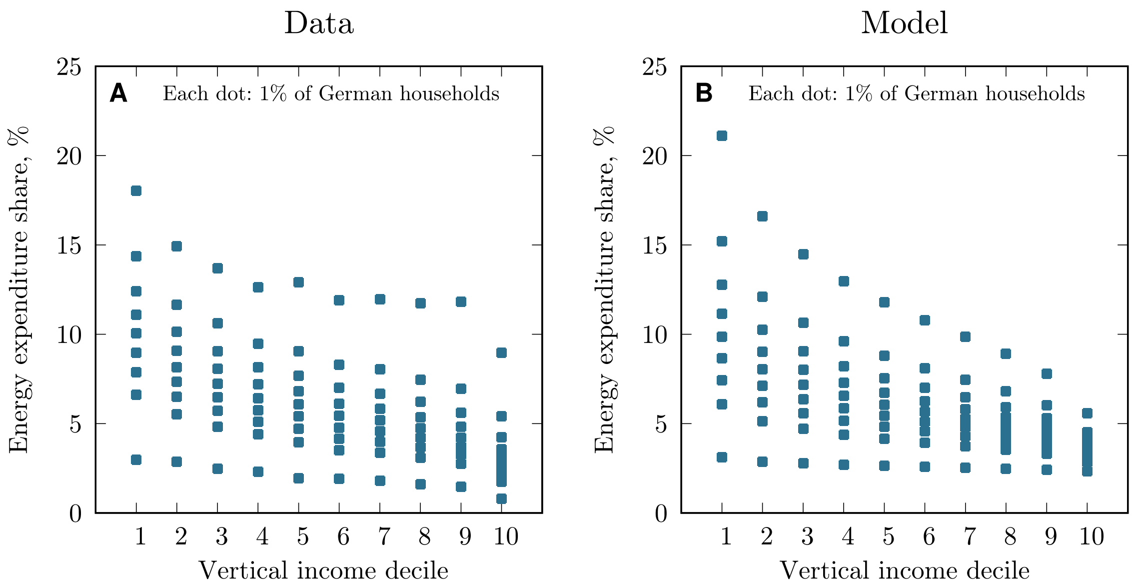


**Fig. 1. Heterogeneity of households.** Panel A: mean expenditure shares in 10 × 10 grid based on EVS 2018 data. Panel B: mean expenditure shares from estimated model.

### *4.2. Model calibration and estimation*

We determine the structural parameters of the utility function based on official German household data that includes information on income, expenditures and labor supply (EVS, 2018). We split households in 10 income deciles, based on both gross incomes and adult-equivalent household expenditures. The latter is a better proxy for permanent-income than annual income, which changes strongly over the life-cycle of an individual.

We set the Frisch elasticity of labor supply to 0.648 (van der Ploeg et al., 2022) and calibrate the dis-utility of labor cost parameter $\psi$ such as to match average per-household labor supply of 33.5 working hours per week in Germany in the absence of (additional) climate policy. Using OLS we then estimate Eq. (24) to determine the baseline labor income tax system in Germany in the absence of optimal tax and climate policy. For simplicity we assume that in the baseline $T(I)$ captures the total difference between households' gross incomes and expenditures. We estimate a uniform marginal transfer of $T_I = 0.5441$ and an uniform absolute lump-sum transfer of $R = 4102.10$ Euro ($R^2 = 0.99$, F-statistic: 7070, $p$-value $< 0.001$).[11] These numbers include not only explicit income taxes but also other social security contribution, transfers and deductibles. In order to specify the exogenous distribution of labor productivity that is in line with the income and expenditure data, we run the baseline numerical model with gross incomes and the estimated labor income tax function (see Table E.4).

For each level of exogenous labor productivity that captures a vertical income decile, we then calculate ten energy expenditure deciles that reflect horizontal inequality in energy efficiency. This creates a grid of 10 × 10 household types that differ in the two dimensions: labor productivity (income) and energy efficiency. To depend less on data outliers, we consider the median income and energy expenditure share within each decile as the value for the specific grid-cell household. The left panel in Fig. 1 shows this heterogeneity for German household data over different income deciles. Although the later analysis will only consider the different energy-efficiency deciles within the median income decile, the full grid is used to estimate the demand function.

We estimate the parameters $\alpha_j$ and $\beta$ in the demand equation (22) with a linear regression using energy expenditure decile dummies that are interacted with household's expenditure $b^j$. Hence, we obtain for each energy-efficiency type the coefficient $\frac{\beta q^j \bar{E}}{\alpha_j}$. For calculating $\alpha_j$, we further need to impose values on $q^j$ and $\bar{E}$. Because the $\alpha_j$ scale with $\bar{E}$, there is an additional degree of freedom and we can set $\bar{E} = 1$ without loss of generality of the model.[12] From the environmental accounting data (Destatis, 2020) and energy price data, we calculated an average energy price (weighted by consumption shares) of $\bar{q} = 462$ Euro/$tCO_2$ (see Table E.2). This allows us to calculate the different energy efficiency deciles (see Table E.3). The common demand parameter is

[11] We are aware of the fact that the German income tax system uses different marginal income tax rates for different tax brackets. In our estimation of $T$ non-linear functional forms did not increase explanatory power and we thus stick to the most simple representation.

[12] To see this, define $\hat{\alpha}_j := \frac{\alpha_j}{\bar{E}}$ and $\hat{\psi} := \psi \bar{E}^{\beta-1}$. Considering the utility function (18), we get $u^j(c^j, E^j, l^j) = u^j(c^j, \alpha_j \tilde{E}^j, l^j) = \frac{\left(c^{j\,\beta}(\alpha_j \tilde{E}^j - \bar{E})^{1-\beta} - \psi \frac{l^{j\,1+\varphi}}{1+\varphi}\right)^{1-\eta}}{1-\eta} = \frac{\left(c^{j\,\beta}(\hat{\alpha}_j \tilde{E}^j - 1)^{1-\beta} - \hat{\psi} \frac{l^{j\,1+\varphi}}{1+\varphi}\right)^{1-\eta}}{1-\eta} \bar{E}^{(1-\beta)(1-\eta)}$. Hence, changes in the value of $\bar{E}$ will only scale the vector $u \in \mathbb{R}^n$ of all households' utility levels by the factor $\bar{E}^{(1-\beta)(1-\eta)}$, given an appropriate recalibration of the parameter $\psi$ to $\hat{\psi}$.

**Table 1**
Calibration of the numerical model.

| Model component | Parameter | Value | Source |
| --- | --- | --- | --- |
| Frisch elasticity of labor supply | $\varphi$ | 0.648 | van der Ploeg et al. (2022) |
| Dis-utility of labor [€/dis-utility of work] | $\psi$ | $1.30313 \times 10^{-5}$ | Calibrated |
| Baseline marginal income tax [%] | $T_I$ | 54 | Estimated, EVS 2018 |
| Baseline lump-sum transfer [€/year] | $R$ | 4102 | Estimated, EVS 2018 |
| Labor productivity [€/h] | $w^j$ | Tab. E.4 | Calculated, EVS 2018 |
| Subsistence energy consumption [€/year] | $\bar{E}$ | 1 | Assumption w.l.o.g. |
| Average energy price [€/tonne $CO_2$] | $\bar{q}$ | 462 | EVS 2018, Destatis 2020 |
| Marginal budget share consumption | $\beta$ | 0.9786 | Estimated, EVS 2018 |
| Substitution elasticity energy production | $\sigma$ | 4 | Calibrated, Destatis 2020 |
| Baseline fossil energy price [€/tonne $CO_2$] | $p_Z$ | 462 | Calibrated, Destatis 2020 |
| Baseline clean energy price [€/tonne $CO_2$] | $p_X$ | 720 | Calibrated, Destatis 2020 |
| Energy efficiency | $\alpha^j$ | Appendix E.3 | Estimated, EVS 2018 |

further estimated to be $\beta = 0.9786$. Note that an unconstrained OLS regression gives a negative estimate for $\alpha^1$ (that is, for the highest energy efficiency type), which would be inconsistent with our model assumptions. We therefore constrain that energy efficiency of the highest efficiency type such that it is twice the second-highest efficiency type, $\alpha^1 = 2\alpha^2$. The estimation of the demand function can explain very well the heterogeneity of energy consumption across income and energy efficiency types, as shown in the right panel in Fig. 1.

For calibrating the energy production sector, we set the elasticity of substitution between fossil and carbon-free energy to $\sigma = 4$.[13] In order to match the share of clean energy production in total primary energy production of 21 percent in 2018 (Destatis, 2020), the clean energy price $p_X$ needs to be set 56 percent higher than the price of fossil energy. To allow for a straight-forward model comparison between a model with and without the mitigation sector with equal final energy prices before climate policy, we set, without loss of generality, $p_Z = p_E = 462$, implying $A = 2.33$. The carbon price needed to reduce carbon emissions by 30 percent is in our model 237 €/t$CO_2$ in the first-best case, which is an average price level compared to the range of 95–410 €/t$CO_2$ that integrated assessment models calculate for a comparable emission target.[14] Table 1 summarizes the empirical calibration of the numerical model.

### 4.3. Policies

For the numerical analysis, we calculate different policies that differ in their welfare implications and their informational requirements.

*Uniform carbon tax and household-specific lump-sum transfers.* This is the first-best policy formalized in Proposition 1, which sets the benchmark for an outcome that maximizes social welfare. It requires the government to be able to fully observe the energy efficiency type of each household.

