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A Welfare Economic Approach to Planetary Boundaries

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Abstract: The crises of both the climate and the biosphere are manifestations of the imbalance between human extractive, and polluting activities and the Earth’s regenerative capacity. Planetary boundaries define limits for biophysical systems and processes that regulate the stability and life support capacity of the Earth system, and thereby also define a safe operating space for humanity on Earth. Budgets associated to planetary boundaries can be understood as global commons: common pool resources that can be utilized within finite limits. Despite the analytical interpretation of planetary boundaries as global commons, the planetary boundaries framework is missing a thorough integration into economic theory. We aim to bridge the gap between welfare economic theory and planetary boundaries as derived in the natural sciences by presenting a unified theory of cost-benefit and cost-effectiveness analysis. Our pragmatic approach aims to overcome shortcomings of the practical applications of CEA and CBA to environmental problems of a planetary scale. To do so, we develop a model framework and explore decision paradigms that give guidance to setting limits on human activities. This conceptual framework is then applied to planetary boundaries. We conclude by using the realized insights to derive a research agenda that builds on the understanding of planetary boundaries as global commons.

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1 Introduction

1.1 The Double Crisis: Climate and Biosphere

The double crisis of biodiversity loss and climate change threatens not only human prosperity but for a substantial share of the global population, it threatens the ability to meet basic human needs (Hallegatte et al. 2016). On a more fundamental level, the double crisis may even threaten long-term human survival (Butler 2018; Kareiva and Carranza 2018). The ongoing climate crisis has already manifested in an increasing number of extreme weather events all over the globe (IPCC 2021). The probability of crossing or having crossed climate tipping points is increasing (Lenton et al. 2019). In the most recent tipping element assessment, 4–5 of the 15–16 known climate tipping elements are likely to cross their tipping points already at the climate planetary boundary level of 1.5 °C (McKay et al. 2022). While the natural science foundations of climate change are increasingly well understood and a large amount of economic research into the damages (Burke et al. 2015; Kalkuhl and Wenz 2020), welfare effects (Carleton et al. 2020), and policy instruments (Creutzig et al. 2021; IPCC 2014) has been conducted, biodiversity, on the other hand, has received much less attention from research. Nevertheless, there is mounting evidence on the severity of the biodiversity crisis, with one million of the estimated eight million plant and animal species being threatened by extinction (IPBES 2019), and an assessed 69% decline of mammal populations since 1970 (WWF 2022). It has become abundantly clear that the ongoing disturbance of key ecosystems through deforestation, land-system change, and overexploitation of natural resources threatens the long-term ability of the biosphere to provide ecosystem services essential to human survival (Millennium Ecosystem Assessment 2005).

The climate and biodiversity crisis are alarming in and of themselves. However, the strong interlinkages and interdependencies between the two crises reinforce one another, amplifying the situation (Dasgupta 2021; Lovejoy and Hannah 2019; Newbold 2018). Climate change accelerates land-system change (IPCC 2019), which is a primary driver of biodiversity loss (Sodhi et al. 2009). The large-scale loss of tropical forests through deforestation is disastrous to biodiversity, but also decreases the capability of the biosphere to absorb CO₂ (Hubau et al. 2020) and to stabilize sub-global climatic processes (Baker and Spracklen 2019).
Ultimately, the double crisis of the climate and the biosphere embodies the fact that human extractive and polluting activities exceed the regeneration rate of natural systems (Dasgupta 2021). In the Anthropocene (Waters et al. 2016), these extractive and polluting activities have reached a pace and scale that threaten life support systems and the stability of the Earth system (Rockström et al. 2021). Thus, planetary boundaries emerge as a necessary framework to guide the development of our world in the Anthropocene (Rockström et al. 2009; Steffen et al. 2015). Scientifically defining a safe operating space on Earth for the biophysical systems and processes that regulate the life support system and system state of the Earth, is justified by the observed combination of resource overextraction and potential interactions and feedbacks that may trigger irreversible changes and crossing of tipping points. These effects put the stability of the Earth system at risk and thereby threaten the ability of the planet to support humanity.

The planetary boundary framework attempts to quantify the extent of human disturbance that regulating bio-physical systems and processes can tolerate without undermining the resilience of the Earth system. Such resilience reflects the ability of the Earth to remain in a Holocene-like interglacial state that can continue to support human development (see the Supplementary Material for more details on the planetary boundary framework and a summary of all planetary boundaries). The stock-taking of the pressure human (economic) activities exert on natural equilibria is a useful and necessary tool to assess and sustainably manage human use of natural resources. Nevertheless, the extent to which this tool has been utilized in economic research has been limited by a lack of integration of the concept into economic theory and methods, though some preliminary attempts have been made: Barbier and Burgess (2017) interpret the planetary boundary framework as representing a strong sustainability perspective (i.e. the processes and systems encapsuled by the planetary boundary framework are non-substitutable with human-made capital) and the space below planetary boundaries as depletable stocks that should be managed in line with the weak sustainability principle. O’Neill et al. (2018) also treat the planetary boundary framework along the lines of strong sustainability and empirically evaluate the capability of countries to provide basic human needs without transgressing nationally downscaled planetary boundaries. Crépin and Folke (2015) suggest that the large uncertainties combined with potentially extremely adverse impacts of transgressing planetary boundaries justifies using precaution in economic policy. Finally, Wagener and de Zeeuw (2021) refer to the planetary boundary framework as an example and motivation in their game-theoretical analysis of cooperative games in the presence of tipping points. However, apart from these dispersed examples, the limited integration with economics is quite surprising: the planetary boundary framework is fundamentally connected to the notion of “resources” and defines global budgets for natural capital flows (e.g. global carbon,
nitrogen, water, and phosphorus flows) that are utilized by humans. Aggregated at a global level, planetary boundaries such as biodiversity and freshwater are common pool resources, i.e. resources that are used by economic agents all over the world and to which access cannot be limited on a global level. As these common pool resources are limited in their availability (set by the planetary boundary level) and their utilization by some reduces the availability for others (e.g. a ton of carbon emitted into the atmosphere reduces the carbon budget for everyone else), planetary boundaries can be understood as global commons. This is a core economic concept. Understanding planetary boundaries as global commons calls for a thorough integration of the concept into (welfare) economic analysis as it implies the need of governance structures to ensure their sustainable utilization.

1.2 Current Economic Approaches to Planetary Boundaries

Typically, when economic analysis is applied to one of the planetary boundaries, cost-effectiveness analysis (CEA) is employed. In CEA, an environmental target is taken as given. This target does not have to be an optimal target in a welfare economic sense. It is set “exogenously” and may be based on results from natural science or derived from political or social considerations, both of which are mostly not reflected in welfare economic analyses. Based on such an exogenous target, a welfare economic analysis is conducted into how to achieve this target at minimal cost. This approach is mainly driven by results from natural science on targets and disregards (economic) damages that are associated with the environmental problem at hand. Therefore, benefits from achieving the target (i.e. damages that are avoided) are not accounted for. When communicating the results to the public or policy makers, the cost of achieving the targets are overemphasized by CEA as it does not contrast them with the (economic) benefits of avoided damages. This, in turn, weakens public and political support for stringent environmental and climate targets. Furthermore, in CEA it is not possible to check whether a lower (as in “more ambitious”) target may be more appropriate. It may be the case that damages below the target become so high compared to relatively low mitigation costs, that it is welfare optimal to stay below the exogenously set target. This possibility cannot be accounted for in CEA.

A second approach that is often used to analyze environmental problems related to planetary boundaries is identifying the damaging economic activity’s optimal, i.e. welfare maximizing trajectory. We refer to this non-marginal welfare analysis as cost-benefit analysis (CBA) as it is an approach of evaluating costs and benefits of specific paths. This kind of analysis often takes the form of integrated assessment models (IAMs) (see, e.g. Schultes et al. 2021 who employ the same conception of CBA). Our definition of CBA has to be distinguished from a marginal cost-benefit analysis
that uses shadow prices resulting from the aforementioned welfare maximization in policy evaluations or project appraisals to evaluate (small) changes on a welfare-optimal path.

CBA in the non-marginal sense weighs the societal costs and benefits associated with an economic activity and identifies a welfare optimal path and level of that activity. In this approach, there is no exogenously given target but only an optimal value for a specific economic activity or environmental indicator.

Part of the costs assessed in a CBA is (non-market) damages that arise due to environmental degradation from the economic activity under scrutiny. This raises an important point: The CBA approach relies on the quantifiability of all costs and benefits associated with a certain economic activity. Thus, this approach is not suitable for cases with unknown damages and/or probability distributions.

For this reason, it seems useful to explicitly distinguish between what is theoretically attainable with CBA and what is practically achievable in real-world applications of CBA. In what we call theoretical CBA, one could incorporate all kinds of tipping points, feedback loops, system states, and endogeneities of natural and social systems that are relevant to the economic activity under scrutiny. Even if there was uncertainty about some aspects, these uncertainties could be addressed and incorporated into the modelling approach using adequate probability distributions.

In contrast, in what we call practical CBA this is however not possible due to data limitations, missing evidence for underlying probability distributions, and limited knowledge about functional relationships. Especially, for ecosystems’ “regulating and maintenance services” (Dasgupta 2021), the extent, consequences, and monetary value of damages are often unclear. Moreover, there is still evidence missing on natural systems’ thresholds and tipping points and the evidence available is often accompanied by large uncertainties. However, practical CBA can only account for “known”, quantified damage and benefit estimations in monetary terms and natural processes for which strong empirical evidence with low uncertainty is available.¹ As a consequence, natural systems’ thresholds for which no or only little empirical evidence is available and related highly uncertain damage and benefit estimates are for the most part unaccounted for in practical CBA analyses. Moreover, a theoretical CBA, that includes all kinds of relevant uncertainties, interdependencies and so on would be from a modelling perspective not sufficiently tractable in practice.

¹ See the technical report of the Interagency Working Group on Social Cost of Greenhouse Gases, US Government (2021) as an example of practical CBA conducted in real world policy applications. See Dietz et al. (2021), Drupp and Hänsel (2021), or Kikstra et al. (2021) as examples of how incorporating previously unaccounted damages, feedbacks or tipping points into CBA lead to higher damage estimates.
These concerns with practical CBA are of special relevance with respect to the large-scale environmental processes that the planetary boundary framework is concerned with. Practical CBAs of such environmental problems are conservative in the sense that they only incorporate empirically founded damage and benefit estimates and probability distributions for which sufficient (economic and natural science-based) evidence is available. Potential catastrophes (damages) as well as miracles (benefits) for which no probability estimates are available are left out of the analysis (see also Sunstein (2021)). As we can reasonably assume a certain level of societal risk aversion, i.e. society weighs damages from catastrophes more heavily than benefits from miracles (comparable to prospect theory on an individual level (Kahneman et al. 1990)), CBA leads to overly optimistic levels of environmentally damaging activities.\(^2\) This is especially concerning in the face of planetary-scale system state changes and the associated potential for catastrophic welfare damages.\(^3\)

In summary, CBA may theoretically be able to address many concerns that are put forward by natural science researchers and is conceptually extremely flexible. However, the fact that CBA is in theory applicable to almost all environmental problems should not conceal the fact that practically it is often not the appropriate tool to derive environmental targets on a global scale.\(^4\)

To overcome the outlined shortcomings of CEA and practical CBA regarding large-scale environmental problems, we propose the pragmatic approach of a unified theory of CBA and CEA.

1.3 A New Approach: Planetary Boundaries Warrant Constraints to Welfare Maximization

The limitations of CEA and (practical) CBA warrant a new approach to welfare economic analyses of planetary boundaries and the associated global commons. In this paper, we conceptualize how to think about planetary boundaries beyond classic CEA and CBA approaches. Our main proposal is to combine the consideration of damages from CBA with an exogenously set target or limit from CEA. Using a stylized,

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2 Assuming equal but unknown probabilities for a miracle or a catastrophe with the same monetary value (either as damage or benefit), societal risk aversion leads to a higher weighting of the catastrophe which would lower the optimal pollution level.

3 Interested readers may refer to Martin and Pindyck (2015) for details on the economics of multiple potential catastrophes and resulting investment strategies of a social planner and to Barro (2006) for more information on the relationship between catastrophic economic disasters and macroeconomic aggregates such as asset prices.

4 See also the very relevant discussion by Stern et al. (2022) on the appropriateness of IAMs (as a tool for non-marginal CBA in our sense) for climate change as one planetary boundary.
but versatile dynamic analytical model, the implications of such an approach will be explored.

Several articles have already employed the proposed approach of a constrained CBA. van der Wijst et al. (2021) compare greenhouse gas (GHG) emission pathways to achieve a 2 °C target with and without considering damages. The authors conduct various analyses with an integrated assessment meta-model and, among other things, compare the results for a pure CEA that only considers mitigation costs with a CEA that also takes into account economic impacts from climate damages. They find higher abatement in the beginning of the time horizon and higher carbon prices if damages are also considered in a constrained optimization. Implicitly, their analysis of adding damages to a CEA corresponds to our approach of a constrained CBA. However, they provide no general motivation for this kind of analysis. Dietz and Vennmans (2019) also compute optimal GHG emission pathways with their analytical IAM in which they add an additional exogenous constraint on temperature to their cost-benefit setup. As a motivation for their analysis, they mention the empirical observation of the ratified Paris Agreement which requires temperature to be kept well below 2 °C above pre-industrial levels. Thus, a pure CBA would not be a sufficient description of real-world circumstances. They conclude that the analytical solution to this problem is the same as for an unconstrained CBA but with an additional boundary condition. Compared to the emission path in a CEA, the constrained CBA emission path starts with lower emissions and lower emission reductions. Schultes et al. (2021) use a large (i.e. detailed) IAM to assess the optimal GHG emission pathway while staying below a temperature limit. Their approach, which they call least-total-cost pathways, is conceptually closest to our more general proposal of a unified theory of CBA and CEA. Schultes et al. (2021) also come closest to an elaborated justification in their motivation for the least-total-cost approach. They argue, due to large uncertainties about the monetized impacts, CBA does often not include climate impacts from crossing tipping points in the climate system.5 In CEA on the other hand, an exogenous temperature limit based on precaution (a “guardrail”) is used to limit the risk of crossing climate tipping points. However, CEA lacks the capability of weighing mitigation costs against climate damages below the limit. Thus, they argue, a combination of both approaches seems reasonable.