*Uniform carbon tax, uniform lump-sum transfer and household-specific marginal labor income tax.* As emphasized in Proposition 1, the marginal labor income tax is zero in the first-best setting. If, however, household-specific lump-sum transfers are not possible, differentiated marginal labor income tax rates can target horizontal inequality and enhance social welfare. Such a policy ensures that households face differentiated incentives to work. An example for such tax differentiation constitutes the German commuting allowance that allows to deduct commuting costs from the taxable income. This implies that households with *ceteris paribus* large commuting distances pay effectively lower marginal income taxes compared to households with lower commuting distances.

*Uniform linear taxation and household-specific energy subsidies.* The policy consists of a uniform carbon tax, a uniform marginal labor income tax, a uniform lump-sum transfer and household-specific energy subsidies. As emphasized in Equation (10), carbon taxes that are differentiated by household type, can be welfare-optimal if household-specific transfers are not possible. Because of the additional mitigation sector in the numerical application, we cannot differentiate carbon prices by households but rather implement household-specific energy subsidies (or taxes). Such a policy ensures that households face differentiated incentives to reduce energy demand (while the carbon tax works entirely in the energy production sector by substituting fossil with clean energy). An example for this type of tax differentiation constitutes the mobility subsidy that is currently discussed in Germany (Held et al., 2021). Contrary to

[13] Based on global input–output data, Papageorgiou et al. (2017) estimate an elasticity of substitution between clean and dirty inputs of approximately 2. As this estimate is based on past production data, it disregards the large role of sector-coupling between the electricity sector and the transport and heating sector, which will play an important role in the future. Moreover, from a physical point of view, energy production technologies are close to perfect substitutes.

[14] Pietzcker et al. (2021b) calculate that carbon prices in the EU-ETS for the European Green Deal emission targets should be around 130 €/t$CO_2$ in 2030; Pietzcker et al. (2021a), using a wide range of models and assumptions on EU-ETS targets and non-EU-ETS targets that achieve aggregate EU climate targets, calculate carbon prices between 95–210 €/t$CO_2$ in the EU-ETS sectors and 210–405 €/t$CO_2$ in non-EU-ETS sectors.

the commuting allowance, the mobility subsidy constitutes a subsidy that depends only on commuting distance and is independent from income.[15] Households with higher commuting distances would therefore face effectively lower energy prices than households with lower commuting distances.

The previous two approaches rely on perfect observability of energy efficiency types (or the absence of any self-selection constraints). If energy efficiency type is not observable or self-selection constraints become relevant, non-linear policies – that is, marginal energy tax rates that increase or decrease in energy consumption – constitute potential alternatives. Such a non-linear tax rule could come close to – but never outperform – a policy with optimally differentiated taxes. For example, a non-linear energy tax could never achieve higher social welfare than an energy tax that is differentiated by (observable) household types since the latter is a more general case of the former.[16] Hence, policies with differentiated taxes also constitute upper bounds regarding the welfare-effects of non-linear tax schemes that do not rely on observable household types. Besides their practical relevance in the German tax system, the welfare effects of differentiated taxes hence constitute a valuable theoretical benchmark for potential non-linear energy taxes.[17]

Finally, we consider a set of linear policies or taxes/subsidies that have rather low institutional and informational requirements as the household type does not need to be observable:

*Uniform linear taxation and subsidies on clean energy production.* The policy consists of a uniform carbon tax, a uniform marginal labor income tax, a uniform lump-sum transfer and a uniform subsidy on clean energy. It thus relies only on linear taxes and has therefore low informational requirements for the government.

*Uniform linear taxation.* The policy consists of a uniform carbon tax, a uniform marginal labor income tax and a uniform lump-sum transfer. It is the simplest revenue-neutral policy in our setting that achieves the given climate target.

### 4.4. Numerical solution technique

We numerically calculate the optimal tax and transfer policies that maximize social welfare, considering the first order conditions determining energy demand, energy production and labor supply. The model is written in the AMPL programming language and solved with the Knitro optimization solver (version 12.4). The modelling data and code is available in the follwing repository: https://www.openicpsr.org/openicpsr/project/179981/version/V1/view.

## 5. Numerical optimization results

In the following, we summarize the results of the numerical optimization of climate policy instruments that achieve a 30% reduction in aggregate carbon emissions. We will first present results for the first-best optimum (uniform carbon taxation and household-specific lump-sum transfers) and then move to purely linear policies with lowest informational requirements.[18] In Sections 5.3 and 5.4 we compare the full set of policies introduced in Section 4.3 with respect to their welfare and labor market effects.

### 5.1. First-best: Household-specific lump-sum transfers

As a first step we consider the first-best optimal system of taxation and redistribution that includes an optimized household-specific labor income tax function and a uniform Pigouvian carbon tax of $\tau_Z^* = 237$ €/t$CO_2$. The optimization is summarized in Fig. 2 and confirms the results formalized in Proposition 1: While the marginal income tax $T_I^j$ is zero across households, horizontal inequality due to heterogeneous energy efficiencies is taken into account by household-specific lump-sum transfers $R^j$.

Panel Fig. 2A shows the transfer payments without climate target (BAU = business as usual), as also in this case differentiated lump-sum transfers are used as an optimal instrument to target horizontal inequality. Note that lump-sum transfers are negative to ensure that the policy is revenue neutral and meets the governmental revenue requirement, without affecting incentives to work.

For the range of (horizontal) inequality aversion considered, panel Fig. 2A implies that even for low inequality aversion (yellow line, $\epsilon = 0.5$) it is socially optimal that less energy-efficient households transfer less of their gross labor income to the government as compared to more energy efficient households. This effect is even more pronounced the higher $\epsilon$, that is, the higher the social preference to care about equity for a given level of private household inequality aversion. Thus, for plausible values of overall inequality aversion, the equity motive dominates the efficiency motive within the equity–efficiency trade-off in allocating transfers to households discussed in Section 3.4.1.

Panel Fig. 2B isolates the additional ($\Delta$) transfer that is socially optimal when the climate target is introduced. It corresponds to the difference in optimal lump-sum transfers between the case with and without climate target and can be interpreted as a

[15] Therefore the mobility subsidy has also different distributional effects as the commuting allowance implies higher tax savings for households with larger marginal tax rates.

[16] Formally, any non-linear energy tax schedule could be reproduced by appropriately chosen household-specific energy taxes.

[17] For the sake of space, we did not include the welfare analysis of non-linear energy taxes in our numerical analysis.

[18] Because of limited space, we do not discuss policies with differentiated taxes in detail in the main text, but offer an extended analysis in Appendices E.1 and E.2.

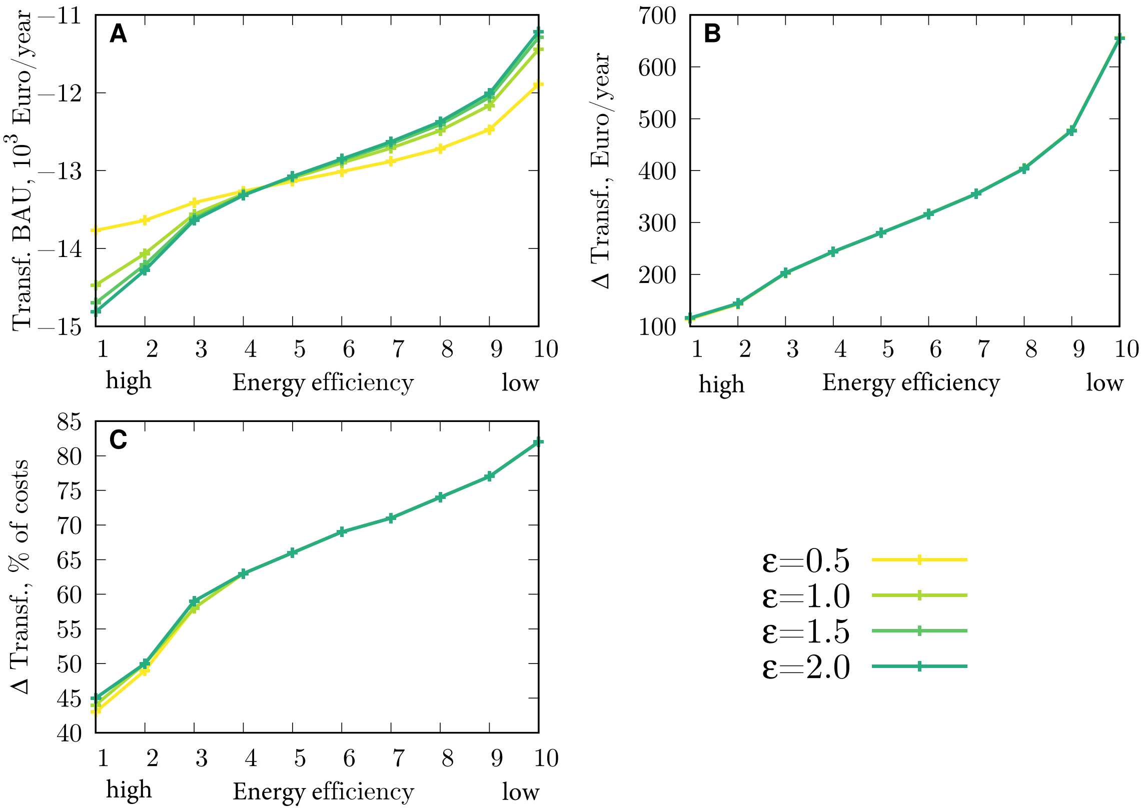


**Fig. 2. Optimal household-specific lump-sum transfers** for the median vertical income decile, all energy-efficiency deciles (1 = highest, 10 = lowest) and inequality aversion $\epsilon = [0.5; 2]$. **A** shows transfers without climate target (BAU-Business as Usual), **B** depicts the additional $\Delta$ transfer when the optimal carbon tax $\tau_Z^* = 237$ €/t$CO_2$ is set to achieve a 30% reduction in fossil energy production and **C** expresses this change as a percentage of households' policy costs.

household-specific climate dividend that accounts for the effect of climate policy on horizontal inequality. Panel Fig. 2C expresses this climate dividend as a percentage of individual households' climate policy costs measured by the respective equivalent variation.

While the lump-sum transfers that target horizontal inequality in the absence of a climate target vary with inequality aversion $\epsilon$, the differentiated climate dividends and covered policy costs are almost independent of $\epsilon$. For the most (least) energy-efficient households the climate dividend amounts to 116 € (655 €) per year covering 45% (82%) of households' policy costs. Thus, irrespective of the societal preferences about increasing horizontal inequality resulting from ambitious climate policy, the climate dividend for the least energy-efficient households should be 5–6 times larger then the dividend for the most efficient household.

### *5.2. Second-best: Uniform linear taxation and subsidies on clean energy production*

The first-best policy serves as the benchmark for the welfare analysis. In the following we provide results for a linear (uniform) policy package, that does not suffer from the high informational requirements of type-specific taxes. The most natural linear policy package consists of a uniform carbon tax combined with an optimized labor income tax system, that is, a uniform lump-sum transfer and a uniform marginal labor income tax. The following section shows that in this case it is generally welfare-enhancing to additionally subsidize the production of clean energy and thereby alleviate the carbon tax burden for energy-intensive households.

Combining uniform carbon taxation with an optimized uniform system of income taxation and subsidies on clean energy production results in an increase of mitigation welfare costs of only 1% (5%) for $\epsilon = 0.5$ ($\epsilon = 2$) as compared to the first-best case. For different degrees of inequality aversion, Fig. 3 disentangles the resulting optimal system of taxation and redistribution: Panel Fig. 3A depicts how the rates of carbon- and marginal labor income tax change due to the introduction of the climate target. Panel Fig. 3B shows how much the two taxes contribute to the additional tax revenue. Panels Fig. 3C and Fig. 3D reveal how that additional revenue should be recycled through uniform lump-sum transfers and subsidies on clean energy production.

The additional ($\Delta$) revenue that the government generates when facing a climate target should predominately come from carbon taxation. With increasing inequality aversion the uniform optimal carbon tax decreases from 233 to 217 €/t$CO_2$, accounting for 97%–88% of total $\Delta$ revenue. The remaining revenue is optimally generated through marginal labor income taxation increasing from 0.02 percentage points for $\epsilon = 0.5$ to 0.09 percentage points for $\epsilon = 2$. Note, that the range of the optimal marginal labor income tax in the BAU scenario without climate target is only 0.06% to 0.32% (not in the figure) reflecting that the numerical analysis

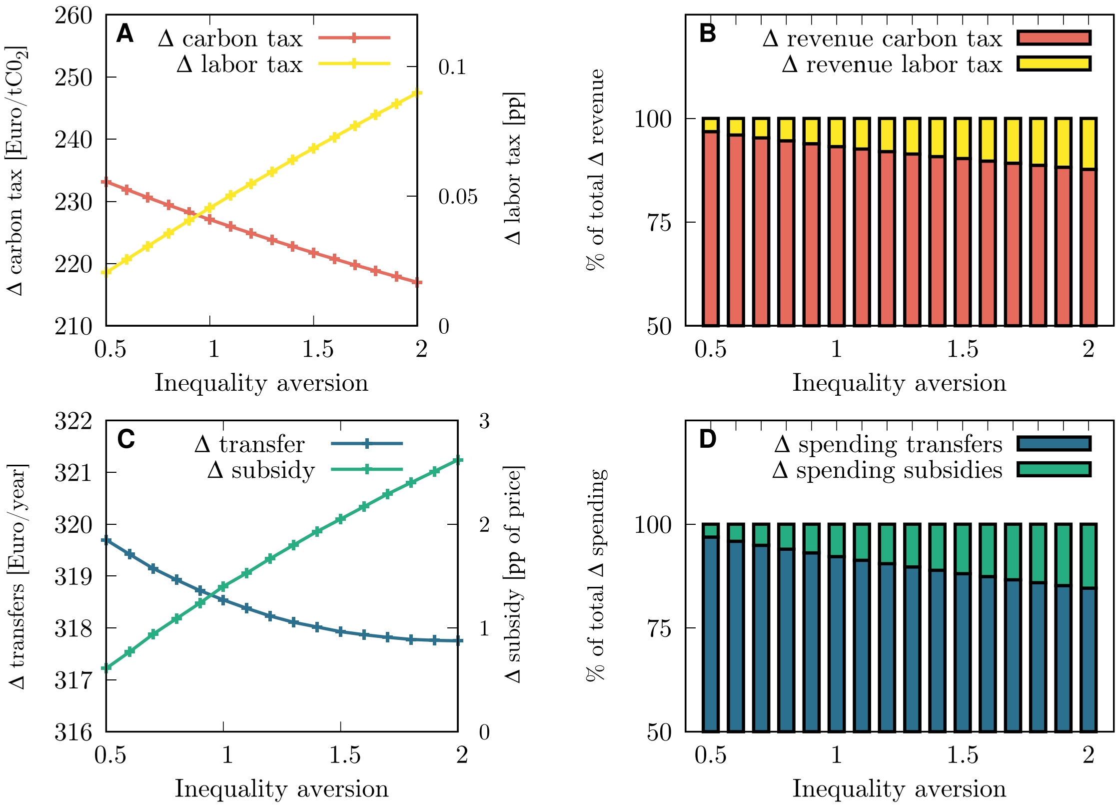


**Fig. 3. Optimal linear taxation and subsidies on clean energy production** for the median vertical income decile and different levels of inequality aversion $\epsilon = [0.5; 2]$. Panel **A** shows the additional ($\Delta$) optimal carbon- and marginal labor income tax due to introducing a climate target that achieves a 30% reduction in total carbon emissions, while **B** shows, in relative terms, how much of the additional tax revenue comes from the carbon tax and from the income tax. Panel **C** shows the additional ($\Delta$) optimal lump-sum transfer and uniform subsidy on clean energy, while **D** depicts the resulting optimal allocation of governmental spending.

disregards heterogeneity in labor productivity. The welfare optimal tax system thus consists of minimal distorting marginal labor income tax rates that increase slightly when a climate target is introduced.

The additional tax revenue generated in the presence of a climate target should be predominately rebated via lump-sum transfers. With increasing inequality aversion the uniform transfer slightly decreases from 320 to 318 €/year accounting for 97%–85% of total governmental spending. The remaining tax revenue is used to subsidize the production of clean energy. The subsidy increases from 0.6 percentage points of the clean energy price $p_X$ for $\epsilon = 0.5$ to 2.6 percentage points for $\epsilon = 2$. The higher the inequality aversion, the higher the resulting share of tax revenue allocated to subsidies on clean energy production reaching 15% for $\epsilon = 2$.

### *5.3. Labor market effects for different policies*

In this section we analyze the effects on endogenous labor supply due to the introduction of the climate target. Since in the numerical model we focus on horizontal inequality within the median income decile, we neglect the effects of endogenous labor supply on vertical income inequality. The following Fig. 4 thus shows how optimal yearly working hours respond to the introduction of a climate target for different policies and when energy efficiency is either low (Fig. 4A), average (Fig. 4B) or high (Fig. 4C).

Without a climate target, welfare-optimal policies already exploit the significant potential to lower marginal labor income taxation relative to the status quo in Germany to which the baseline model is calibrated. This already boosts endogenous labor supply by between 60% and 66% for all policies and energy-efficiency deciles. Fig. 4 shows that the additional introduction of a climate target affects labor supply by only between 0.6% and 1.4% for all policies and energy-efficiency deciles, that is, climate policy has only modest quantitative effects on labor supply. Nevertheless, we find interesting qualitative differences across policies and efficiency deciles. Equation (23) shows that optimal labor supply increases in households' energy efficiency, energy- and numeraire consumption, while it decreases in the marginal labor income tax. Panels Fig. 4A and Fig. 4B reflect that the negative effect of low to average energy efficiency quantitatively dominates the positive effect from lower household-specific marginal labor income taxes and transfers that increase energy and numeraire consumption. As a result household specific lump-sum transfers minimize negative effects on labor supply while household-specific marginal labor income taxes result in the highest drop in labor supply. For the most energy efficient households, however, negative effects on labor supply are lowest for household-specific energy subsidies and highest for a system of uniform taxation and redistribution (Fig. 4C).