Even though some literature already employed our proposed approach of a constrained CBA, neither of the aforementioned studies applied the approach of a

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5 In our view, practical CBA attempts never include all climate impacts from crossing climate tipping points. Practical CBA may reasonably attempt to include short-term impacts of crossing tipping points, but this is never done beyond the year 2100. However, the full social impacts of crossing tipping points may not be felt within the next 500 years (e.g. 10 m of sea-level rise from the full melting of Greenland and the West-Antarctic Ice Shields will only realize within such a very long time horizon).
constrained CBA to planetary boundaries in general, but only in the context of climate change, and without explicitly providing a detailed justification and discussion of why a combination of CBA and CEA is advisable. Thus, we aim to provide a rationale not only for the context of climate change but for the planetary boundary framework on a general level. Thereby, we bridge the gap between the natural science derived planetary boundaries and economic approaches to global commons. Note, this is not a classical research paper that develops a model and applies it to specific cases. It rather aims to synthesize and conceptualize how to integrate the planetary boundary framework into economics. Doing so, many gaps become apparent that need to be filled by future research. Thus, a high-level summary of this future research agenda is presented at the end of this paper.

The remainder of this paper is structured as follows: Section 2 develops a theoretical model framework that generalizes CEA and CBA. The model framework takes into account damages while approaching a set limit as well as the adherence to this limit. Insights about optimal pathways and first policy implications are derived from this framework. Section 3 provides the rationales for setting a limit to welfare optimization, as is done in the model framework. Section 4 outlines possible extensions of the model framework to accommodate interdependencies between planetary boundaries and regional–global interlinkages. Section 5 presents a classification of planetary boundaries with economic terminology. Preliminary thoughts on policy instruments are also elaborated. The section concludes with envisioning a research agenda that builds on the understanding of planetary boundaries from the perspective of global commons. Section 6 summarizes and discusses the results.

2 A Unified Theory of Cost-Benefit and Cost-Effectiveness Analysis

This section presents a unified theory of CBA and CEA by means of a formally derived model framework. In this framework, welfare measured by discounted net benefits from a certain economic activity is maximized. The economic activity directly contributes to human welfare, but also causes damages by straining a natural system. The natural system regenerates from the strain with a certain rate. This regeneration rate depends on how intact the natural system is and decreases with accumulating exposure to the economic activity. The model framework also includes an exogenously set limit to the accumulating exposure that safeguards the natural system against too much strain. This limit constrains the welfare maximization. Rationales that may motivate such a limit are discussed in Section 3.
To avoid confusion, it should be noted that we use the term “limit” purposefully to set it apart from the term “boundary” as in planetary boundaries. The exogenously set limit introduced in this section is not equivalent to a planetary boundary. In Section 2 we abstract from specific limits like planetary boundaries, political, or environmental targets on purpose and use the term “limit” as a placeholder for more concrete concepts (such as planetary boundaries). Section 3 will substantiate these concepts with decision paradigms on how such a limit may be chosen.

2.1 Model Framework

In the following, we develop a stylized dynamic model framework to formalize concepts that combine key considerations from the planetary boundaries and the welfare economics literature. The model starts with the canonical stock-pollution problem (e.g. Keeler et al. 1972) and is modified by various features. Let $X(t)$ be the flow variable of a human activity that affects the long-run state of natural systems. We consider $X(t)$ to be a side-product of certain economic activities like carbon emissions from fossil fuel use or other types of pollutants, though in general it can refer to any human activity that impacts natural systems, such as water use, habitat conversion or resource use. Pollution arises because an associated economic activity constitutes a direct benefit to human well-being, $B(X(t))$. Benefits increase with diminishing marginal returns, i.e. $B'(X(t)) > 0, B''(X(t)) < 0$. We disregard issues of measurement here and assume a money-metric approach (e.g. willingness to pay) that translates well-being to the monetary value of a consumption bundle. Let $Z(t)$ be the stock variable reflecting the state of a natural system. For illustration purposes, we consider $Z(t)$ to be a stock of pollution that accumulates in natural compartments by:

$$
\dot{Z}(t) = X(t) - \delta(Z(t))Z(t)
$$

where $\delta(Z(t))$ denotes a natural regeneration rate that may itself be dependent on the state of the natural system. An example for a stock-dependent regeneration rate is the carbon cycle. In a balanced system, removal of CO$_2$ from the atmosphere matches emissions. If the system is pushed out of balance, e.g. due to human CO$_2$ emissions, the stock of carbon in the atmosphere increases. So far it is only 44% of this human-emitted CO$_2$ that has accumulated in the atmosphere (contributing to the 1.2 °C global mean surface temperature rise we have seen so far). The remaining 56% has been absorbed in intact ecosystems on land and in the ocean. With rising saturation, the compensation capacity of oceans and increased biomass production is gradually exhausted through the loss of resilience and extreme climate events triggering positive feedbacks (such as forest fires and disease that cause rising loss of...
carbon) – and with it, the capability of the system to regenerate from the additional emission input diminishes. Note that $\delta(Z(t))$ also denotes damages to the regenerative capacity when $\delta(Z(t)) < 0$ (i.e. the higher the pollution stock, the lower the regenerative capacity of the natural system to absorb and recover from pollution). Following the example of the carbon cycle, the increasing stock of carbon in the atmosphere induces global warming, which triggers new sources of carbon emissions that are unrelated to human activities (e.g. thawing permafrost) which further diminishes the capability of the system to regenerate.

The pollution stock $Z(t)$ also affects human well-being as nature affects productivity, human health, etc., through its various amenities and ecosystem services. This is represented as a damage-function $D(Z(t))$ that measures welfare damages quantified as willingness to pay to reduce pollution levels. Again, we understand $D(Z(t))$ as a broad measure of market and non-market damages in terms of willingness-to-pay that could also include premiums related to risk aversion in case of uncertain damage estimates. Finally, we introduce the upper limit $\bar{Z} \geq Z(t) \forall t$, which reflects an exogenously set limit to the pollution stock (e.g. a political or environmental target). As mentioned above, $\bar{Z}$ should not be understood to be a planetary boundary at this point. The rationales for choosing a limit $\bar{Z}$, such as the precautionary principle will be discussed in Section 3. As $Z(t)$ increases with the pollution flow $X(t)$, and regenerative capacity of the natural system $\delta(Z(t))$ decreases, the limit $\bar{Z}$ is approached. Thereby, the limit imposes a scarcity.

For identifying welfare-optimal paths of pollution $X^*(t)$, intertemporal welfare as discounted net benefits ($NB$), needs to be maximized:

$$\max_{X(t)} NB := \int_0^\infty [B(X(t)) - D(Z(t))] e^{-\rho t} dt$$

subject to

$$\dot{Z}(t) = X(t) - \delta(Z(t))Z(t) \quad (3)$$

$$Z(0) = Z_0 \quad (4)$$

$$Z(t) \leq \bar{Z} \forall t \quad (5)$$

$$\lim_{t \to \infty} \mu(t)Z(t)e^{-\rho t} = 0 \quad (6)$$

where $\rho$ denotes the (consumption) discount rate. In general, the consumption discount rate is endogenous and depends on risk or inequality aversion (Ramsey rule), risk changes (Dietz et al. 2018), and the substitutability between market and non-market goods (Drupp and Hänsel 2021; Sterner and Persson 2008). Also, growth rates and damages from pollution will be influenced by the exogenously set limit which
will also have implications for the consumption discount rate. However, here, it is
assumed to be constant for analytic simplicity.

This model set-up illustrates a pollution problem. However, not all environ-
mental problems refer to pollution problems. Thus, it is noteworthy that the pollu-
tion problem modeled above can be transformed into a depletion problem where $Z$ is
the initial stock of natural resources (biodiversity, fresh water, habitat size, etc.) and
$Y (t) := Z - Z(t)$ the remaining budget.\(^6\)

This very general formulation of the problem allows the case of exhaustible and
renewable resources (with initial availability $Z$) to be considered, as well as a stock-
pollutant problem with an environmental target $Z$. A large set of environmental
problems can thereby be modeled, including those problems that are considered in
the planetary boundary framework.

2.2 Optimality Conditions and Shadow Prices

Using optimal control theory, we can derive the optimality conditions for the
pollution problem with and without an exogenous limit. Without exogenous limit,
the problem is

$$
\max_{X(t)} \int_0^\infty \left[ B(X(t)) - D(Z(t)) \right] e^{-\rho t} dt
$$

(7)

subject to

$$
\dot{Z}(t) = X(t) - \delta(Z(t))Z(t)
$$

(8)

$$
Z(0) = Z_0
$$

(9)

$$
\lim_{t \to \infty} \mu(t)Z(t)e^{-\rho t} = 0.
$$

(10)

The current-value Hamiltonian of this problem is

$$
H_c = B(X(t)) - D(Z(t)) + \mu(t) [X(t) - \delta(Z(t))Z(t)].
$$

(11)

The maximum principle requires the following conditions (assuming an interior
solution):

$$
\frac{\partial H_c}{\partial X(t)} = \frac{\partial B(X(t))}{\partial X(t)} + \mu(t) = 0 \quad \iff \quad \mu(t) = \frac{\partial B(X(t))}{\partial X(t)}
$$

(12)

6 Section 2.2 presents the derivation of optimality conditions for a pollution problem. Appendix A1
details the same derivations for a depletion problem.
\[ \frac{\partial H_c}{\partial \mu (t)} = \dot{Z}(t) = X(t) - \delta(Z(t))Z(t) \]  
(13)

\[ \dot{\mu}(t) = \rho \mu(t) - \frac{\partial H_c}{\partial Z(t)} = \rho \mu(t) + \frac{\partial D(Z(t))}{\partial Z(t)} + \mu(t) \left[ \frac{\partial \delta(Z(t))}{\partial Z(t)} Z(t) + \delta(Z(t)) \right] \]  
(14)

From this follows the growth rate of the shadow price \( \mu(t) \) (omitting time dependency \( t \) for better readability):

\[ \hat{\mu} = \frac{\mu}{\mu} = \rho + \delta(Z) + \frac{\partial \delta(Z)}{\partial Z} \frac{Z}{Z} - \frac{\partial D(Z)}{\partial B(X)} \frac{Z}{Z}. \]  
(15)

We can show that the growth path of the shadow price is qualitatively similar if we include an exogenously set limit. Then, the problem is

\[ \max_{X(t)} \int_0^\infty [B(X(t)) - D(Z(t))]e^{-\rho t} dt \]  
(16)

subject to

\[ \dot{Z}(t) = X(t) - \delta(Z(t))Z(t) \]  
(17)

\[ Z(0) = Z_0 \]  
(18)

\[ Z(t) \leq \bar{Z} \quad \forall \ t \]  
(19)

\[ \lim_{t \to \infty} \mu(t)Z(t)e^{-\rho t} = 0. \]  
(20)

As we introduced an additional state-space constrained here, we use the current-value Lagrangian

\[ L_c = H_c + \lambda(t) \left[ Z - Z(t) \right] \]

\[ = B(X(t)) - D(Z(t)) + \mu(t) [X(t) - \delta(Z(t))Z(t)] + \lambda(t) \left[ \bar{Z} - Z(t) \right] \]  
(21)

with the following conditions (assuming an interior solution):

\[ \frac{\partial L_c}{\partial X(t)} = \frac{\partial B(X(t))}{\partial X(t)} + \mu(t) = 0 \iff \mu(t) = -\frac{\partial B(X(t))}{\partial X(t)} \]  
(22)

\[ \frac{\partial L_c}{\partial \mu(t)} = \dot{Z}(t) = X(t) - \delta(Z(t))Z(t) \]  
(23)

\[ \dot{\mu}(t) = \rho \mu(t) - \frac{\partial L_c}{\partial Z(t)} = \rho \mu(t) + \frac{\partial D(Z(t))}{\partial Z(t)} + \mu \left[ \frac{\partial \delta(Z(t))}{\partial Z(t)} Z(t) + \delta(Z(t)) \right] + \lambda(t) \]  
(24)
\[
\frac{\partial L_c}{\partial \lambda(t)} = Z - Z(t) \geq 0, \quad \lambda(t) \geq 0, \quad \lambda(t) \frac{\partial L_c}{\partial \lambda(t)} = 0. \tag{25}
\]

We define \( \bar{t} \) as the point in time, where the limit is reached \( Z(\bar{t}) = Z \). Time \( \bar{t} \) may be indefinite if the limit is never reached \( Z(t) < Z \forall t \). For \( t \in [0, \bar{t}) \), the constraint in Eq. (19) is nonbinding, \( Z - Z(t) > 0 \) and thus, as in Eq. (25) required, \( \lambda(t) = 0 \). From this follows the growth rate of the shadow price \( \mu(t) \) (again, omitting time dependency \( t \) for better readability):

\[
\hat{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta(Z) + \frac{\partial \delta(Z)}{\partial Z} Z - \frac{\partial D(Z)}{\partial B(X)} \frac{\partial B(X)}{\partial X}. \tag{26}
\]

For \( t \in [\bar{t}, \infty) \), the constraint in Eq. (19) is binding, \( Z - Z(t) = 0 \), and thus, as in Eq. (25) required, \( \lambda(t) > 0 \). The growth rate of the shadow price \( \mu(t) \) then is:

\[
\hat{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta(Z) + \frac{\partial \delta(Z)}{\partial Z} Z - \frac{\partial D(Z)}{\partial B(X)} \frac{\partial B(X)}{\partial X} + \lambda \frac{\lambda}{\mu}. \tag{27}
\]

The variable \( \mu(t) \) is the co-state variable for the stock \( Z(t) \); it denotes a shadow price that measures the social cost of the pollution. That means, it measures the marginal change in social welfare for a marginal change in \( Z \). The condition in Eqs. (12) and (22) reflects the fundamental optimality principle that marginal social costs \( \mu(t) \) need to equal marginal benefits of pollution \( B(X(t)) \).