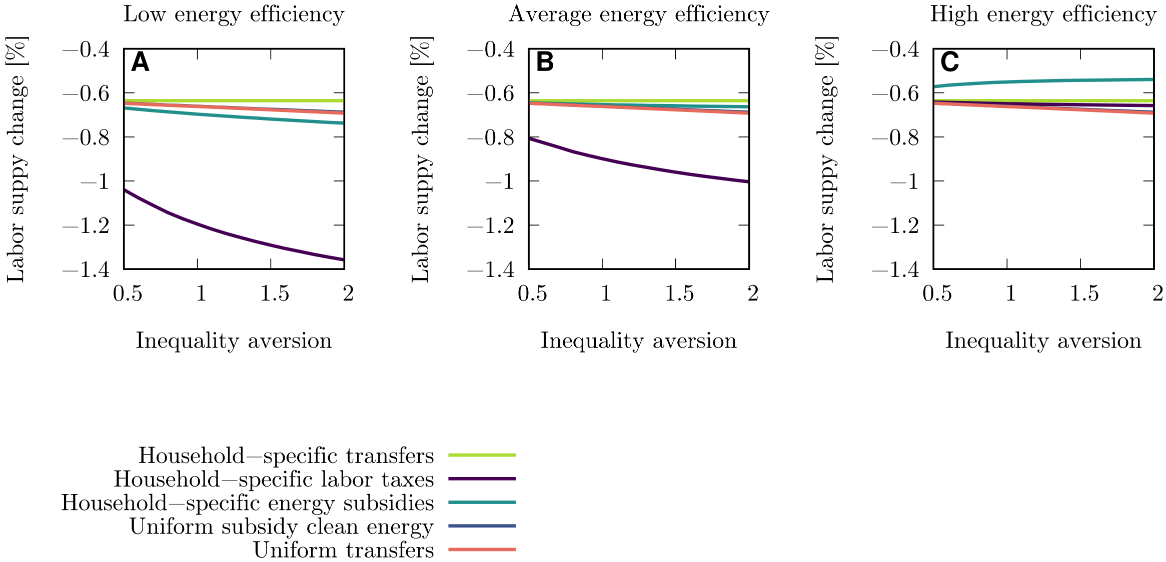


**Fig. 4. Labor supply changes, relative to the case without climate target.** Results for the median vertical income decile and efficiency decile 10 (Panel **A**: low energy efficiency), efficiency decile 5 (Panel **B**: average energy efficiency) and efficiency decile 1 (Panel **C**: high energy efficiency).

### 5.4. Mitigation welfare costs for different policies

In a last step we now compare the suggested policy instruments in terms of their implied impact on monetized aggregate social welfare by calculating the increase in mitigation welfare costs relative to the first-best optimum (see also Fig. 5).

The first-best solution to tackle horizontal inequality can be achieved by household-specific redistribution of the additional revenue generated from climate policy via lump-sum transfers. In this case the social welfare impact due to the introduction of a climate target can be minimized at an average of −0.4% of total household net income across the considered range of inequality aversion. If, in turn, the optimal system of taxation is uniform across households, mitigation welfare costs increase by 2%–12% depending on the level of inequality aversion.

Between these two options the government has a number of policy instrument packages at its disposal resulting in lower mitigation welfare cost as compared to a uniform system of taxation. Household-specific marginal labor income tax rates – which are conceptually similar to the existing German commuting allowance – lead to an increase in mitigation welfare cost of only between 0.9% and 1.1% as compared to the first-best case. Differentiated energy consumption subsidies – which correspond conceptually to a mobility subsidy based on commuting distance – result in slightly higher increases in welfare cost between 0.9% and 2.3%.

In case the government cannot implement any of the preceding household-specific policy packages, it can still dampen the adverse effects of carbon taxation on hardship households by implementing subsidies on clean energy production. This will help to reduce the carbon-intensive lifestyle of households by accelerating the transformation towards a carbon-free energy system. In this case the increase in mitigation welfare costs relative to the first-best case amounts to 1.1% (4.6%) for $\epsilon = 0.5$ ($\epsilon = 2$).

## 6. Conclusions

In this paper we have developed a welfare-theoretic model of optimal taxation and redistribution when households face ambitious climate policy and differ not only in labor productivity, but also in their endowments with energy efficient capital used to convert energy to well-being. This is, for example, the case for commuting households or households living in badly insulated homes or households living in areas with poor access to public transportation infrastructure (like rural areas). While these conditions can change in the very long-run due to investments, they are rather inflexible in the short to medium-run. Our approach allows to derive optimal climate and tax policy in a first-best setting (with perfect information by governments) and various second-best settings (with imperfect information by governments) that take into account the heterogeneity in energy efficiencies — and the associated horizontal distributional effects of climate policy.

The key findings can be summarized as follows. Within a standard social welfare framework, horizontal inequality effects are already accounted for. Nevertheless, it is not always socially optimal to reduce horizontal inequality due to the equity–efficiency trade off. If the ability of households to convert scarce resources into well-being outweighs normative welfare weights related to horizontal inequality, energy-efficient households receive larger transfers. Our numerical application of the welfare-theoretic model shows, however, that under plausible assumptions on inequality aversion, the equity motive dominates the efficiency motive and carbon-intensive households should receive larger redistributive resources to combat horizontal inequality.

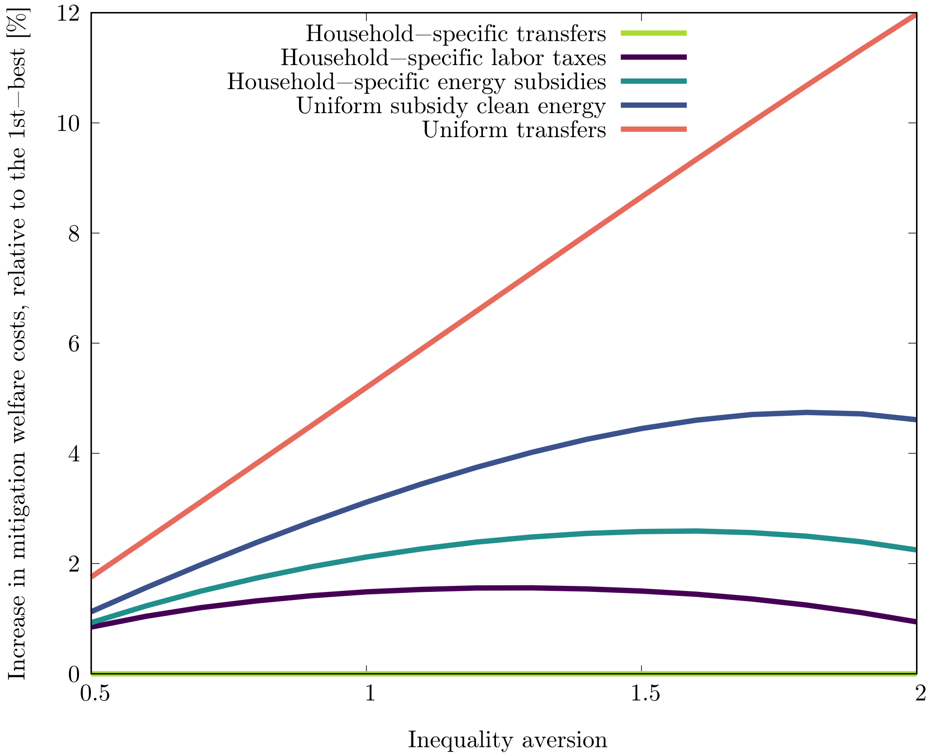


**Fig. 5. Increase in mitigation welfare costs.** Percentage change relative to the first-best case (household-specific lump-sum transfers and uniform carbon tax).

When the government can observe the energy efficiency type, the first-best policy that maximizes social welfare and optimally addresses the equity–efficiency trade-off is a uniform carbon tax which is combined with household-specific lump-sum payments. Setting differentiated tax rates on labor income or energy use – as partly implemented in current tax systems – creates additional welfare losses compared to the first-best. Such policies also provide an upper bound welfare benchmark for non-linear policies where tax rates change with income or energy consumption.[19] Non-linear energy subsidies require, however, a fraudproof monitoring of personalized energy use. This may be easy for grid-based energy consumption (electricity and natural gas), but might also involve substantial transaction costs for monitoring gasoline, fuel or heating oil consumption.

Compared to these approaches, linear policies – that is, uniform taxes or subsidies on factors – have the lowest informational and administrative challenges. As they are not well targeted to address household heterogeneity, they also perform worse in terms of social welfare. Recycling all revenues from carbon pricing back to households on an equal-per capita base would be the most straightforward approach in this setting. Using some revenues from carbon pricing to subsidize clean energy, however, increases welfare further because energy-intensive households benefit from cheaper (and cleaner) energy. Horizontal equity concerns may therefore constitute a new second-best rationale for clean energy policies, besides technological innovation issues (Fischer and Newell, 2008; Kalkuhl et al., 2012; Lehmann, 2012). In our calibrated numerical model, we show that the majority of carbon pricing revenues should still be transferred back to households on an equal per capita base.

With our numerical analysis, we can quantify the trade-offs between informationally demanding policies vs. simpler linear policy mixes. When social inequality aversion is small, the social-welfare adjusted mitigation costs of achieving a 30% reduction in carbon-intensive energy production increase only marginally, by less than 2 percent. When inequality aversion is large, linear policies increase the costs of reducing emissions by more than 10 percent. It can therefore be valuable to identify targeted transfers wherever this is possible at low administrative and incentive costs. In our numerical analysis, optimal targeted lump-sum transfers from introducing a carbon price of 237 EUR/t$CO_2$ are five to six times higher (654–655 €/year) for energy-intensive households compared to energy-efficient households (114–116 €/year). Examples for observable energy efficiency types are energy certificates of buildings or commuting distances — but targeted transfers should be designed to avoid perverse incentives that prevent the adoption of better technologies. This could be done by linking transfers to conditions at a specific closing date or by phasing out transfers over time.

[19] Such policies satisfy self-selection constraints as households do not have incentives to pretend to be another household type.

These insights put existing climate policy packages also into a new light. The introduction of a national carbon price in Germany in 2021 was, for example, combined with reductions in energy prices, large subsidies for carbon-saving and low-carbon technologies (like electric vehicles, heat pumps, building insulation) and the introduction of a temporary long-distance commuting allowance which can be deduced from the income tax (Edenhofer et al., 2020). From a horizontal equity point of view, subsidy programs – consuming three quarters of the carbon price revenues – seem to be exaggerated. Other programs like the temporary long-distance commuting allowance, in turn, could conceptually be understood as a targeted transfer to particularly energy-intensive households.

Our unique contributions are twofold. First, we show how horizontal inequality due to heterogeneous households' energy efficiencies can be integrated into a welfare-theoretic optimal taxation model. Second, we discuss what this implies for the design of optimal climate policy. While our theoretic model provides a consistent approach to incorporating both vertical and horizontal distributional effects, we focused in the numerical analysis on horizontal inequality only. Therefore, we shed light on this previously neglected dimension of inequality in the most transparent way. In addition, vertical equity effects might conceptually be easier to tackle, because vertical heterogeneity is – by definition – correlated to an observable variable (income). Hence, vertical effects can be addressed by a modified income tax system in a rather direct way as shown in our theoretic model and other contributions, such as Jacobs and van den Ploeg (2019). Nevertheless, extending also our numerical model to combine vertical and horizontal distributional effects, would be an interesting avenue for future research and a natural next step to complement our understanding of optimal policies with different dimensions of inequality. In such a setting, carbon pricing will then not only impact the distribution of costs within income groups but also – through non-linear Engel curves – across income groups. Ultimately, quantitatively disentangling vertical *and* horizontal equity effects can lay the foundation for a consistent and rational debate about fair climate policy and the just transition.

## Appendix A. Proofs for Section 3

### A.1. Social planner problem

The social planner's Lagrangian is

$$L^{SP} = W(u^1, \dots, u^n) + \mu(E^* - \sum_j \tilde{E}^j) + \gamma(\sum_j w^j(1 - z^j) - c^j - p_E \tilde{E}^j) \tag{26}$$

and the first order conditions are

$$0 = W_u^j u_E^j \alpha_j - \mu - \gamma p_E \quad \forall j \tag{27}$$

$$0 = W_u^j u_c^j - \gamma \quad \forall j \tag{28}$$

$$0 = W_u^j u_z^j - \gamma w^j. \tag{29}$$

The social planner chooses an allocation that balances households' welfare weights $W_u^j$, their marginal utilities $u_E^j$, $u_c^j$ and $u_z^j$ and their energy efficiencies $\alpha_j$. Thus, normative distributional social preferences are balanced with efficiency in consumption.

### A.2. Proof of Lemma 1

**Proof.** The Lagrangian of the government's optimization problem is

$$L^G = W(v^1, \dots, v^n) + \mu(E^* - \sum_j \tilde{E}^j) + \gamma \sum_j \left( \underbrace{w_j l_j - b_j}_{T(I^j)} + t_E^j \tilde{E}^j \right). \tag{30}$$

For the government, setting household-specific transfers is equivalent to choosing the levels of $l^j$ and $b^j$ (Aronsson and Sjögren, 2018). The first order condition for the case of a given income tax and transfer system is

$$L^G_{t_E^j} = W_v^j v_q^j - \mu \tilde{E}_q^j + \gamma \left( t_E^j \tilde{E}_q^j + \tilde{E}^j \right) = 0. \tag{31}$$

The first-order conditions for a given carbon tax are

$$L^G_{b^j} = W_v^j v_b^j - \mu \tilde{E}_b^j + \gamma \left( t_E \tilde{E}_b^j - 1 \right) = 0 \tag{32}$$

$$L^G_{l^j} = -W_v^j v_z^j + \mu \tilde{E}_z^j + \gamma \left( w^j - t_E \tilde{E}_z^j \right) = 0. \tag{33}$$

Equation (10) in the main text follows directly from FOC (31). In order to derive the marginal income tax formula, we solve (32) for $v_b^j$ and (33) for $v_z^j$. We then divide the expression for $v_z^j$ by the expression for $v_b^j$ and use that $MRS_{z,b}^j = v_z^j / v_b^j$. Thus

$$MRS_{z,b}^j = \frac{\gamma \left( w^j - t_E^j \tilde{E}_z^j \right) + \mu \tilde{E}_z^j}{\gamma \left( 1 - t_E^j \tilde{E}_b^j \right) + \mu \tilde{E}_b^j}. \tag{34}$$

We then divide both sides of the equation by the denominator on the RHS, use $\widehat{\tilde{E}^j_z} = \tilde{E}^j_z - \tilde{E}^j_b MRS^j_{z,b}$ to denote how increased leisure time affects the household's conditional demand for energy, divide by $\gamma$ and rearrange to obtain

$$\widehat{\tilde{E}^j_z}\left(t^j_E - \frac{\mu}{\gamma}\right) = \left(w^j - MRS^j_{z,b}\right). \tag{35}$$

We derive another condition for optimal labor supply by maximizing the household's indirect utility function with respect to $l^j$ and $b^j$ subject to $z^j = 1 - l^j$ and $b^j = w^j l^j - T(\underbrace{w^j l^j}_{I^j})$. The first-order condition reads

$$v^j_b w^j (1 - T^j_I) = v^j_z, \tag{36}$$

where $T^j_I = T_I(w^j l^j)$ is the marginal income tax rate that household $j$ faces.

We use equations (35) and (36), as well as $MRS^j_{z,b} = v^j_z / v^j_b$ to derive the marginal income tax formula as given in Proposition 1. □

*A.3. Proof of Proposition 1*

**Proof.** Similar to the proof for Lemma 1, we obtain first-order conditions

$$L^G_{t_E} = \sum_j W^j_v v^j_q - \mu \sum_j \tilde{E}^j_q + \gamma \left(t_E \sum_j \tilde{E}^j_q + \sum_j \tilde{E}^j\right) = 0 \tag{37}$$

$$L^G_{b^j} = W^j_v v^j_b - \mu \tilde{E}^j_b + \gamma \left(t_E \tilde{E}^j_{b^j} - 1\right) = 0 \tag{32}$$

$$L^G_{l^j} = -W^j_v v^j_z + \mu \tilde{E}^j_z + \gamma \left(w^j - t_E \tilde{E}^j_z\right) = 0 \tag{33}$$

The only difference to the proof of Lemma 1 is equation (37). Hence, the formula for the optimal individual marginal income tax rate does not change.

It remains to be shown that by setting $t_E = \frac{\mu}{\gamma}$, the government can achieve the first-best. Thus, we compare the government's FOCs (37), (32) and (33) with the social planner's FOCs (27) - (29). If the government sets $t_E = \frac{\mu}{\gamma}$, then (32) and (33) are equivalent to (28) and (29). Moreover, household behavior as given by Eq. (7) implies that

$$\gamma \alpha_j \frac{u^j_E}{u^j_c} = \gamma p_E + \mu. \tag{38}$$

Since (28) holds, we can substitute it into (38), which closes the proof. □

*A.4. Proofs for Section 3.4.2*

Proposition 3

**Proof.** By eliminating $p_E$ from the energy producer's first-order conditions (14) and (15), we can obtain a relation between tax and subsidy.

$$\tau_Z = \frac{\tilde{E}_Z}{\tilde{E}_X}(p_X - s_X) - p_Z \tag{39}$$

The governments Lagrangian is

$$L^G = W(v^1, \ldots, v^n) + \mu(Z^* - Z) + \gamma\left(\tau_Z Z + nT - s_X X\right) \tag{40}$$

and the first order conditions are

$$L^G_{\tau_Z} = \sum_j W^j_v v^j_q q^j_{\tau_Z} - \mu Z_{\tau_Z} + \gamma(\tau_Z Z_{\tau_Z} + Z - s_X X_{\tau_Z}) = 0 \tag{41}$$

$$L^G_T = -\sum_j W^j_v v^j_b + \mu Z_b + \gamma(-\tau_Z Z_b + n + s_X X_b) = 0 \tag{42}$$

$$L^G_{s_X} = \sum_j W^j_v v^j_q q^j_{s_X} - \mu Z_{s_X} + \gamma(\tau_Z Z_{s_X} - s_X X_{s_X} - X) = 0 \tag{43}$$

From (39) and (42), we get

$$s_X = \frac{1}{\Omega}\left[-\frac{\mu}{\gamma} + \left(\frac{\sum_j W^j_v \lambda^j}{\gamma} - n\right)\frac{1}{Z_b} + \frac{\tilde{E}_Z}{\tilde{E}_X} p_X - p_Z\right], \tag{44}$$

where we have defined $\Omega := \frac{X_b}{Z_b} + \frac{\tilde{E}_Z}{\tilde{E}_X}$.

This expression is non-zero in general, when the government is constrained to equal per capita lump-sum transfers $T$. □

Corollary 1

**Proof.** Substitute $\tau_Z = \frac{\mu}{\gamma}$ in (39) and then plugging the resulting equation into (44) closes the proof. □

## Appendix B. Optimal taxation with uniform carbon tax

How is the optimal tax system affected when the government is constraint by implementing a uniform carbon tax on all households, i.e. $t_E^j = t_E$ for all $j$?