There are two special cases where the shadow price has a very clear interpretation:

1. If the limit is (asymptotically) nonbinding (i.e. \( Z(t) < Z \forall t \)), \( \mu \) represents the social cost of pollutant \( X \), i.e. the discounted marginal damages of a marginal increase in pollution. For carbon emissions, \( \mu \) refers to the social cost of carbon that measures the marginal damage of one additional ton of \( \text{CO}_2 \). This interpretation of \( \mu \) is equivalent to the CBA in environmental economics where optimal emissions \( X(t) \) are determined such that marginal damages \( \mu(t) \) equal marginal costs \( B(X(t)) \). Hence, CBA is a special case of a missing or nonbinding limit.

2. If the limit is (asymptotically) binding (i.e. \( \lim_{t \to \infty} Z(t) = Z \)) and social damages are disregarded, i.e. set \( D'(Z) = 0 \) in Eq. (27), the additional constraint in Eq. (19) is not needed. The changed transversality condition \( \lim_{t \to \infty} Z(t) = Z \) would be sufficient to ensure the adherence to the limit. Then, \( \mu \) represents the user cost of consuming a marginal unit of an exhaustible resource \( X \) and follows the usual Hotelling price path. The user cost is a scarcity price that measures the foregone benefits of using
a resource today rather than in the future. In the classical exhaustible resource model, the market price has to equal the user cost for a dynamically efficient resource use. This interpretation of $\mu$ is also associated with CEA in which an environmental target is achieved at least cost, disregarding environmental benefits. Hence, CEA is a special case of a binding limit and disregarded environmental damages.

These two special cases illustrate the major shortcomings of the CBA and the CEA approaches: In practice, CBA cannot incorporate constraints to the state space ("limits") that are motivated by concerns about system change, large impacts, or catastrophic damages if the evidence base is weak, not quantified, or extreme uncertainties are involved. CEA does not disclose benefits in the form of environmental (social) damages avoided. The conceptual framework outlined above therefore proposes a pragmatic approach to address the shortcomings of CBA and CEA regarding environmental problems on a planetary scale. It provides a more general approach that incorporates both the cost-benefit and the cost-effectiveness approach into one unified theory.

However, there is also a third case that deserves special attention: The limit is (asymptotically) binding (i.e. $\lim_{t \to \infty} Z(t) = \bar{Z}$) and social damages are accounted for. In this case, we have to differentiate whether $X(t)$ is an essential good or not. If $X(t)$ is an essential good, marginal benefits from pollution, $B'(X(t))$, go towards infinity as $X(t)$ goes to zero. Again, the changed transversality condition $\lim_{t \to \infty} Z(t) = \bar{Z}$ would imply a $\mu(0)$ such that the growth path of $\mu$ is consistent with the limit. If $X(t)$ is non-essential (i.e. there exists a perfect backstop technology), the additional constraint is needed. For $t < \tilde{t}$, the shadow price of the limit is equal to zero, $\lambda = 0$. At $\tilde{t}$, when the limit becomes binding, the shadow price jumps to a positive value. It then measures the value of relaxing the limit by a marginal unit. As long as the limit is nonbinding, this value is zero. However, as soon as the limit becomes binding, there can be additional welfare gains by relaxing the limit. At $\tilde{t}$, $Z(\tilde{t}) = \bar{Z}$ and the growth rate of the stock must be zero $\dot{Z}(t > \tilde{t}) = 0$. From this follows, that the steady state pollution level equals the regeneration rate, $\dot{X}(t > \tilde{t}) = \delta(\bar{Z})\bar{Z} = \text{const.}$, and that marginal benefits from the steady state pollution level are constant as well, $B(\bar{X}) = \text{const}$. As $\mu(t) = -B(X(t))$, the co-state variable $\lambda$ must jump from zero to a positive value, as

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7 A resource is called "essential" if marginal productivity – and thus, the factor price – converges to infinity when the resource supply approaches zero. This is, for example, the case in a Cobb–Douglas production function or in a CES production function with imperfect substitutability. This terminology can also be transferred to utility from specific environmental services (e.g. drinking water, food intake).
soon as the limit becomes binding such that $\bar{\mu} = 0$ and thus $\mu$ remains constant (see Eq. (27)). Simultaneously, the pollution level $X(t)$ may jump downwards to the regeneration rate: $X(t) = \delta(Z)Z$.

The central value of the unified theory is the shadow price of pollution, $\mu$. In combination with the shadow price of the limit, $\lambda$, it is a generalization of the social cost and the user cost concept.

### 2.3 Insights About Optimal Pathways

In the optimality condition in Eq. (27), $\bar{\mu}$ is the growth rate of the optimal price of pollution, $X$.\(^8\) Thus, the optimality condition describes the price path of $X$. Let $\eta(X) := \frac{B''(X)}{B'(X)} X$. Because of diminishing marginal benefits, $\eta(X) < 0$. Taking the time derivative of the first-order condition $\mu(t) = -B'(X)$, we can translate the growth rate of the shadow price, $\mu$, directly into a growth rate of the pollution level:

$$\bar{\mu} = \frac{\dot{\mu}}{\mu} = \frac{-B''(X)X}{-B'(X)} = \frac{B''(X)X}{B'(X)} \frac{\dot{X}}{X} = \eta(X) \frac{\dot{X}}{X} = \eta(X) \bar{\mu}$$

From this, several insights can be derived. First, consider a model without welfare damages, $D(Z)$, and without regeneration damages (i.e. $D'(Z) = 0$ and $\delta'(Z) = 0$). Then, the optimality condition in Eq. (27) becomes:

$$\bar{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta(Z) \quad \Rightarrow \quad \bar{X} = \frac{\rho + \delta(Z)}{\eta(X)}$$

This is the usual Hotelling rule: the discounted price along the efficient usage path of a finite resource is constant. It describes a dynamically optimal use of the pollution flow $X$ below the limit $\bar{Z}$. As $\eta(X) < 0$, pollution levels have to fall over time.

Now, consider regeneration damages due to an increasing pollution stock, $\delta'(Z) < 0$. This means that the higher the pollution stock, the lower the regenerative capacity of the natural system. Then, the optimality condition in Eq. (27) becomes:

$$\bar{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta(Z) + \delta'(Z) \times Z \quad \Rightarrow \quad \bar{X} = \frac{\rho + \delta(Z) + \delta'(Z) \times Z}{\eta(X)}$$

The term $\delta'(Z) \times Z$ is negative and thus subtracted from the usual Hotelling rule. This implies that the price path becomes flatter, i.e. growing at a lower rate but starting at a higher initial price level (see Figure 1). A flatter price path, in turn, implies a flatter

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\(^8\) For better readability, we omit the index $t$ from now on when the variables’ time dependency is clear from the context.
Figure 1: Optimal shadow price $\mu$ and pollution flow $X$ over time to stay below the limit $\bar{Z}$. 

Lower regeneration rate $\delta$ would imply lower steady state value of $\bar{X}$, higher marginal benefits $B' (\bar{X})$, and thus higher shadow price $\mu$.

Optimal price path according to:
- Standard CEA (Hotelling rule): $\hat{\mu} = \rho + \delta$
- CEA incl. regeneration damages: $\hat{\mu} = \rho + \delta + \delta'$
- Unified theory: $\hat{\mu} = \rho + \delta + \delta' + \frac{\delta'}{B'}$

Optimal pollution/extraction path according to:
- Standard CEA (Hotelling rule): $\hat{\mu} = \rho + \delta$
- CEA incl. regeneration damages: $\hat{\mu} = \rho + \delta + \delta'$
- Unified theory: $\hat{\mu} = \rho + \delta + \delta' + \frac{\delta'}{B'}$
pollution path. As the cumulative amount of pollution is fixed by $\bar{Z}$, a flatter pollution path implies a lower initial pollution level and therefore more abatement early on. Hence, consideration of damages to the regeneration rate demands a stronger mitigation response than the standard model with a constant regeneration rate $\delta(Z)$.

The same observation holds when welfare damages, i.e. $D'(Z) > 0$, are also considered. This flattens the Hotelling price path even more, implying an even stronger reduction in short-term pollution levels.

This result can be compared to a pure CBA without an exogenously given upper limit $Z$. In such, the constraint in Eq. (5) would be dropped. As the stock then would not need to be smaller or equal to the limit for all $t$, pollution levels $Z$ could become higher than $\bar{Z}$ and are potentially unbounded (depending on the benefit function $B(X)$). This means that CBA does not account for any limit (including planetary boundaries) and thus, does not prevent economies from transgressing them. To yield a welfare optimal result, CBA requires that all benefits and damages are correctly specified. In face of the extremely large uncertainties and unknown probability distribution concerning potential global system state changes, this is unlikely to be achieved.

### 2.4 Pollution Levels and Shadow Prices in the Long Run

In the long run, pollution levels cannot exceed the regenerative capacity if the pollution stock should remain below its limit $\bar{Z}$. This implies for $t \to \infty, X(t) \leq \delta(\bar{Z})\bar{Z}$. The pollution level $\tilde{X}$ that implies zero accumulation of pollution is also called the steady-state pollution level, as $\tilde{X} = \delta(\bar{Z})\bar{Z}$.

This constitutes a steady state that is consistent with the limit $\bar{Z}$. If a steady state exists, the corresponding shadow price has to be constant as well as $\mu = B'(\tilde{X}) = \text{const}$. Hence, the growth rate of the shadow price shown in Eq. (26) applies only in the transitional period before the steady state pollution level is achieved (see Figure 1).

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9 As discussed in Section 3.2, it could also be optimal to stay below the limit, as environmental damages increase too strongly before the limit is reached (see Figure 3, panel a). In that special case of a nonbinding limit, the corresponding (optimal) steady state level is determined from Eq. (26) by substituting the steady state condition $\tilde{X} = \delta(Z^*)\bar{Z}^*$ and by again setting $\dot{\mu} = 0$, which implicitly define $\tilde{X}$ by solving

$$\dot{\tilde{X}}(Z^*) = B'(\delta(Z^*)Z^*)\left(\rho + \delta(Z^*) + \delta'(Z^*)\times Z^*\right)$$

If $Z^* < Z$, it will be optimal never to approach the limit $Z$. In that case, the steady-state level is influenced by marginal damage considerations.

10 We disregard questions on the stability of the steady state. Typically, dynamic economic problems (like the Ramsey model but also the stock-pollutant problem) exhibit saddle point stability.
One implication of the aforementioned considerations is that low regeneration rates also imply very small pollution levels in the long run. In particular, when regeneration rates are zero, it will be impossible to stay below \( \bar{Z} \) when pollution levels do not converge to zero. Therefore, the regenerative capacity \( \delta(\cdot) \) determines the magnitude of pollution levels (or resource use) in the long run. With low regenerative capacity, the shadow price has to increase accordingly over time. When \( X \) is an essential resource (or non-avoidable pollution by-product of an essential good production), marginal benefits increase without bound when \( X \) converges to zero. Thus, when \( \delta = 0 \) and \( X \) is essential, the shadow price of pollution needs to increase to infinity to stay below the limit. Conversely, when a substitute to \( X \) exists, the increase in \( \mu \) is bounded when pollution or resource use is reduced through the substitute. In this case, even with zero regenerative capacity, the shadow price will only increase up to the point where it equals the price of the substitute, after which it remains constant.

### 2.5 Implications for Environmental Taxes

Generally, in this model framework, the budget \( Y(t) = \bar{Z} - Z(t) \) is considered to be a common pool resource that can be utilized. A common pool resource is characterized by non-excludability and rivalry. This means, one cannot exclude agents (individuals, firms, or countries) from using the resource. At the same time, usage of the budget by one agent decreases the size of the budget for all others. For example, on a global level one cannot prevent someone from emitting CO2. However, each ton of CO2 that is emitted reduces the available carbon budget. Similarly, at a global level, all societies depend on the functioning of critical biomes, such as forest systems, wetlands, and glaciers that provide stability in terms of, e.g. flows of water and nutrients and stocks of carbon. The ecosystem services provided by the functioning of critical biomes also make these biomes global commons.

Human economic activity \( X \) exploits the budget given by the limit \( \bar{Z} \). This budget is a global common pool resource for which a governance regime is needed to prevent overexploitation and to provide incentives for optimal use. In a decentralized economy, agents maximize the (private) benefits from resource use \( B(X) \), but disregard the environmental damages or the environmental budget. The market price of using the resource will therefore usually be lower than \( \mu \) because resource use is subject to open-access problems or pollution externalities. Hence, \( \mu \) has a central role for policy intervention as it denotes the optimal level of a Pigouvian tax. A tax that equals the shadow price \( \mu \) can in this case ensure that resource use and pollution levels follow optimal pathways. The shadow price \( \mu \) therefore determines the value of an optimal environmental tax or a price in a cap-and-trade system that internalizes environmental damages and ensures that the limit is respected. It
constitutes therefore a generalization of the Pigouvian tax that only reflects marginal damages.

The path of price $\mu$ is flattened when accounting for regenerative capacity and welfare damages, implying strong early mitigation action is favorable. To stay within the limit, strong mitigation in the near future is required (suggesting less need in the far-distant future). This translates into higher initial levels of environmental taxes.