The Lagrangian of the government's optimization problem is

$$L^G = W(v^1, \dots, v^n) + \mu(E^* - \sum_j \tilde{E}^j) + \gamma \sum_j \Big( \underbrace{w_j l_j - b_j}_{T(I^j)} + t_E \tilde{E}^j \Big) \tag{45}$$

The first-order conditions are the same as in the proof of Proposition 1, that is, equations (37), (32) and (33). Applying Roy's identity to (37) we can derive the optimal uniform carbon tax on all households,

$$t_E{}^* = \frac{\mu}{\gamma} + \frac{\sum_j W_{v^j} \frac{\lambda^j}{\gamma} \tilde{E}^j}{\sum_j \tilde{E}^j_q} - \frac{\sum_j \tilde{E}^j}{\sum_j \tilde{E}^j_q}. \tag{46}$$

Again, we can think about conditions for the optimal uniform carbon tax to lie below the Pigouvian level $\frac{\mu}{\gamma}$. This is the case iff

$$\sum_j W_{v^j} \lambda^j \tilde{E}^j > \gamma \sum_j \tilde{E}^j \tag{47}$$

We can interpret (47) by first considering the simple case of a utilitarian welfare function and identical households. Then, the condition collapses to

$$\lambda > \gamma \tag{48}$$

implying that the tax is below the Pigouvian level if the private marginal benefit of consumption is higher than the social marginal benefit of tax income. This is the fourth motive discussed above in Section 3.4.1. With heterogeneous households, but identical welfare weights, (47) becomes

$$\sum_j \lambda^j \frac{\tilde{E}^j}{\sum_i \tilde{E}^i} > \gamma. \tag{49}$$

If the weighted average of households' private marginal benefit of consumption is higher than the marginal social benefit of public funds, the optimal tax is below the Pigouvian level. Here, the weights are determined by relative energy demand. Finally, if different welfare weights are introduced, the level of the carbon tax depends on the normative standpoint of the government. Equilibria with relatively high welfare weights will then tend to coincide with optimal carbon tax rates below the Pigouvian level. This is the first motive discussed above in Section 3.4.1.

The marginal income tax formula stays unaffected and is still given by (9). Since the uniform carbon tax only considers the weighted average of households' marginal utility of income, the income tax schedule ensures that differences in both skill and in energy efficiency are taken into account.

## Appendix C. Efficiency enhancing investments by households

In the long run, households can make investments in efficiency-enhancing capital.

### C.1. Households

Households $j$ live for two periods $t = 1, 2$. They work only in the first period, hence leisure time is set to 1 in the second-period sub-utility function. Households have the following utility function, in which second-period sub-utility is discounted at rate $\rho$.

$$U^j = u(c_1^j, \tilde{E}_1^j, 1 - l^j) + \frac{1}{1+\rho} u(c_2^j, \tilde{E}_2^j, 1)$$

Given labor income $I^j = w^j l^j$, households choose how much final good $c_t^j$ and energy services $\tilde{E}_t^j$ to consume in each period and how much of their income they want to invest $x^j$. These efficiency enhancing investments can be subsidized at rate $s$. Income not spent in period 1 is saved for period 2 and earns interest at the exogenously given fixed rate $r$. The optimization problem, thus, is

$$\max_{c_1^j, c_2^j, \tilde{E}_1^j, \tilde{E}_2^j, x^j} U^j$$

$$\text{subject to} \quad w^j l^j - T(w^j l^j) = c_1^j + q_1^j \tilde{E}_1^j + \frac{1}{1+r} \left( c_2^j + q_2^j \tilde{E}_2^j + x^j (1-s) \right)$$

The first order conditions are

$$u_{c_1^j} = \lambda^j$$
$$u_{c_2^j} = \frac{1+\rho}{1+r}\lambda^j$$
$$u_{E_1^j}\alpha_{1,j} = \lambda^j \frac{q_1}{\alpha_{1,j}}$$
$$u_{E_2^j}\alpha_{2,j} = \frac{1+\rho}{1+r}\lambda^j \frac{q_2}{\alpha_{2,j}}$$
$$u_{E_2^j} f' \tilde{E}_2^j = \frac{1+\rho}{1+r}\lambda^j (1-s).$$

These implicitly determine demand functions, which yield the indirect utility function $v^j(b^j, q_1, q_2, s, z^j)$. Maximization of this function, subject to $z^j = 1 - l^j$ and $b^j = w^j l^j - T(w^j l^j)$ yields the same condition (36) as in the static case.

### C.2. Social planner economy

Analogously to the short-run described above, the social planner now chooses an allocation of numeraire consumption, energy services consumption and investments in energy efficiency to maximize welfare. Based on Germany's target based approach, we assume that there are two distinct quantity targets $E_t^*$ in each period t. The Lagrangian, hence, is

$$L^{SP} = W(u^1, \dots, u^n) + \mu_1(E_1^* - \sum_j \tilde{E}_1^j) + \mu(E_2^* - \sum_j \tilde{E}_2^j) + \dots$$
$$\dots + \gamma \left[ \sum_j I^j - c_1^j - p_1 \tilde{E}_1^j - \frac{1}{1+r}\left( c_2^j + p_2 \tilde{E}_2^j + x^j \right) \right]. \tag{50}$$

The first order conditions are

$$0 = W_u^j u_{E_1^j} \alpha_{1,j} - \mu_1 - \gamma p_1 \tag{51}$$
$$0 = W_u^j u_{E_2^j} \frac{\alpha_{2,j}}{1+\rho} - \mu_2 - \gamma \frac{p_2}{1+r} \tag{52}$$
$$0 = W_u^j u_{c_1^j} - \gamma \tag{53}$$
$$0 = W_u^j u_{c_2^j} \frac{1}{1+\rho} - \gamma \frac{1}{1+r} \tag{54}$$
$$0 = W_u^j u_{E_2^j} \tilde{E}^j f'(x_0^j + x^j) \frac{1}{1+\rho} - \gamma \frac{1}{1+r} \tag{55}$$
$$0 = W_u^j u_z^j - \gamma w^j. \tag{56}$$

### C.3. The role of the government in a decentralized market economy

The government maximizes social welfare using taxes, subsidies and transfers, subject to its budget constraint:

$$\sum_j T(w^j l^j) + t_{E_1}^j \tilde{E}_1^j + \frac{1}{1+r}\left( t_{E_2}^j \tilde{E}_2^j - s x^j \right) = 0 \tag{57}$$

The Lagrangian is

$$\mathcal{L}^G = W + \mu_1(E_1^* - \sum_j \tilde{E}_1^j) + \mu_2(E_2^* - \sum_j \tilde{E}_2^j) + \gamma \sum_j T(I^j) + t_1^j \tilde{E}_1^j + \frac{1}{1+r}\left( t_2^j \tilde{E}_2^j - s x^j \right)$$

*Uniform carbon tax, general income tax function, linear investment subsidy.* The first order conditions are

$$\sum_j W_u^j v_{q_1} - \mu_1 \sum_j \tilde{E}_{1,t_1}^j - \mu_2 \sum_j \tilde{E}_{2,t_1}^j + \gamma \sum_j \left( t_1 \tilde{E}_{1,t_1}^j + \tilde{E}_1^j + \frac{1}{1+r} t_2 \tilde{E}_{2,t_1}^j \right) = 0$$
$$\sum_j W_u^j v_{q_2} - \mu_1 \sum_j \tilde{E}_{1,t_2}^j - \mu_2 \sum_j \tilde{E}_{2,t_2}^j + \gamma \sum_j \left( t_1 \tilde{E}_{1,t_2}^j + \frac{1}{1+r}(t_2 \tilde{E}_{2,t_2}^j + \tilde{E}_2^j) \right) = 0$$
$$W_v^j v_b^j - \mu_1 \tilde{E}_{1,b}^j - \mu_2 \tilde{E}_{2,b}^j - \gamma \left( 1 - t_1 \tilde{E}_{1,b}^j - \frac{1}{1+r}(t_2 \tilde{E}_{2,b}^j - s x_b^j) \right) = 0 \qquad \forall j$$
$$W_v^j v_z^j - \mu_1 \tilde{E}_{1,z}^j - \mu_2 \tilde{E}_{2,z}^j - \gamma \left( w^j - t_1 \tilde{E}_{1,z}^j - \frac{1}{1+r}(t_2 \tilde{E}_{2,z}^j - s x_z^j) \right) = 0 \qquad \forall j$$
$$\sum_j W_v^j v_s - \mu_1 \sum_j \tilde{E}_{1,s}^j - \mu_2 \sum_j \tilde{E}_{2,s}^j + \gamma \sum_j \left( t_1 \tilde{E}_{1,s}^j + \frac{1}{1+r}(t_2 \tilde{E}_{2,s}^j - s x_s^j - x^j) \right) = 0.$$

We can rearrange these to obtain a linear system of equations in the carbon tax rates.

$$t_1 + t_2 \frac{1}{1+r} \frac{\sum_j \tilde{E}^j_{2,t_1}}{\sum_j \tilde{E}^j_{1,t_1}} = \frac{\mu_1}{\gamma} + \frac{\mu_2}{\gamma} \frac{\sum_j \tilde{E}^j_{2,t_1}}{\sum_j \tilde{E}^j_{1,t_1}} - \frac{\sum_j W^j_v v_{q_1}/\gamma + \sum_j \tilde{E}^j_1}{\sum_j \tilde{E}^j_{1,t_1}}$$

$$t_1 + t_2 \frac{1}{1+r} \frac{\sum_j \tilde{E}^j_{2,t_2}}{\sum_j \tilde{E}^j_{1,t_2}} = \frac{\mu_1}{\gamma} + \frac{\mu_2}{\gamma} \frac{\sum_j \tilde{E}^j_{2,t_2}}{\sum_j \tilde{E}^j_{1,t_2}} - \frac{\sum_j W^j_v v_{q_2}/\gamma + \sum_j \frac{\tilde{E}^j_2}{1+r}}{\sum_j \tilde{E}^j_{1,t_2}}$$