3 Decision Paradigms for Choosing a Limit

Up to this point, the limit $Z$ was simply assumed to be given. It was used as a placeholder abstracting from specific real-world boundaries or targets that may be exogenously chosen by decision makers or derived from natural science results. However, reflecting our proposal of the unified theory of CBA and CEA, there must be an underlying rationale that justifies setting a limit that constrains welfare optimization. Thus, this section explores decision paradigms that warrant setting such a limit. In this process, several limits are introduced that replace the notational placeholder $Z$ in the formal model framework above.

First, we discuss the precautionary principle as a reason for constraining welfare optimization.\textsuperscript{11} The resulting limit is the lower end of a so-called “domain of ambiguity”, within which non-negligible probabilities of catastrophic outcomes exist.\textsuperscript{12} Second, we consider how the presence of tipping points in combination with the application of the precautionary principle provides another reason not to exceed a certain limit. Lastly, we examine the case of when using a limit in terms of a proxy variable may be appropriate. This may be the case when the true natural system that exhibits tipping points or may otherwise change with catastrophic outcomes is unobservable. If the proxy variable only reflects the true state of the system with a measurement error, it is sensible to set the limit of the proxy variable conservatively and update it as new knowledge is advanced. This is essentially what underlies the planetary boundary framework. Planetary boundaries are proxy boundaries based on a precautionary principle that are set at the lower end of a scientifically defined uncertainty zone about tipping points or thresholds of large-scale change. The

\textsuperscript{11} The term precautionary principle is not strictly defined but in general refers to taking rigorous action to avoid certain negative outcomes which might or might not be uncertain. The precautionary principle may take the form of the maxi–min principle. Generally, application of a precautionary principle seems especially sensible in the presence of possible outcomes that involve catastrophic and irreversible harm (Sunstein 2021). For in-depth discussions of the precautionary principle see, e.g. Sunstein (2021), Nordhaus (2011), and Stirling (2017).

\textsuperscript{12} See Section 3.2 for a more detailed explanation of the term ‘domain of ambiguity’.
precaution is motivated by the highly uncertain, but non-negligible probability of catastrophic welfare damages in the wake of a shift of the Earth system to a new state.

3.1 (Unconstrained) CBA

The unconstrained CBA approach does not justify an externally chosen limit $\bar{Z}$. In a CBA, damages $D(Z)$ measure certainty-equivalent damages derived from a willingness to pay or willingness to accept approach. In other words, they can be interpreted as risk-adjusted expected damage. This includes all kinds of uncertainties for which meaningful probability distributions can be derived (from biophysical and economic models) and applied. Risk aversion could also be reflected in the damage function. Welfare is measured in this assessment by discounted net benefits, $NB$. Net benefits are defined as the discounted difference between the benefits measured in monetary units arising from the economic activity $X$ and the damages measured in monetary units arising from the increase of the pollution stock $Z$ that occur each period (see Eq. (2)).

In practical CBA applications, the upper limit of the pollution stock is the value of $Z$ that is associated with the welfare-optimal limit of the pollution stock, $Z_{CBA}$ (see Figure 2). As discussed in Section 1.2, practical CBA is very likely to be overly

![Figure 2](image_url): Optimal value of $Z$ according to standard cost-benefit analysis. $NB$ denotes net benefits. The temporal dimension is disregarded here for illustrative purposes.
optimistic in its assessment of $Z_{CBA}$ when applied to large scale environmental problems as encapsuled in the planetary boundary framework due to its quantification bias.

### 3.2 Uncertain, Nonlinear, and Irreversible Social Damages

Naturally, the question arises, how to deal with the quantification bias of practical CBA in the face of potentially catastrophic social welfare damages due to large-scale environmental changes. One approach is to add uncertainties, tipping points, feedback effects, etc. to the analysis as more and more knowledge becomes available. However, this approach faces two limitations: First, it depends on the availability of knowledge which is by definition in the case of highly uncertain, nonlinear, and irreversible social damages very limited and waiting for more knowledge becoming available (this has been a main argument of proponents of climate delay (Lamb et al. 2020)) may mean waiting for disaster. Second, while there is (especially, concerning climate change) a large body of literature about many different aspects that can be added to practical CBA, combining all those into a comprehensive CBA quickly becomes infeasible from a numerical modelling perspective.

An alternative approach is to resort to exogenous limits that constrain the state-space of practical CBA. Related to limits (and especially with regard to planetary boundaries), it is often argued that they can be reconciled with CBA through the introduction of highly convex damage functions where marginal damages approach infinity as the limit is approached. Economic work on estimating damages is typically based on empirical analyses in which moderate deviations of environmental conditions around past trends are considered (Auffhammer 2018; Dell et al. 2014; Hsiang 2016). However, such an approach allows one only to identify causal impacts of environmental change on human well-being within the environmental variability that humanity has experienced. However, it is unclear and uncertain to what extent these relationships can be extrapolated to large-scale changes in environmental conditions. Extrapolation and out-of-sample prediction of damages is particularly problematic if damages are highly nonlinear.

This is the case for the environmental problems encapsuled in the planetary boundary framework. They are associated with large-scale, nonlinear, and irreversible change. Especially, the “danger zone” beyond planetary boundaries is a domain of ambiguity. For our purposes, “ambiguity” refers to cases in which the

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13 For example, there are studies on threshold dynamics related to tipping points, uncertainty about tipping points as well Bayesian learning in the presence of such uncertainty (e.g. Cai et al. 2015; Hwang et al. 2017; Lemoine and Traeger 2014, 2016b; Lontzek et al. 2015).
combination of risk (in terms of irreversible and potentially catastrophic impacts rather than moderate consumption losses) and uncertainty (of when that impact may occur) become extremely large. In such cases, it is impossible to derive and apply probability distributions (Crépin and Folke 2015), either because of epistemic uncertainty or fat tails (see also Lemoine and Traeger 2016a; Traeger 2014). However, as recent economic theory uses expected welfare, it relies on existing probability distributions and converging expected values. Without probability distributions, the application of standard expected welfare maximization is not possible.

This problem of recent welfare economic theory is addressed, if the analysis is constrained to a state space, where uncertainty is relatively low.14 Stern et al. (2022) call this a “guardrail approach” in the context of climate change. There may be a certain arbitrariness in the exact location of the applied limit. However, in principle, it still seems plausible that the research community is able to form a consensus on which damages and impacts have known (subjective) probability distributions and which do not. The former can be captured in an expected welfare CBA approach; the latter are taken into account by constraining the analysis with an exogenous limit. A prominent example is the 1.5 °C limit in climate science. Another example is the planetary boundary framework. The limited past responses of environmental systems to human disturbance can serve as an indicator for future responses (Liski and Salanié 2020), i.e. that resilience is gradually lost is providing a stronger argument for relatively conservative limits to human activities stressing global life support systems.

Following this line of argument, let us assume that we enter the domain of ambiguity for some $Z > Z_{PP}$, i.e. we consider that there is an unknown but non-negligible probability of severe catastrophic impacts for $Z > Z_{PP}$ (the PP in $Z_{PP}$ stands for precautionary principle).15 The uncertainty about catastrophic impacts is not to be confused with uncertainties about damage estimates for which probability distributions can be assigned. The uncertainties in the domain of ambiguity refer to

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14 See also Weitzman (1974) who argues that in situation with higher uncertainty about damages relative to the uncertainty about benefits, a quantitative limit seems preferable compared to price instruments.

15 The limit $Z_{PP}$ should not be understood as a planetary boundary. In the framework by Rockström et al. (2009) and Steffen et al. (2015) planetary boundaries are set at the lower end of a zone of uncertainty defined by the scientific assessment of risks of crossing global thresholds or triggering feedback processes, regional thresholds, and strong interlinkages with global thresholds of other planetary boundaries. This is different from choosing the limit $Z_{PP}$ as is done here. The choice of $Z_{PP}$ is motivated by the non-negligible probability of catastrophic social and welfare damages in the domain of ambiguity that begins at $Z_{PP}$. Whether the domain of ambiguity about these catastrophic damages begins before planetary boundaries are crossed or not, is unimportant. Accordingly, the adherence to the limit $Z_{PP}$ is not motivated by thresholds in Earth system processes, but rather by a willingness to pay to avoid the domain of ambiguity.
fundamental uncertainties in a Knightian sense about the damage function itself. From this, the question arises: Should $Z_{PP}$ become a boundary to human economic activity, even if practical CBA suggests a less stringent boundary?

If the answer were yes in any case, this would constitute an unreserved application of the precautionary principle. It would call for the unconditional adherence of the boundary $Z_{PP}$ to eliminate the non-negligible probability of severe catastrophe.

Whether this is sensible, however, depends on the costs of keeping the pollution stock $Z$ below $Z_{PP}$. Importantly, the general understanding is that the precautionary principle should only be applied if the costs of precaution are not excessively high (Gardiner 2006; Pindyck 2011; Sunstein 2021). At the same time, economics unfortunately cannot provide a definitive answer to the question, whether the cost of precaution is too high. Instead, society and its institutions have to deliberate and weigh whether limiting economic activity in the face of ambiguity about catastrophic impacts from environmental change is desired. Nevertheless, based on welfare economic theory an approximate assessment of the cost of keeping $Z$ below $Z_{PP}$ can be conducted. This assessment cannot answer the question whether the limit $Z_{PP}$ should become a limit to human economic activity but it can inform societies’ institutions that are tasked with deciding on policy instruments.

We propose, the assessment of the cost of precaution to take the form of comparing welfare for the constrained case, $Z \leq Z_{PP}$, with welfare for the unconstrained cost-benefit case, $Z = Z_{CBA}$. The constrained case, $Z \leq Z_{PP}$, corresponds to an application of the unified theory of CEA and CBA. The unconstrained case, $Z = Z_{CBA}$, corresponds to a practical CBA which takes into account known and quantified data and uncertainties. This approach is pragmatic in the sense that the true level of the cost of precaution cannot be assessed as a theoretical CBA with all relevant damages etc. is not possible. However, comparing welfare as described here yields the best available estimate for the cost of precaution.

As in Eq. (2), welfare is measured in this assessment by discounted net benefits, $NB$. Comparing net benefits for the constrained case, $NB_{Z \leq Z_{PP}}$, and for the unconstrained cost-benefit case, $NB_{Z = Z_{CBA}}$, yields the cost of precaution that incur when constraining $Z$ to $Z_{PP}$. Importantly, the calculation of $NB_{Z = Z_{CBA}}$ must include avoided (certainty-equivalent) damages, as otherwise the costs of applying the precautionary principle would be biased upwards. Furthermore, it must be kept in mind that the unconstrained practical CBA that is used to calculate $NB_{Z = Z_{CBA}}$ most likely does not include all relevant damages. It is therefore typically overly optimistic regarding $Z_{CBA}$ and overly pessimistic regarding the cost of precaution.

Two stylized cases related to the assessment of the cost of precaution can be identified, as shown in Figure 3. In panel (A), damages from pollution outweigh benefits before the domain of ambiguity is reached. Thus, the optimal level of $Z$ is
**Figure 3**: Assessment of the adequacy of the precautionary principle. The temporal dimension is disregarded here for illustrative purposes.
below the domain of ambiguity \( (Z_{CBA} < Z_{PP}) \). No application of the precautionary principle is required. In panel (B), the unconstrained CBA yields an optimal level of \( Z \) within the domain of ambiguity \( (Z_{CBA} > Z_{PP}) \) and there is an associated welfare loss – and thus cost of precaution – with adhering to the limit \( Z_{PP} \).

Based on the assessment of the cost of precaution, societies’ institutions must decide whether limiting pollution to \( Z \leq Z_{PP} \) is appropriate. On a theoretical basis, this is the case if the difference between \( NB_{Z=Z_{PP}} \) and \( NB_{Z=Z_{CBA}} \) is not larger than societies’ willingness to pay to avoid the domain of ambiguity in which catastrophic damages may occur (see e.g. Martin and Pindyck 2015 for more details on the willingness to pay to avert catastrophes). If this willingness to pay to avoid the domain of ambiguity is higher than the cost of precaution, it is rational to stay below \( Z_{PP} \). However, in this regard, we again face a limit of economic theory. It seems impossible to accurately measure societies’ willingness to pay to avoid the domain of ambiguity. Thus, the decision cannot be based on economic rationale but must be found in a political process. It is a trade-off between unquantifiable risks and only partially quantifiable costs. The resulting decision may be ad-hoc and time inconsistent. This may be unsatisfactory from a scientific perspective, but it is important to keep in mind that the question at hand is not a natural science question but one of social consideration. The task of economics as scientific community is to provide as much information points to these social considerations as possible but not to be prescriptive.

### 3.3 Nonlinear Natural System Change, Tipping Points, and System Collapse

A connection remains between planetary boundaries and the limit \( Z_{PP} \) set by the precautionary principle. The planetary boundary framework differs from conventional environmental pollution or depletion problems in the possibility of large-scale, self-amplifying effects that induce irreversible environmental degradation if human pressure surpasses certain thresholds. The motivation for setting planetary boundaries is therefore deeply linked to concerns about strong nonlinear dynamics that might arise in the interplay of several natural systems when planetary boundaries are crossed. This aspect is integrated in our framework in a straight-forward way.

Let us introduce a threshold, \( Z_{TIP} \), beyond which the regeneration rate becomes negative, i.e. \( \delta < 0 \) for \( Z > Z_{TIP} \). This means, even without further pollution \( (X = 0) \), the pollution stock increases \( (\dot{Z} > 0) \) once \( Z_{TIP} \) is crossed. This implies a continued degeneration of the natural system because the regeneration capacity of biophysical systems is lost (e.g. positive feedbacks causing abrupt melting of icesheets, collapse of...
coral reef systems, and dieback of tropical rainforests). Such dynamics entail that when $Z_{TIP}$ is surpassed, it is impossible to stay below any limit $\bar{Z} > Z_{TIP}$.\(^{16}\) Note that $Z$ is a high-level control variable of a natural system. The described behavior of $\delta$ may, however, also occur locally and does not imply that the regeneration rate cannot become positive again for another natural system. This complex behavior may lead to multiple steady states (see also Tahvonen and Salo 1996).

In the context of the climate problem, various thresholds $Z_{TIP}$ have been considered for which system dynamics tip irreversibly after crossing this critical value – so-called tipping points (Lenton et al. 2008). The consideration of tipping points introduces a natural science-based rationale for setting a limit to the pollution stock, $Z$.\(^{17}\) While the choice of $Z_{PP}$ follows an economic rationale of weighting costs, benefits, and risks of high levels of pollution, degradation, or global warming, the tipping point $Z_{TIP}$ might become binding long before $Z_{PP}$ is reached. Hence, nonlinear system dynamics might require that pollution and degradation are limited to lower levels than are warranted by cost-benefit-risk considerations that are based on willingness to pay to avoid the domain of ambiguity and ignore such tipping points. The resulting limit that prevents exceeding $Z_{PP}$ is therefore $\min\{Z_{TIP}, Z_{PP}\}$, as it is impossible to stay below any limit $\bar{Z} > Z_{TIP}$.

In this context, CBA would evaluate welfare levels with and without crossing tipping points to determine which pathway is globally optimal (e.g. Tahvonen and Withagen 1996). As noted above, in the context of deep uncertainties, when a precautionary principle is applied, adherence to any limit of $Z$ has to pass the test of whether it incurs exorbitantly high welfare costs. This is true for limiting $Z$ to $Z_{TIP}$ as well. Compared to Figure 3, the value of $Z_{CBA}$ and $Z_{PP}$ (and thus the domain of ambiguity) remain unchanged in Figure 4. In panel (A), the welfare-optimal value of

16 Note that this behavior will of course not be sustained infinitesimally as then for $t \to \infty$, the pollution stock $Z$ would grow exponentially to infinity. If a global subsystem that is faced with depletion such as the Amazon rainforest or the Greenland Ice Sheet is concerned, the regeneration rate may stay negative until the state of the subsystem is zero, i.e. the rainforest or ice sheet has disappeared. In contrast, on a global level, there will be a state $Z > Z > Z_{TIP}$ for which $\delta(Z) > 0$, again. This means, there will be a new global steady state (which is most probably much less favorable to humans than the current one).

17 Still, the tipping point $Z_{TIP}$ is not a planetary boundary as defined by Rockström et al. (2009) and Steffen et al. (2015), even though the planetary boundary quantifications are informed by tipping point assessments (i.e. set at the precautionary end of the scientific uncertainty range, which includes probabilities of crossing tipping points as well as undermining long-term resilience of the Earth system). The rationale behind limiting $Z \leq Z_{TIP}$ is based on the willingness to pay to avoid the domain of ambiguity for $Z \geq Z_{PP}$. The tipping point $Z_{TIP}$ becomes the upper limit only due to the inevitability of entering the domain of ambiguity in the presence of self-amplifying processes once the tipping point is crossed. Still, the tipping point $Z_{TIP}$ may also give reason for a planetary boundary. However, as noted above, planetary boundaries are set at the lower end of a zone of uncertainty.
Figure 4: The cost of precaution in the presence of tipping points. The temporal dimension is disregarded here for illustrative purposes.
Z, Z_{CBA}, is below the domain of ambiguity. However, due to a tipping point \( Z_{TIP} < Z_{CBA} \) that leads to self-amplifying processes, this welfare-optimal value of \( Z \) cannot be maintained. Instead, with a certain time-lag, \( Z \) will degrade into the domain of ambiguity once \( Z_{TIP} \) is crossed. Thus, to avoid the domain of ambiguity, there is a cost of precaution as \( Z \) needs to be limited to \( Z_{TIP} < Z_{CBA} \). This cost of precaution is even higher in panel b of Figure 4. As \( Z_{CBA} \) is within the domain of ambiguity, there is already a cost of precaution, just as in panel (B) of Figure 3. However, this cost is further amplified by the tipping point \( Z_{TIP} \) in Figure 4. Note, as the graphs in Figure 4 do not depict the temporal dimension, a direct comparison with Figure 3 would be stylized.

### 3.4 Threshold Crossing and Non-Observable System States

Thus far, certain knowledge about natural system state \( Z \) and tipping point \( Z_{TIP} \) was assumed in the developed model framework. Uncertainty about damage estimates was accounted for implicitly by including risk premiums in the damage function. The fundamental uncertainties about catastrophic impacts that may materialize with a non-negligible probability in the domain of ambiguity give reason for limiting \( Z \) to below \( Z_{PP} \). In the presence of tipping points and self-amplifying processes, it would be limited to \( Z_{TIP} \). In the context of climate change, integrated assessment models have already included threshold dynamics related to tipping points, uncertainty about tipping points as well Bayesian learning in the presence of such uncertainty (e.g. Cai et al. 2015; Hwang et al. 2017; Lemoine and Traeger 2014, 2016b; Lontzek et al. 2015). We refrain from endogenizing uncertainty and learning in our model framework as this would unnecessarily complicate the conceptual ideas. In this sense, we follow a pragmatic approach.

So far, we have assumed that the threshold \( Z_{TIP} \) is not crossed – such that \( Z = Z_{TIP} \) in Eq. (5). Assuming certain knowledge of the natural system state and the location of thresholds and tipping points in the model framework is not in line with the planetary boundary framework. Planetary boundaries are set to avoid a zone of uncertainty around thresholds or to avoid large-scale changes in one planetary boundary that make the crossing of other boundary thresholds more likely. Thus, in the following, we formally relate planetary boundaries as defined by Rockström et al. (2009) and Steffen et al. (2015) to the model framework.

Thus far, we have assumed that the value of \( Z_{TIP} \) is known and that we can perfectly observe the state of a natural system \( Z \). Now, suppose that we cannot observe the state of the natural system \( Z \), but only a proxy variable \( W \). For example, we may use tree cover, habitat size, or species extinction as proxy \( W \) instead of the true natural system state \( Z \), which represents biosphere integrity (the resilience of
ecosystems to shocks and stress and their capacity to uphold ecological functions). Other examples might be ice sheet volume, flow, and resulting ice mass loss estimates (W) instead of the stability of the Greenland Ice Sheet (Z). Aragonite saturation of mean surface ocean (W) could be used instead of the health of coral reef ecosystems (Z). The proxy W is linked to the true state Z, but it might react to changes in the true state with a delay. For example, initiation of large-scale ice sheet instability of the West Antarctic Ice Sheet (WAIS) cannot be observed directly (Z) and it is unclear whether the accelerating mass loss of the Amundson Sea sector of West Antarctica is a delayed indicator (W) that such a destabilization has already occurred (Feldmann and Levermann 2015).

This means that a system of interest might have crossed a threshold resulting in shifts in feedback, making a state change unavoidable, even if full impact is only felt after decades or centuries. For example, a 1.5 °C global mean surface temperature rise may only cause 50 cm of global sea level rise by 2100. It will, however, commit all future generations to 2 m of sea level rise (IPCC 2021). It might also mean that the system is irreversibly damaged before it becomes visible in terms of, e.g. decreasing tree cover or glacier melting. Formally, this can be expressed as:

$$W = \gamma(Z) + \varepsilon$$

where \(\gamma\) captures the (known) functional form of the relationship between \(W\) and \(Z\), which (depending on the system of concern) may also incorporate a delayed response of the proxy variable to the true system state. We assume here (without loss of generality) that \(W\) is measured in units of \(Z\) equivalents.\(^{18}\) We define \(\varepsilon\) to be an error term with \(E[\varepsilon] = 0\). It follows that for \(W < \gamma(Z_{TIP})\), it remains unclear whether the threshold has already been crossed. It could be, that \(Z > Z_{TIP}\), but due to the error in the observable proxy, \(W\) is still below \(Z_{TIP}\) for a considerable period of time. Clearly, due to this, \(W_{PB}\) should be chosen to be lower than \(\gamma(Z_{TIP})\).

In such a setting, it is sensible to define an observable proxy boundary

$$W_{PB} = E[\gamma(Z_{TIP})] - \pi$$

where \(\pi\) is a risk premium subtracted (or added depending on whether we have a pollution or depletion problem) to the expected value of the tipping point measured in the proxy variable \(W\) (the PB in \(W_{PB}\) stands for planetary boundary). The risk premium depends on society’s risk aversion, the degree of uncertainty about the relationship between \(Z\) and \(W\), and the variance of the measurement error \(\varepsilon\). However, if \(\pi\) is chosen too high and thus \(W_{PB}\) is set too low, welfare costs might be excessive, as discussed above. Thus, to set the boundary \(W_{PB}\), a Bayesian approach seems to be very useful. In this case, \(W_{PB}\) is chosen based on some prior beliefs about the system and key

\(^{18}\) Otherwise, we would just need to apply a linear conversion factor.
structural parameters (like $\gamma, Z_{TIP}, \delta(\cdot)$). After some time observing the response of $W$ to pollution $X$, one can refine and update these parameters, and thus, $W_{PB}$. It is important to stress that the proxy boundary $W_{PB}$ is not fixed, as is the case with $Z_{TIP}$. It is a preliminary boundary that is necessary due to the unobservability of $Z$ and dependent on the risk premium $\pi$. However, as new knowledge is advanced and the relationship between natural system state and proxy becomes better known, it may be adjusted. Thus, in the deterministic welfare optimizing framework, the true but unobservable limit $Z_{TIP}$ would be replaced with $W_{PB}$. Continuously updating $W_{PB}$ as learning occurs, is a way to cautiously approach true tipping points. Hence, by setting $W_{PB}$ as a boundary (replacing $Z$ in our framework above), the welfare optimizing pathways would again prevent crossing $W_{PB}$. If the limit becomes binding, the shadow price $\lambda$ would reflect the value of $W_{PB}$, accordingly.

Figure 5 illustrates this reasoning. The natural system state is unobservable and the proxy $W$ reacts stochastically to changes in the true state. Thus, uncertainty exists about when $Z_{TIP}$ is crossed (grey gradient in Figure 5). The exact location of

Figure 5: Applying a proxy boundary due to the unobservability of the natural system state. The temporal dimension is disregarded here for illustrative purposes.
$Z_{TIP}$ may also be uncertain. Both uncertainties support setting a conservative proxy boundary $W_{PB}$ at the lower end of the zone of uncertainty.

If for some reason – like natural events or missing policy intervention – the proxy boundary was crossed, a system collapse might not follow with certainty, as the proxy boundary was chosen conservatively. The optimal speed to return to the boundary $W_{PB}$ depends in this case on the expected likelihood that the tipping point has not already been crossed and on the willingness to pay to avoid entering the domain of ambiguity. If pollution levels are below the regeneration rate, $W_{PB}$ can be reached after some time. The speed at which $W_{PB}$ is reached will also reveals some information about the underlying system dynamics. This could be used to update the boundary $W_{PB}$ following a Bayesian approach.

In this sense, the boundary $W_{PB}$ corresponds to a planetary boundary as defined by Rockström et al. (2009) and Steffen et al. (2015). The presence of local and global thresholds in the (unobservable) natural system state and the aim to avoid the domain of ambiguity (which is associated with catastrophic welfare damages) out of precaution, justifies setting a boundary at the lower end of the zone of uncertainty. The planetary boundary $W_{PB}$ is a proxy boundary for the true threshold in the natural system state. It accounts for the uncertainties on the position of $Z_{TIP}$ as well as delays in the response of the proxy to changes in the natural system. Note that not all planetary boundaries are based on tipping points: some are motivated by their strong influence on tipping points of other planetary boundaries. Others may be associated with large-scale change without tipping behavior that might nevertheless lead to catastrophic welfare losses. For such planetary boundaries, $W_{PB}$ is set in relation to $Z_{PP}$, which refers to the lower end of the domain of ambiguity. While insights from natural sciences are crucial to identify tipping points and other (more complex) nonlinear dynamics, the economic welfare theoretic approach outlined above identifies optimal pathways and shadow prices (that serve as benchmark for economic policy instruments like environmental taxes) to remain within safe Earth-system boundaries.

### 3.5 Synthesis: The Reversal of the Burden of Proof

In Section 2, the model framework was introduced. The placeholder $\bar{Z}$ was used to denote a limit that was set as a constraint in our welfare optimization. In Section 3, decision paradigms were introduced that justify several potential limits that may replace $\bar{Z}$ in an application of the model framework.

Two points that follow from the discussion of decision paradigms are particularly relevant: First, CBA is only a special case of the general framework developed in Section 2. For it to be applicable, CBA must reflect all relevant damages. Otherwise, it
underestimates welfare losses and justifies a limit that is too high. Second, there is a reversal of the burden of proof. If $Z_{CBA}$ is greater than the planetary boundary ($W_{PB}$), a tipping point ($Z_{TIP}$), or the lower end of the domain of ambiguity ($Z_{PP}$), not setting a limit to $Z$ – and thus choosing $Z_{CBA}$ as the result of an unconstrained welfare optimization – can only be justified, if at all, by considering the precautionary principle to cause excessive costs. Thus, CBA would only be used for welfare optimization if either limiting human activities is extremely expensive, or it is believed that there are only extremely small probabilities of catastrophe involved in the domain of ambiguity (see Gardiner 2006, pp. 51–52 on the last point).