Comparing these with the social planners FOCs yields expressions for the optimal carbon tax rates that are similar to the static case:

$$t_1^* = \frac{\mu_1}{\gamma}$$

$$t_2^* = (1+r)\frac{\mu_2}{\gamma}$$

$$s^* = 0$$

Finally, to obtain an expression for the marginal income tax rate, we use (36) again, i.e. that $T^j_I = 1 - \frac{v^j_z}{v^j_b}\frac{1}{w^j}$. From the governments FOCs, we see that

$$\frac{v^j_z}{v^j_b} = \frac{\mu_1 \tilde{E}^j_{1,z} + \mu_2 \tilde{E}^j_{2,z} + \gamma \left[ w^j - t_1 \tilde{E}^j_{1,z} - \frac{1}{1+r}\left( t_2 \tilde{E}^j_{2,z} - s x^j_z \right) \right]}{\mu_1 \tilde{E}^j_{1,b} + \mu_2 \tilde{E}^j_{2,b} + \gamma \left[ 1 - t_1 \tilde{E}^j_{1,b} - \frac{1}{1+r}\left( t_2 \tilde{E}^j_{2,b} - s x^j_b \right) \right]} = MRS^j_{z,b}.$$

Using the notation $\widehat{Q^j_z} := Q^j_b MRS^j_{z,b} - Q^j_z$ for $Q = \tilde{E}^j_1, \tilde{E}^j_2, x^j$, we obtain

$$T^{j*}_I = \left( t_1 - \frac{\mu_1}{\gamma} \right) \frac{1}{w^j} \widehat{\tilde{E}^j_{1,z}} + \left( \frac{t_2}{1+r} - \frac{\mu_2}{\gamma} \right) \frac{1}{w^j} \widehat{\tilde{E}^j_{2,z}} - \frac{s}{1+r} \widehat{x^j_z} \frac{1}{w^j} = 0.$$

In the social optimum, the marginal income tax is zero. The government uses individualized lump-sum transfers to households to balance its budget and to solve the distributional problem.

*Uniform carbon tax, uniform lump-sum transfer, linear investment subsidy.* With this more limited set of policy instrument, the term $T(w^j l^j)$ in (57) becomes $-nR$, where $R$ is the uniform lump-sum transfer that each household receives. As a result of this optimization problem, we obtain the following system of equations from the first order conditions.

$$\begin{pmatrix} 0 & \gamma \sum \tilde{E}_{1\,b}^{\,j} & \gamma \frac{\tilde{E}_{2\,b}^{\,j}}{1+r} & -\gamma \frac{\sum x^j_b}{1+r} \\ 0 & \gamma \sum \tilde{E}_{1\,q_1}^{\,j} & \gamma \frac{\tilde{E}_{2\,q_1}^{\,j}}{1+r} & -\gamma \frac{\sum x^j_{q_1}}{1+r} \\ 0 & \gamma \sum \tilde{E}_{1\,q_2}^{\,j} & \gamma \frac{\tilde{E}_{2\,q_2}^{\,j}}{1+r} & -\gamma \frac{\sum x^j_{q_2}}{1+r} \\ 0 & \gamma \sum \tilde{E}_{1\,s}^{\,j} & \gamma \frac{\tilde{E}_{2\,s}^{\,j}}{1+r} & -\gamma \frac{\sum x^j_s}{1+r} \\ n & -\sum \tilde{E}_1^{\,j} & -\frac{\tilde{E}_2^{\,j}}{1+r} & \sum x^j \end{pmatrix} \begin{pmatrix} R \\ t_1 \\ t_2 \\ s \end{pmatrix} = \sum_j \begin{pmatrix} W^j_v v^j_b - \mu_1 \tilde{E}_{1\,b}^{\,j} - \mu_2 \tilde{E}_{2\,b}^{\,j} - \gamma \\ W^j_v v^j_{q_1} - \mu_1 \tilde{E}_{2\,q_1}^{\,j} - \mu_2 \tilde{E}_{2\,q_1}^{\,j} + \gamma \tilde{E}^j_1 \\ W^j_v v^j_{q_2} - \mu_1 \tilde{E}_{1\,q_2}^{\,j} - \mu_2 \tilde{E}_{2\,q_2}^{\,j} + \frac{\gamma}{1+r} \tilde{E}_2^{\,j} \\ W^j_v v^j_s - \mu_1 \tilde{E}_{1\,s}^{\,j} - \mu_2 \tilde{E}_{2\,s}^{\,j} - \frac{\gamma}{1+r} x^j \\ 0 \end{pmatrix}$$

Setting $t_1 = \frac{\mu_1}{\gamma}$ and $t_2 = \frac{\mu_2}{\gamma}(1+r)$ and applying Roy's Identity yields an expression for the linear subsidy.

$$s = (1+r)\frac{\sum_j W^j_v \lambda^j/\gamma - 1}{\sum_j x^j_b}. \tag{58}$$

In the social optimum, $W^j_v \lambda^j = \gamma$ for all $j$ and the subsidy would be zero. Using the second best set of linear uniform policy instruments, however, does not guarantee that this condition is fulfilled. Hence, in the second best, a non-zero subsidy on efficiency enhancing investments may be optimal.

## Appendix D. Curvature of the social welfare function

Households' utility $u^j$, social welfare $W(u)$ and the combined concavity parameter $\epsilon$ are given by

$$u^j(c^j, \tilde{E}^j) = \frac{\left( c^{j\,\beta} \left( \alpha_j \tilde{E}^j - \bar{E} \right)^{1-\beta} \right)^{1-\eta}}{1-\eta},$$

$$W(u) = \sum_j \frac{u^{j\,1-\zeta}}{1-\zeta},$$

$$\epsilon = \eta + (1-\eta)\zeta.$$

**Table D.1**
Relationship between the curvature of the individual household utility function $\eta$, governmental inequality aversion $\zeta$ and the combined concavity of the social welfare function $\epsilon$ determining social inequality aversion. Note that $\zeta$ needs to be negative to ensure positive values for $\epsilon$ (Kaplow, 2010). W.l.o.g. we use $\epsilon$ for $\zeta = 0$ for solving the numerical model in this paper.

| $\eta$ | $\epsilon$ for $\zeta = 0$ | $\epsilon$ for $\zeta = -1$ | $\epsilon$ for $\zeta = -2$ |
|---|---|---|---|
| 0.5 | 0.5 | – | - |
| 0.6 | 0.6 | 0.2 | - |
| 0.7 | 0.7 | 0.4 | 0.1 |
| 0.8 | 0.8 | 0.6 | 0.4 |
| 0.9 | 0.9 | 0.8 | 0.7 |
| 1 | 1 | 1 | 1 |
| 1.1 | 1.1 | 1.2 | 1.3 |
| 1.2 | 1.2 | 1.4 | 1.6 |
| 1.3 | 1.3 | 1.6 | 1.9 |
| 1.4 | 1.4 | 1.8 | 2.2 |
| 1.5 | 1.5 | 2 | 2.5 |
| 1.6 | 1.6 | 2.2 | 2.8 |
| 1.7 | 1.7 | 2.4 | 3.1 |
| 1.8 | 1.8 | 2.6 | 3.4 |
| 1.9 | 1.9 | 2.8 | 3.7 |
| 2 | 2 | 3 | 4 |

**Table E.2**
Energy prices and consumption shares. Price data is taken from BMWI (Entwicklung von Energiepreisen und Preisindizes, https://www.bmwi.de/Redaktion/DE/Artikel/Energie/energiedaten-gesamtausgabe.html). Emission-weighted consumption shares for households are derived from the environmental national accounts (Destatis, 2020).

| | Price | Unit | Price (ct/kwh) | Emission factor (tCO2/TJ) | Price (€/tCO2) | Consumption share |
|---|---|---|---|---|---|---|
| Electricity | 30.19 | ct/kwh | 30.19 | 130.00 | 645.11 | 0.22 |
| Natural gas | 6.53 | ct/kwh | 6.53 | 55.90 | 324.55 | 0.18 |
| Coal | 4.57 | ct/kwh | 4.57 | 97.75 | 129.86 | 0.08 |
| Heating oil | 69.40 | €/100l | 7.73 | 77.65 | 276.52 | 0.12 |
| Gasoline | 1.46 | €/l | 16.31 | 73.10 | 619.60 | 0.20 |
| Diesel | 1.32 | €/l | 13.23 | 74.00 | 496.57 | 0.16 |
| Heat | 23.28 | €/GJ | 8.38 | 63.89 | 364.32 | 0.04 |

**Table E.3**
Estimated $\beta$ and energy efficiency levels $\alpha^j$ for energy efficiency type $j$ from a constrained regression of Eq. (22) with $\alpha^1 = 2\alpha^2$ on a $10 \times 10$ grid of households. F-Stat = 154.55, Prob > F = 0.0000, Root MSE=0.0091.

| Efficiency type $j$ | Term | Coefficient | Std. err. | t | $p$-value | Estimated $\alpha_j$ |
|---|---|---|---|---|---|---|
| 1 | $\beta q/(\alpha^1 b^i)$ | 89.92 | 28.87 | 3.11 | 0.0025 | 5.007 |
| 2 | $\beta q/(\alpha^2 b^i)$ | 179.83 | 57.74 | 3.11 | 0.0025 | 2.504 |
| 3 | $\beta q/(\alpha^3 b^i)$ | 365.25 | 57.59 | 6.34 | 0.0000 | 1.233 |
| 4 | $\beta q/(\alpha^4 b^i)$ | 492.10 | 57.59 | 8.54 | 0.0000 | 0.915 |
| 5 | $\beta q/(\alpha^5 b^i)$ | 604.09 | 57.59 | 10.49 | 0.0000 | 0.745 |
| 6 | $\beta q/(\alpha^6 b^i)$ | 717.16 | 57.59 | 12.45 | 0.0000 | 0.628 |
| 7 | $\beta q/(\alpha^7 b^i)$ | 837.29 | 57.59 | 14.54 | 0.0000 | 0.538 |
| 8 | $\beta q/(\alpha^8 b^i)$ | 987.14 | 57.59 | 17.14 | 0.0000 | 0.456 |
| 9 | $\beta q/(\alpha^9 b^i)$ | 1213.26 | 57.59 | 21.07 | 0.0000 | 0.371 |
| 10 | $\beta q/(\alpha^{10} b^i)$ | 1759.70 | 57.59 | 30.55 | 0.0000 | 0.256 |
| | $1 - \beta$ | 0.02 | 0.00 | 9.89 | 0.0000 | |