Figure 6 synthesizes the considerations presented in Section 3. The decision tree guides a decision maker through the process of choosing a limit that may apply to the intertemporal welfare optimization problem presented in Eqs. (2)–(6). To address the shortcomings of practical CBA with regard to environmental problems as captured by the planetary boundary framework, the welfare maximization of CBA may be constrained with an exogenously set limit. There are several rationales for setting such a limit: 1) a willingness to pay to avoid the domain of ambiguity, in which the probability of catastrophic welfare damages is non-negligible; 2) the presence of tipping points, which when crossed lead to a collapse of the stability of natural systems; and 3) the unobservability of the natural system and the need to rely on proxy variables to assess its true state. Which of these rationales is most appropriate depends on the environmental problem at hand. The table in Figure 6 presents an overview and short explanation of the potential limits. The arrow below the decision tree illustrates the ordering of the potential limits by their restrictiveness. The decision tree summarizes the evaluations involved in (potentially) choosing a constraint for welfare optimization.

Let us take climate change as an example. The natural system $Z$ of interest is in this case the stability of the climate system. The first question in the decision tree asks whether this natural system exhibits a tipping point. Because it does, this tipping point may apply as a potential constraint to welfare optimization. Then, the tree leads to the question of whether $Z$ is observable. Unfortunately, the stability of the climate system can only be inferred from proxy variables such as the behavior of carbon sinks and the realization of weather events. We cannot explicitly observe a tipping point in the climate system itself. Thus, a proxy variable $W$ (e.g. temperature rise above pre-industrial levels) has to be chosen. On this proxy variable, a boundary is conservatively set such that as long as this proxy boundary is not crossed, it is unlikely for the climate system to tip according to scientific assessment. This is equivalent to the planetary boundary for climate change. Depending on the exact specification, practical CBA may conclude that the amount of climate change that maximizes welfare in terms of discounted net benefits ($Z_{CBA}$) is above the planetary boundary for climate change ($W_{PB}$). In such a case, the cost of precaution is assessed.
Figure 6: Decision tree to guide the choice of a limit that should be set on human activities straining natural systems.
by comparing net benefits of the unconstrained (cost-benefit) case with the net benefits of the constrained case using the planetary boundary as a limit. There are only two cases in which the welfare optimal value of global warming yielded by practical CBA ($Z_{CBA}$) may be used to guide human activity: 1) the welfare optimal value of global warming is below the planetary boundary ($Z_{CBA} < WPB$) as damages become too high even before the boundary is reached; and 2) the welfare optimal value of global warming is above the planetary boundary ($Z_{CBA} > WPB$) and the difference between the net benefits of the unconstrained case and the net benefits of the constrained case is exorbitantly large. In other words, the cost of precaution is excessive. Otherwise, the welfare maximizing path constrained by the planetary boundary $WPB$ must be chosen. The same procedure may be applied to other planetary boundaries such as nitrogen or land-system change.

4 Model Extensions: Accounting for Interlinkages

The model developed in Section 2 can be extended in various ways to reflect more complex relationships among natural systems, benefits, and damages. Two potential extensions are especially relevant for the application of the model framework to planetary boundaries. First, including interlinkages among planetary boundaries. This would take the form of including the state of other planetary boundaries into the transition function of the planetary boundary in focus. More specifically, one could model these interlinkages by making the regeneration rate $\delta(\bullet)$ dependent on multiple stocks that represent the state of several natural systems relevant for planetary boundaries. A second, more complex potential extension, include regional–global interlinkages. This case is examined below.

While some planetary boundaries, such as climate change, create damages on a global scale and the spatial distribution of the polluting activity does not matter, damages from other planetary boundaries, such as nitrogen and biosphere integrity, manifest at a regional or even subregional scale. Still, those planetary boundaries have regional–global interlinkages. The developed model framework allows for the integration of this aspect. To do so, the index $i$ is introduced to denote regions. The exact definition of “regions” is irrelevant in this general framework. In further applications of the model framework, “regions” could be defined in a way that best serves the specific modelling aim. Generally, the index $i$ can denote any sub-global spatial aggregation. This may range from highly aggregated sets characterized by biophysical properties, e.g. biomes (tropical forest, temperate grasslands etc.) or tipping elements (AMOC, WAIS, etc.), to more regional environmental systems like anoxic coastal zones or river delta wetlands (Baltic Sea, Yellow River delta), to even smaller units like farmland at the scale of hectares with high nitrogen loading or
pixel-based classifications. Spatial aggregations along socially constructed lines like nation states are also conceivable.

In the following, we will use the concept of a proxy boundary as described above and relate the model to planetary boundaries. For this, we assume $n$ regions and introduce the subscript $i = \{1, \ldots, n\}$ to denote regions. $B_i(X_i)$ are regional benefits from regional pollution flows that increase the stock variable $W$, which is a proxy for the natural system state and serves as the stock variable in the planetary boundary framework. We distinguish between damages from global pollution stocks $D_i(W)$ and damages from regional pollution stocks $G_i(W_i)$. The global pollution stock is given by the sum of regional pollution stocks, $W = \sum_i W_i$. The pollution is limited by the global planetary boundary $WPB$ and regional sub-boundaries $WPB_i$.

### 4.1 Globally Optimal Results

From a global, aggregate welfare perspective, the intertemporal optimization problem to be solved is

$$\max_{X_1, \ldots, X_n} \int_0^\infty \sum_i [B_i(X_i(t)) - D_i(W(t)) - G_i(W_i(t))] e^{-\rho t} dt$$

subject to

$$\dot{W}(t) = \sum_i X_i - \delta(W(t))W(t)$$

$$\dot{W}_i(t) = X_i - \varepsilon_i(W_i(t))W_i(t) - \delta_i(W(t))W_i(t) \quad \forall i = \{1, \ldots, n\}$$

$$WPB - W(t) \geq 0 \quad \& \quad WPB_i - W_i(t) \geq 0 \quad \forall t$$

$$\lim_{t \to \infty} \mu(t)W(t)e^{-\rho t} = 0, \quad \lim_{t \to \infty} \lambda_i(t)W_i(t)e^{-\rho t} = 0$$

The first constraint (Eq. (34)) is the transition function of the global stock. The second constraint (Eq. (35)) is the transition function of the regional stock and, in fact, consists of $n$ constraints, one for each region $i$. The global pollution stock regenerates with rate $\delta$, while the regional pollution stock regenerates with a local rate $\varepsilon_i$ which is dependent on the local pollution stock and a local rate $\delta_i$ which is dependent on the global pollution stock. Disaggregating the regional regeneration capacity into a component depending on the local pollution stock ($\varepsilon_i(W_i)$) and a second component depending on the global stock ($\delta_i(W)$) increases the flexibility of the modelling framework.

While not necessarily applicable to all planetary boundaries, there are examples that justify this setup. For example, the resilience and regenerative capacity of the marine biosphere integrity depends on globally interconnected food webs that relate
to global stocks (e.g. krill and phytoplankton stocks form the basis of the food chain and may influence resilience of the biosphere in region \( i \) while not reproducing in region \( i \) but, e.g. in Antarctic waters) as well as regional ecosystem states that relate to regional stocks. Similarly, regional aerosol loading and the capacity of the regional biosphere and climatic conditions to reduce aerosol loading are also influenced by background aerosol loadings that comes from globally dispersed and sometimes far away sources (e.g. volcanic eruptions may increase global background aerosol loadings, which influence the capacity of the regional vegetation and (micro-)climate, e.g. through cloud formation, to reduce local aerosol loading) (Kristiansen et al. 2016).

Using optimal control theory, we obtain the following optimality conditions (see Appendix A2 for the full derivations) before either of the limits becomes binding:

\[
-B'_i(X_i) = \mu + \lambda_i \quad \forall \ i = 1, \ldots, n
\]  

(38)

\[
\hat{\lambda}_i \frac{\dot{\lambda}_i}{\lambda_i} = \rho + \varepsilon_i + \delta_i + \varepsilon'_i \times W_i - \frac{G_i(W_i)}{B'_i(X_i) + \mu} \quad \forall \ i = 1, \ldots, n
\]  

(39)

\[
\hat{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta + \delta' \times W - \frac{\sum D'_i(W)}{B'_i(X_i) + \lambda_i} + \sum \left[ \frac{\lambda_i}{\mu} \times \delta'_i \times W_i \right]
\]  

(40)

Compared to the purely global model presented in Section 2, we now distinguish between the global price, \( \mu \), and the regional prices, \( \lambda_i, \ i = 1, \ldots, n \). For each region, the sum of the global and the regional shadow price must equal the marginal benefits of pollution, \( B'_i(X_i) \) (Eq. (38)). The path of the regional price, \( \hat{\lambda}_i \) (Eq. (39)) is very similar to the results from the global model above (Eq. (26)), but now, the regeneration rate is split in a regional (\( \varepsilon_i \)) and a global component (\( \delta_i \)). As in Section 2.3, the path of the regional price, \( \hat{\lambda}_i \), becomes flatter compared to the usual Hotelling rule due to damages to the regenerative capacity (\( \varepsilon'_i < 0 \)) and due to economic damages from an increasing pollution stock (\( G'_i(W_i) \)), and there is a marginal rate of substitution between damages and benefits. However, in the marginal rate of substitution between regional damages and regional benefits in Eq. (39), the shadow price of the global boundary, \( \mu \), is added to the marginal regional benefits. Thus, there is a global component in the shadow price of regional pollution, steepening the price path. This pushes regional decision makers toward later but larger increases in abatement.

As in Section 2.2, Eq. (26), \( \hat{\mu} \) is the growth rate of the optimal global shadow price of pollution, \( X_i \) (Eq. (40)). When regional-global interlinkages are taken into account, the global shadow price path has the additional component \( \sum \left[ \frac{\lambda_i}{\mu} \times \delta'_i \times W_i \right] \). This term considers the regeneration rate of the regional boundary (times the ratio of the regional and global price \( \frac{\lambda_i}{\mu} \)). Thus, the global price is further flattened by the damages
to the regional regenerative capacity, $\delta_i < 0$, implying stronger initial policy action (i.e. earlier abatement) on the global scale. The relative level of regional and global shadow prices reveals the relative importance of regional and global boundaries, pressing for action on the more concerning scale. This provides information to political decision makers on which scale, regional or global, political capital should be spent.

Importantly, it becomes apparent that the optimal growth rates of the global and regional prices interact as $\hat{\lambda}_i = \hat{\lambda}_i (\mu)$ and $\hat{\mu} = \hat{\mu} (\lambda_i)$. Hence, for an optimal management of planetary boundaries, the regional dimension cannot be separated from the global dimension as both need to be determined simultaneously. This shows that regional and global boundaries need to be aligned in the governance of planetary boundaries.

It is worth noting that this extension of the model framework could also be modified to reflect cross-interdependencies among planetary boundaries that manifest at different levels. This would essentially be a combination of the two potential extensions outlined at the beginning of this section. In such a case, regional–global interactions as well as interdependencies among planetary boundaries on both levels would be modelled. There are many examples that demonstrate the relevance of such model extensions. For instance, there is rising evidence that interhemispheric differences in aerosol loading, in particular when transgressing the aerosol planetary boundary in the northern hemisphere, may shift the inter-tropical convergence zone southwards and thereby cause reductions in monsoon rainfall in the southern hemisphere (Donohoe et al. 2013). Similarly, land-use change, such as deforestation in the Amazonas, creates regional damages and benefits and is also linked to global damages from climate change as the regeneration rate for carbon is diminished. This highlights the necessity to align regional and global boundaries within one environmental problem dimension and across different boundaries.

### 4.2 Non-Cooperative Regional Optimization

Accounting for regional–global interlinkages in the model framework also allows us to investigate non-optimal regimes. In the previous section, we assumed global cooperation ensuring that the global, aggregate welfare optimum is achieved through the maximization of the sum of all regional net benefits. However, in the absence of such cooperation, each region would maximize its net benefits disregarding external effects on other regions. The intertemporal optimization problem to be solved then is for each region $i$:

$$\max_{X_i} \int_0^\infty \left[ B_i (X_i (t)) - D_i (W (t)) - G_i (W_i (t)) \right] e^{-\rho t} dt$$ (41)

subject to
\[ \dot{W}(t) = \sum_i X_i(t) - \delta(W(t))W(t) \quad (42) \]

\[ \dot{W}_i(t) = X_i(t) - \varepsilon_i(W_i(t))W_i(t) - \delta_i(W(t))W_i(t) \quad (43) \]

\[ W_{PB} - W(t) \geq 0 \quad \& \quad W_{PB,i} - W_i(t) \geq 0 \quad \forall \ t \quad (44) \]

\[ \lim_{t \to \infty} \mu(t) W(t) e^{-\rho t} = 0, \quad \lim_{t \to \infty} \lambda_i(t) W_i(t) e^{-\rho t} = 0 \quad (45) \]

The first constraint is the same as above, the second constraint now truly is only one constraint instead of \( n \) constraints as above. Using optimal control theory, we obtain the following optimality conditions (see Appendix A2 for the full derivations) before either of the limits becomes binding:

\[ -B_i'(X_i) = \mu + \lambda_i \quad (46) \]

\[ \tilde{\lambda}_i = \frac{\dot{\lambda}_i}{\lambda_i} = \rho + \varepsilon_i + \delta_i + \varepsilon_i' \times W_i - \frac{G_i'(W_i)}{E_i'(X_i) + \mu} \quad (47) \]

\[ \tilde{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta + \delta' \times W - \frac{D_i'(W)}{E_i'(X_i) + \lambda_i} + \frac{\lambda_i}{\mu} \times \delta_i \times W_i \quad (48) \]

The conditions in Eqs. (46) and (47) remain unchanged compared to the results with global cooperation. However, the growth path of the global shadow price in Eq. (48) disregards marginal damages as well as the regional shadow prices of all other regions \( j \neq i \). This means the last two terms in Eq. (48) are smaller than in the cooperative result in Eq. (40):

\[ \frac{D_i'(W)}{E_i'(X_i) + \lambda_i} < \frac{\sum_i D_i'(W)}{E_i'(X_i) + \lambda_i} \quad \text{and} \quad \frac{\lambda_i}{\mu} \delta_i W_i < \frac{\sum_i \lambda_i}{\mu} \delta_i W_i. \]

Thus, the growth rate of the global shadow price is higher, and the price path is steeper in the non-cooperative result. This leads to delayed abatement on the global scale of the planetary boundary.

As the growth rate of the regional shadow price, \( \tilde{\lambda}_i \), depends on the global shadow price \( \mu \), the regional shadow price path flattens compared to the cooperative result. This pushes regional decision makers to early strong abatement on the regional scale but too little and too late abatement on the global scale.

## 5 Applying the Model Framework to Planetary Boundaries

Thus far, we have developed a unified theory of CBA and CEA. The model framework outlined above considers marginal damages that are inflicted on human wellbeing as
well as deliberately set limits to calculate the shadow price of human activities that strain planetary boundaries. This section clarifies the application of the model framework to specific Earth system problems captured by the planetary boundary framework. To do so, we first define the human activities that strain planetary boundaries and the potential to substitute with activities that inflict less harm. Thereafter, we classify planetary boundaries with economic terminology and give examples for what lies behind the mathematical notation used in the model framework.

5.1 Planetary Boundary Straining Activities and the Potential for Substitutes

The planetary boundary framework captures the fact that human economic activities alter natural systems and strain the resilience of fundamental Earth system processes to remain in their current equilibrium. These human activities were denoted as $X$ in the model framework above. In general, $X$ can refer to water use, habitat conversion, resource use, side-products of certain economic activities like carbon emissions from fossil fuel use, or other types of pollutants. In the context of planetary boundaries, specific human activities that particularly strain natural system equilibria can be identified.

Table 1 contains all nine planetary boundaries and sub-components. The column “Proxy Variable” is included to clarify how the boundaries are quantified in the planetary boundary framework (Steffen et al. (2015) call these variables “control variable”). The proxy variables can be interpreted as corresponding to $W_{PB}$ in the model framework above. For each boundary, the main human activities that strain that boundary are listed. Additionally, the substitutability of these activities is assessed. See Appendix A4 for the references used for the assessment in Table 1. Note that the substitutability assessment does not refer to the benefits associated with remaining below planetary boundaries. Those benefits are the avoided damages $D(Z)$.$^{19}$ Instead, the assessment looks at the extent to which human activities that strain planetary boundaries are essential or to which a substitute can be utilized in the near future.

The column “Main activities altering natural system” illustrates which measurable variables could be used for $X$ when numerically applying the model

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19 Some ecosystem services are essential as physical capital and innovation cannot substitute these services (e.g. ecosystem service that are essential for food production or drinking water provision). In such cases, marginal damages increase to infinity when ecosystem services deteriorate (see, e.g. Drupp and Hänsel 2021; Sterner and Persson 2008).
**Table 1:** Human activities straining planetary boundaries and their substitutability.

<table>
<thead>
<tr>
<th>Planetary boundary</th>
<th>Proxy variable</th>
<th>Main activities altering natural system</th>
<th>Substitutability</th>
</tr>
</thead>
</table>
| Climate change     | Atmospheric CO₂ concentration (in ppm) or energy imbalance at top-of-atmosphere (in W/m²) | - Fossil fuel burning  
- Agriculture  
- Deforestation | Yes – high to medium  
- High for fossil fuel burning  
- Medium to low for agriculture and deforestation |
| **Biosphere integrity** | Genetic diversity | Extinction rate (in extinctions/million species-years) | - Habitat loss through  
- Land-system change (esp. deforestation)  
- Pollution (esp. N and P)  
- Overexploitation  
- Non-native species introduction | Yes – medium to low  
- Medium to low for habitat loss  
- Medium for overexploitation and non-native species |
|                   | Functional diversity | Biodiversity intactness index (in % of pre-industrial levels) | - Habitat loss through  
- Land-system change (esp. deforestation)  
- Pollution (esp. N and P)  
- Harvesting (hunting, poaching, etc.) | Yes – medium to low |
| Land-system change | Area of forested land (in % of original forest cover) | - Deforestation  
- Wet-/Peatland conversion  
- Soil sealing (urbanization and infrastructure building) | Yes – medium to low  
- Medium to low for deforestation and wet-peatland conversion  
- Low for soil sealing |
| Freshwater use     | Consumptive blue water use (in km³/year) | - Agricultural irrigation  
- Industrial water use  
- Household water use | No (but renewable resource) |
| **Biochemical flows** | Phosphorous | Flow from freshwater systems into ocean (in million tons (Tg)/year) | - Application of synthetic fertilizer | No |
Table 1: (continued)

<table>
<thead>
<tr>
<th>Planetary boundary</th>
<th>Proxy variable</th>
<th>Main activities altering natural system</th>
<th>Substitutability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>Industrial and intentional biological fixation (in</td>
<td>Application of fertilizer</td>
<td>Yes – medium</td>
</tr>
<tr>
<td></td>
<td>million tons (Tg)/year)</td>
<td>– Fossil fuel burning</td>
<td>– Low for fertilizer</td>
</tr>
<tr>
<td></td>
<td></td>
<td>– Fossil fuel burning</td>
<td>– High for fossil fuel burning</td>
</tr>
<tr>
<td>Ocean acidification</td>
<td>Aragonite saturation state of mean surface ocean</td>
<td>CO₂ emissions from</td>
<td>Yes – high to medium</td>
</tr>
<tr>
<td></td>
<td>(in % of the pre-industrial levels)</td>
<td>– Fossil fuel burning</td>
<td>– High for fossil fuel burning</td>
</tr>
<tr>
<td></td>
<td></td>
<td>– Agriculture</td>
<td>– Medium to low for deforestation and</td>
</tr>
<tr>
<td></td>
<td></td>
<td>– Deforestation</td>
<td>industrial processes</td>
</tr>
<tr>
<td>Atmospheric aerosol</td>
<td>Aerosol optical depth</td>
<td>Fossil fuel burning</td>
<td>Yes – high</td>
</tr>
<tr>
<td>loading</td>
<td></td>
<td>– Deforestation through burning</td>
<td></td>
</tr>
<tr>
<td>Stratospheric ozone</td>
<td>Stratospheric O₃ concentration (in % from pre-</td>
<td>Release of CFCs and HCFCs</td>
<td>Yes – high</td>
</tr>
<tr>
<td>depletion</td>
<td>industrial level)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Novel entities</td>
<td>n. a.</td>
<td>Chemical pollution</td>
<td>?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>– GMOs</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>– Release of heavy metals</td>
<td></td>
</tr>
</tbody>
</table>

To avoid misunderstanding with Earth system/ecological science, we want to highlight that the substitutability assessment in the last column refers to the substitutability of the human activities straining planetary boundaries and not to the benefits associated with remaining below planetary boundaries. See Appendix A4 for references.
framework developed in this paper. The column “Substitutability” is strongly related to Section 2.4, which outlines long-run pollution levels and shadow prices. Section 2.4 established that if substitutability is not given or is very low for a human activity and regeneration rates of the planetary boundary are zero, the shadow price will go toward infinity when the boundary is approached. In contrast, the shadow price of a planetary boundary with high substitutability of the straining activity will not surpass the price of the substitute, as the straining activity will be substituted once it is cheaper to do so. The substitutability assessment is subject to large uncertainties, as the future availabilities of new technologies that may enable substitutions are difficult to predict. However, it is still possible to conduct some general assessment based on the nature of the problem.

*Climate change* and *stratospheric ozone depletion* are, for example, to a large part characterized by the release of a by-product of production into the atmosphere. These emissions are not a necessary or unavoidable economic activity – they can be mitigated at a cost. Land for agricultural production on the other hand, is very much needed to feed the growing human population. It is difficult to substitute the associated land conversion that strains *Biosphere integrity* and *Land-system change*. There might be potential to increase the efficiency of agricultural land-use, but there is no substitute for a certain (large) surface area needed in agricultural production. This limited substitutability implies that shadow prices (and thus optimal environmental pricing) could become very large for land-converting activities.

### 5.2 Classification by Economic Terminology

To further illustrate the application of the model framework to planetary boundaries, Table 2 classifies planetary boundaries with economic terminology and shows the quantities behind the mathematical notation used above. The classification does not claim to account for the full complexity of planetary boundaries. Instead, it is intended to facilitate discourse and research centered on planetary boundaries by fleshing out the generalized model framework developed above. For the references used for the assessment in Table 2, see Appendix A5.

The classification of planetary boundaries into *open access resource versus pollution problem* is closely linked to the formulation of the model framework as a depletion or pollution problem. If the planetary boundary is an open access resource, the budget formulation using $Y$ is more suitable (see Appendix A1). If the planetary boundary is rather based on a pollution problem, the formulation using $Z$ as a stock variable is more fitting. The term “open access resource” describes resources whose utilization cannot be restricted and is rivalrous, meaning that utilization of the resource by one agent leads to less availability for other agents. Thus, open access
Table 2: Classification of planetary boundaries along model terminology.

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</thead>
<tbody>
<tr>
<td>Climate change</td>
<td>P</td>
<td>S</td>
<td>Slow (CO₂) to medium (Methane)</td>
<td>Yes</td>
<td>Yes, T</td>
<td>Yes, T</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Climate response to CO₂ pulse approx. constant over centuries</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>- Half-life time of methane in the atmosphere is 12 years</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>- 1.5–2 °C (disintegration of Greenland and West Antarctic ice sheets as starters of domino effect)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>- See Lenton et al. (2008) for further tipping points</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biosphere integrity</td>
<td></td>
<td></td>
<td>Slow</td>
<td>Yes</td>
<td>Yes</td>
<td>n. a.</td>
</tr>
<tr>
<td>Genetic diversity</td>
<td>Depends on underlying driver of species extinction</td>
<td>S</td>
<td>Slow</td>
<td>No</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- One lineage split every two million years</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>- 3–6 species/year for complete biota</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Functional diversity</td>
<td>Open Access [O], Pollution [P]</td>
<td>S</td>
<td>Slow to medium</td>
<td>No</td>
<td>Yes</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Highly variable depending on ecosystem</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Rainforest may recover within 100–300 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Land-system change</td>
<td>Open Access [O]</td>
<td>S</td>
<td>Slow to medium</td>
<td>No</td>
<td>Yes</td>
<td>N</td>
</tr>
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Table 2: (continued)

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</thead>
<tbody>
<tr>
<td>Freshwater use</td>
<td>O</td>
<td>F – surface S – fossil</td>
<td>Fast</td>
<td>No</td>
<td>Yes</td>
<td>N</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biochemical flows</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phosphorous</td>
<td>P</td>
<td>S</td>
<td>Slow to medium</td>
<td>No</td>
<td>Yes</td>
<td>N</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>P</td>
<td>S</td>
<td>Fast to medium</td>
<td>No</td>
<td>Yes</td>
<td>N</td>
</tr>
<tr>
<td>Ocean acidification</td>
<td>P</td>
<td>S</td>
<td>Slow</td>
<td>Yes</td>
<td>Yes</td>
<td>T</td>
</tr>
</tbody>
</table>

- Surface freshwater is renewable resource and usually regenerates within years
- However, fossil freshwater does not regenerate at all
- Up to 10,000 years in deep ocean
- Ten years half-life time for pyrophosphates, 20 years half-life time for mono- and di-esters in freshwater bodies
- Globally, denitrification fluxes seem to match anthropogenic nitrogen fixation
- Regionally, accumulation in terrestrial biomass and soils is likely
- Dissolved carbon remains >10,000 years in deep sea
- 350 ppm CO₂ (mass loss of coral reefs)
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric aerosol loading</td>
<td>P</td>
<td>F</td>
<td>Fast</td>
<td>No</td>
<td>Yes</td>
<td>T</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Atmospheric aerosol lifetime is typically less than a month</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stratospheric ozone depletion</td>
<td>P</td>
<td>S</td>
<td>Medium</td>
<td>Yes</td>
<td>275.5 O₃ concentration in Dobson units</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>18–31% recovery in 31 years; full recovery expected within 80 years (1980–2060)</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

See Appendix A5 for references on the assessments in the columns “Regeneration rate of system”, “Global tipping points”, and “Regional thresholds or tipping points”.

resources are vulnerable to overexploitation through externalities. Pollution problems arise from externalities as well. However, in this case the utilization of a resource is not at the heart of the planetary boundary, but rather the by-products that create damages.

The stock versus flow dimension is a common distinction in the characterization of environmental economic problems. Stock problems refer to damages created by the accumulation of emission stocks or the loss of benefits through the depletion of environmental stocks. Flow problems refer to damages created by the flow of emissions. Once the flow stops, damages fall to zero as well. As damages almost never drop to zero instantaneously once emission flows stop, the stock-flow distinction is rather a continuum than two distinct categories. Still, it is often useful to model environmental problems either as purely flow or purely stock problems.

The distinction between stock and flow problems is very closely related to the regeneration rate of a system. The slower the regeneration rate, the more pollutants accumulate, or stocks deplete. A sufficiently high regeneration rate implies that the system can be well approximated by a flow-pollutant problem. For a long-term steady state, pollution or depletion must equal the regenerative capacity of the natural system in the absence of substitutes for the polluting or depleting activity. The combination of low substitutability and slow regeneration necessitates the quickest reduction of the boundary straining activity. Even though we present an overall assessment of the regeneration rate, the classification may be highly variable in sub-systems of planetary boundaries. We denote a regeneration rate as fast if the underlying processes have an approximate time scale of less than 10 years. Medium refers to time scales of less than 100 years, while systems that regenerate on time scales over 100 years are classified to be slow.