## Appendix E. Additional data for calibration

### E.1. Household-specific marginal labor taxes

Here we present more detailed numerical optimization results for the policy case that combines a uniform carbon tax, a uniform lump-sum transfer and household-specific marginal labor income taxes. This case also serves as an illustration of the results from the theoretical model summarized in Proposition 2. Fig. E.1 depicts optimal marginal labor income tax rates differentiated by energy-efficiency type for the median vertical income decile. The optimal system of taxation is linear, except for the marginal labor

**Table E.4**
Calibrated skill-levels $w^j$.

| Inome Decile | Wage/hour [€] |
|---|---|
| 1 | 12.50 |
| 2 | 15.90 |
| 3 | 18.82 |
| 4 | 21.40 |
| 5 | 23.80 |
| 6 | 25.96 |
| 7 | 28.56 |
| 8 | 31.57 |
| 9 | 35.67 |
| 10 | 49.25 |

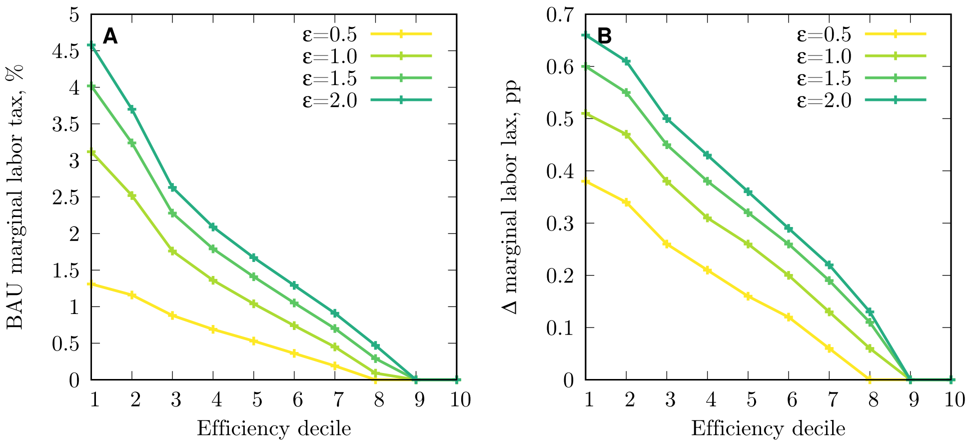


**Fig. E.1. Optimal household-specific marginal labor taxes** for the median vertical income decile, all energy-efficiency deciles (1=high, 10=low) and inequality aversion $\epsilon = [0.5; 2]$. Panel **A** shows the optimal marginal labor tax without climate target (BAU - Business as Usual), **B** plots percentage points (pp) change of the marginal labor income tax when the optimal carbon tax is set to achieve a 30% reduction in fossil energy production. Note the different y-ranges when comparing panel A and B.

income tax rates, which specifically target horizontal inequality stemming from heterogeneity in energy-efficiency.[20] Compared to the first-best optimum mitigation welfare costs increase by only 0.9% (1.6%) for $\epsilon = 0.5$ ($\epsilon = 1.3$).

For the case without climate target, Fig. E.1A depicts the optimal marginal labor tax rate for each efficiency decile and for different degrees of inequality aversion. Also in the absence of a climate target differentiated marginal labor taxes are used to address household heterogeneity. Social optimally requires that the most energy-efficient households pay higher marginal tax rates as compared to very energy-intensive households. Further, the higher inequality aversion, the larger the wedge between the tax rate for the most efficient and the tax rate for the least energy-efficient household. For high inequality aversion ($\epsilon = 2$) this difference amounts to 5.2 percentage points.

Panel E.1B presents the percentage points change in the household-specific marginal labor income tax rate due to the introduction of the climate target. The allocation dynamics of marginal labor taxes between deciles follows a similar pattern and reasoning as compared to the case without climate target. The wedge between the tax paid by the most efficient household and the tax paid by the least efficient household amounts to 0.38 percentage points for $\epsilon = 0.5$ and 0.66 percentage points for $\epsilon = 2$. Similarly to the case of household-specific lump-sum transfers, we thus find support for a dominance of the equity motive in governmental redistributive policy: For plausible values of inequality aversion, households that are rather energy-intensive receive a preferential treatment by the government in form of a lower marginal labor income tax rate.

### *E.2. Household-specific energy consumption subsidies*

Here we present more detailed numerical optimization results for the policy case of uniform linear taxation and household-specific energy subsidies. This policy contains a uniform carbon tax, a uniform marginal labor income tax, a uniform lump-sum transfer and household-specific energy subsidies. Fig. E.2 depicts optimal energy consumption subsidies differentiated by energy-efficiency type for the median vertical income decile. Since the optimal system of carbon and labor taxation is linear, energy subsidies

[20] The uniform carbon tax ($\Delta$) amounts to 238 (241) €/t$CO_2$ for $\epsilon = 0.5$ ($\epsilon = 2$) to achieve the climate target. The carbon tax revenue is redistributed to household through (i) uniform lump-sum transfers of 402 (494) €/year for $\epsilon = 0.5$ ($\epsilon = 2$), and (ii) household-specific marginal labor income tax rates.

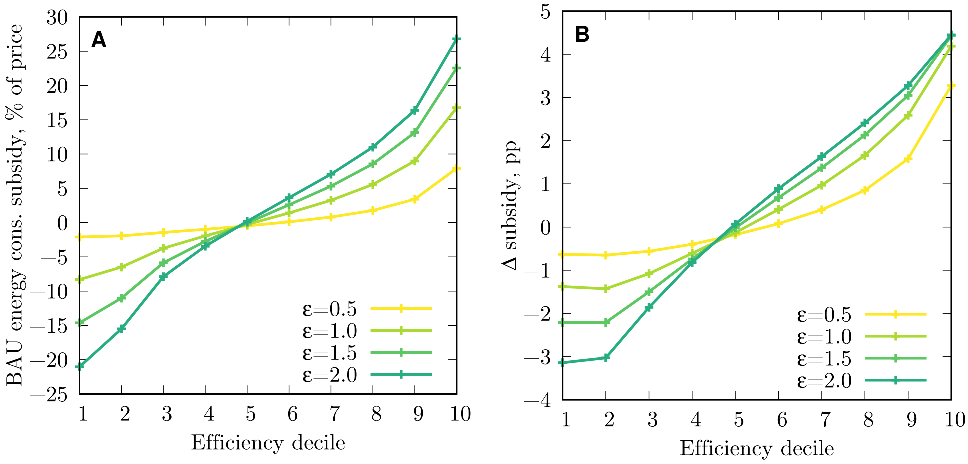


**Fig. E.2. Optimal household-specific energy consumption subsidies(+)/taxes(-)** for the median vertical income decile, all energy-efficiency deciles (1=high, 10=low) and inequality aversion $\epsilon = [0.5; 2]$. Panel **A** shows the optimal energy subsidy (+) or tax (-) without climate target (BAU - Business as Usual), while **B** plots the percentage points (pp) change of the subsidy when the optimal carbon tax is set to achieve a 30% reduction in fossil energy production. Note the different y-ranges when comparing panel A and B.

subsidies specifically target horizontal inequality stemming from heterogeneity in energy-efficiency.[21] Compared to the first-best optimum mitigation welfare costs increase by only 0.9% (2.6%) for $\epsilon = 0.5$ ($\epsilon = 1.6$).

For the case without climate target, Fig. E.2A depicts the optimal energy subsidy (+) or tax (-), measured in percent of the energy price $p_E$, for each efficiency decile and different degrees of inequality aversion. Even in the absence of a climate target there is a strong redistributive motive to address household heterogeneity through differentiated energy subsidies and taxes. Up to the median energy-efficiency decile subsidies are negative for 94% of the depicted combinations of energy-efficiency type and inequality aversion. This means that the vast majority of rather energy-efficient households pay energy taxes, while energy consumption subsidized for the remaining rather carbon-intensive households, above the median energy-efficiency decile. Similarly to the case of household-specific lump-sum transfers, we thus find support for a dominance of the equity motive in governmental redistributive policy: For plausible values of inequality aversion, households that are rather energy-intensive receive a preferential treatment by the government in form of a higher energy subsidy (or lower energy tax). Further, the higher inequality aversion, the larger the wedge between the tax of the most efficient and the subsidy for the least energy-efficient household. For high inequality aversion ($\epsilon = 2$) this difference amounts to more that 50% of the baseline energy price.

Panel E.2B presents the percentage points change in the household-specific energy subsidy due to the introduction of the climate target. The allocation dynamics of subsidies between deciles follows a similar pattern and reasoning as compared to the case without climate target. The wedge between the energy tax paid by the most efficient household and the subsidy received by the least efficient household amounts to 3.9 percentage points for $\epsilon = 0.5$ and 7.6 percentage points for $\epsilon = 2$.

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[21] The uniform carbon tax amounts to 239 (244) €/t$CO_2$ for $\epsilon = 0.5$ ($\epsilon = 2$) to achieve the climate target. The marginal labor income tax is zero and carbon tax revenue is redistributed to household through (i) uniform lump-sum transfers of 309 (290) €/year for $\epsilon = 0.5$ ($\epsilon = 2$), and (ii) household-specific energy subsidies.

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