The columns on tipping points are related to $Z_{TIP}$ in the model framework. Not all planetary boundaries have global tipping points. Still, limits in the sense of $W_{PB}$ as described in Section 3.3 are also set for the remaining boundaries in the planetary boundary framework as sub-global thresholds are present and interact with the planetary boundaries that exhibit global tipping points. For example, land and ocean carbon sinks are made possible through biosphere integrity and intact lands. If these sinks had not absorbed 56% of human induced CO$_2$ emissions since industrialization, the global mean temperature would have already exceeded 1.5 °C, pushing the climate planetary boundary into the high-risk zone of crossing thresholds. The planetary boundary for land-system change is a key example of a boundary without evidence of a planetary scale tipping point, but with strong interactions with other boundaries. Tree cover in major biomes is strongly interlinked with the climate effects of land surface cover (Steffen et al. 2015).

For example, loss of tree cover in the Amazon rainforest above a certain threshold through fire, drought, and deforestation may result in the tipping point for
the “Amazon dieback” to be triggered (Malhi et al. 2009). This would have climate effects beyond the Amazon region (Lenton et al. 2008). The latest evidence shows that the Brazilian part of the Amazon rainforest has already tipped from a net carbon sink to a carbon source (Gatti et al. 2021). This provides strong support for a planetary boundary approach – staying below the land use boundary set at $W_{PB} < Z_{TIP}$ to avoid that the climate boundary crosses its global tipping point $Z_{TIP}$.

The last column contains the distinction between national versus transnational damages to clarify incentive structures. It relates to marginal damages $D'(Z)$ in the model framework and is based on the authors’ own judgement. The column differentiates between terrestrial boundaries in which the boundary straining activity mostly generates marginal damages in the same area as the pollution or depletion is conducted and boundaries where damages occur in a greater area than that in which the polluting or depleting activity is located. Planetary boundaries with mostly national damages generate stronger incentives for regional action. Planetary boundaries with mostly transnational damages require transnational cooperation for effective governance and are subject to free-rider problems. Especially, for planetary boundaries that relate to flows, the categories “national” and “transnational damages” are not distinct. For example, nitrogen accumulates in water bodies that may be transnational and freshwater extracted from rivers may affect all adjoining countries further downstream. However, even in those cases, damages may (mostly) occur at the national level. Either because the polluting country is a riparian state of the transnational waterbody (e.g. Baltic Sea) or damages occur along the whole cascade of the pollution flow and not only in the accumulating water body (e.g. nitrogen accumulating in soils where it is applied as fertilizer as well as entering transnational streams via rainwater runoff (Meter et al. 2016)).

As Steffen et al. (2015) highlight, there is a hierarchy inherent to the planetary boundary framework. Climate change and biosphere integrity are so-called core boundaries. The other planetary boundaries are essential for the resilience of those core boundaries, but their transgression may not lead to a new state of the Earth system. The hierarchical effects between planetary boundaries such as indirect influences on regeneration rates or interconnectedness of tipping points among multiple boundaries has been disregarded in the assessment presented in Table 2.

### 5.3 Preliminary Thoughts on the Implications for Governance

Several insights for policy instruments can be derived from the model framework. Planetary boundaries impose scarcities on economies (Barbier and Burgess 2017). Due to their characteristic of a common pool resource, they need to be actively governed. Otherwise, the budget implied by planetary boundaries will be
overexploited and prices associated with planetary boundaries will be too low (see also Barbier and Burgess 2017). Thus, policy instruments are needed that address these externalities. Environmental taxation or indirect pricing instruments (obtained from permit trading systems) could be used at the core of a broad policy mix to internalize damages that occur before planetary boundaries are reached, and to ensure adherence to those boundaries. Accounting for damages and adhering to boundaries flattens the price paths associated with the activities that strain planetary boundaries (see Section 2.3). This implies that early and strong mitigation is favorable to starting with low ambition levels and ramping up steeply at later time periods.

Environmental taxes can conceptually be implemented in a straight-forward way by setting tax rates equal to shadow prices. Indirect pricing instruments like permit trading systems, however, need to account for potential intertemporal effects when banking or borrowing of permits is possible. While this intertemporal flexibility reduces permit price volatility and the associated welfare losses, the price increase in the permit market does not consider changes in environmental damages unless intertemporal trading ratios are introduced (Kalkuhl and Edenhofer 2014; Kling and Rubin 1997; Leiby and Rubin 2001). Hence, a standard permit trading scheme would mirror the problems of the conventional CEA in which cumulative emission targets are set without considering environmental damages. Near-term mitigation would be too low and too many mitigation efforts would be shifted into the future.

It is important to point out, that the model framework developed in this paper is an optimal one. Beyond the optimal path, only marginal damages of an additional unit of an activity that strains planetary boundaries can be assessed. In such non-optimal regimes, there is either no shadow price guiding such an activity or there are environmental taxes that do not correspond to the shadow price of the straining activity. Then, the planetary boundary may be crossed.

Planetary boundaries that generate regional problems should be treated accordingly and not framed simply as global problems. A global framing of planetary boundaries conceals the need for regional action. It unduly stresses the need for global cooperation to successfully address the environmental problem. Moreover, it distracts from regional benefits that arise from regional mitigation (see Section 4). Activities straining planetary boundaries such as nitrogen or land-system change may create significant regional damages that should create incentives for strong local action. In such cases, transnational cooperation and governance is only of limited use. Thus, political capital may be more effectively used by emphasizing the regional dimension of the planetary boundary and implementing policy instruments accordingly. Furthermore, the extension of the model framework to include regional–global interlinkages shows the necessity of designing policy instruments
such that regional and global boundaries are aligned. The existence of regional damages (vs. transboundary damages) implies regionally differentiated environmental prices or taxes. Hence, optimal environmental policies on freshwater use, fertilizer application, or habitat conversion should reflect a regional price component as well as a global component due to linkages with global natural systems.

Furthermore, marginal damages from planetary boundary straining activities are welfare relevant before planetary boundaries are reached or crossed. The environmental degradation related to planetary boundaries creates social costs independent of the adherence to the boundary itself. These social costs are often inappropriately accounted for in the evaluation of policy instruments. As a result, benefits from environmental policies are underestimated and the costs of staying below planetary boundaries are inflated.

Finally, the order of magnitude for steady state levels of pollution or depletion may vary greatly between planetary boundaries (see Section 5.2). The combination of low substitutability of the human activities that strain planetary boundaries and very low regeneration rates of the natural system that underlies the planetary boundary particularly needs attention. Low regeneration rates mean that the planetary boundary is approached more quickly, and low substitutability prevents switching the boundary straining activities to less harmful alternatives. In consequence, to stay below the boundary, environmental taxes must increase strongly, and a steady state – with potential net zero pollution levels – must quickly be reached (see Section 2.4). The exact point in time at which the steady state is reached is dependent on the location of the boundary and the marginal damages that occur below the boundary.

5.4 Unfolding a Broader Research Agenda

We have developed a conceptual model that integrates natural science based planetary boundaries and welfare economic theory. We classified the planetary boundaries along the lines imposed by the model framework to clarify the economic and incentive structure of planetary boundaries. The conceptual model together with our classification of the economic and incentive structure of planetary boundaries can help shape a comprehensive research agenda on aspects related to system dynamics, economics, public policy, and governance.

Building the natural science base: Regeneration rates, tipping points, system states, and linkages among planetary boundaries need to be explored.

The effective governance of planetary boundaries rests on a better understanding of how human activities like emissions, habitat conversion, etc. change natural systems. These changes include dynamics related to regenerative capacity, including
potential global and/or regional tipping points as well as relations between true system states and observable proxies. Modelling such dynamics and relationships provides a basis for larger integrated assessment models that could be used to explore pathways that prevent crossing planetary boundaries. However, to build such models, the natural science basis regarding the operationalization of interlinkages between regional and global boundaries as well as among multiple boundaries must be expanded. Principles and procedures for updating the planetary boundaries, proxy variables, and associated uncertainty ranges according to new scientific insights could play an important role in such a structured scientific learning process.

Assessing the damages: A comprehensive evaluation of economic, natural capital, and regenerative capacity damages is needed.

A better understanding is needed of the damages caused by the changes human activities induce in natural systems. This includes a more comprehensive and empirically founded understanding of economic damages that occur as planetary boundaries are approached. This is crucial to appropriately consider quantifiable benefits that are realized as the result of environmental conservation. However, a comprehensive damage assessment should not only focus on damages to direct economic activity but also take into consideration damages to natural capital, and to the stability and regenerative capacity of natural systems.

Governing planetary boundaries: Governance and incentive structures including strategic interactions must be explored.

To enable humanity to effectively govern the global commons implied by the planetary boundaries, effective global governance structures that enable monitoring, implementation, enforcement, and evaluation need to be designed. For this, consideration of strategic interactions in the governance of planetary boundaries is important. Research using game-theoretic tools can help one analyze incentives for national governments to consider policies that address planetary boundaries. Planetary boundaries are linked to various local or regional environmental and natural resource problems, but they also entail transboundary spillovers, creating complex incentive structures. Research could therefore focus on the design of international agreements, including trade policy aspects and international transfers to balance costs and benefits across countries. One starting point would be to account for the hierarchical nature of planetary boundaries with climate change and biodiversity as core boundaries and global processes. They should be targeted by international treaties. Other planetary boundaries could then be governed on the national and regional multinational level, involving only those countries that are affected by a specific degradation problem (e.g. the Baltic Sea adjacent countries on the eutrophication of the Baltic Sea).

In summary, a more thorough integration of the natural science-based planetary boundary framework into economic analysis yields a broad research agenda. Tackling this agenda needs input from natural science as well as social science
communities. Close collaboration and interdisciplinary work have the potential to fill the knowledge gaps that still hamper the much-needed effective governance of planetary boundaries and global commons.

**6 Conclusion**

In this paper we have bridged the gap between natural science-based planetary boundaries and welfare economic theory. To do so, we developed a stylized model framework that generalizes cost-benefit analysis (CBA) and cost-effectiveness analysis (CEA) into a unified theory. The model framework could be applied to existing numerical, large-scale integrated assessment and economic models. However, it does not fully endogenousize uncertainty, learning, and decision making under dynamic risk because of the complexity this would introduce. The central characteristic of the model framework is the combined consideration of an exogenously set limit (“boundary”) to environmental damaging activity together with the marginal damages that occur while approaching that limit. In this setting, the central values that result from the constrained welfare maximization yield the shadow price of a planetary boundary. It is a generalization of the social cost and user cost concepts. Accounting for marginal damages while approaching the planetary boundary, and simultaneously adhering to the boundary, both flatten the path of the shadow price. For policy, this implies that – in the presence of marginal damages from environmental damaging activities and a planetary boundary that should be respected – strong and early mitigation is favorable to weak or delayed action.

We also explored several decision paradigms that provide rationales for setting a limit to environmentally damaging human activities. These decision paradigms include practical CBA as a special case of the unified theory, a precautionary principle in the presence of ambiguity about catastrophic welfare damages, natural science-derived tipping points, and lastly, the unobservability of natural system states that require a proxy boundary. The planetary boundary framework is an example of such proxy boundaries. A decision tree was presented to synthesize the relationships between those potential limits. Importantly, it emphasizes that environmental problems that may lead to large scale, nonlinear change should only be governed in accordance with CBA if there is no ambiguity, if the likelihood of catastrophe is extremely small, or if there is an excessive cost of precaution.

It should be noted that there are challenges associated with the basic concepts presented here. First, combining CBA and CEA – as we propose in our unified framework – is challenging from a conceptual and communicative perspective. Our approach involves “corner solutions” in which either the CBA or the CEA framework is binding in the long run, depending on the assessment of environmental damages when respective boundaries are approached. Moreover, while it is important to base
targets or boundaries on the argument of deep uncertainty about potential catastrophic impacts, one should also emphasize the quantifiable economic and welfare gains due to avoided damages. This may help to mobilize early and more ambitious action, which would be well grounded on empirically founded economic welfare arguments. A further challenge of the model framework is related to the governance of planetary boundaries in the presence of interlinkages. Because of the interlinkages between global and regional levels as well as among multiple planetary boundaries, an isolated analysis on a specific boundary can be problematic. On the other hand, a full analysis on all aspects can be overstraining and is not operational on a policy level. Finally, the optimal utilization of the environmentally damaging activity in the model above rests on an efficiency criterion. Distributional aspects, incentive structures, and issues of global cooperation have been excluded.

While keeping those challenges in mind, there are several opportunities that are also relevant. Ultimately, more research is needed on economic and policy aspects of planetary boundaries that incorporate synergies and trade-offs. Such research could eventually identify which activities and which policy instruments have the highest impact on our ability to remain within our safe operating space. This information would be extremely important to guide policymaking on the national and international levels to maintain living conditions on Earth for humankind. The model framework presented in this paper could lay the groundwork for motivating economists to include planetary boundaries in their research agendas and to go beyond classic theoretical approaches to help govern our global commons.

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Appendix

A1: Model framework and optimality conditions of depletion problem

The pollution problem introduced in the main text can be transformed into a depletion problem where \( \bar{Z} \) is the initial stock of natural resources (biodiversity, fresh water, habitat size, etc.) and \( Y(t) = \bar{Z} - \bar{Z}(t) \) the remaining budget. Welfare damages are then similarly transformed to \( \tilde{D}(Y(t)) = \tilde{D}(\bar{Z} - \bar{Z}(t)) \). The overall optimization problem with the adjusted transition function then reads:
\[
\max_{X(t)} \int_{0}^{\infty} \left[ B(X(t)) - \tilde{D}(Y(t)) \right] e^{-\rho t} dt \quad (A1)
\]

subject to

\[
\dot{Y}(t) = -X(t) + \delta(\bar{Z} - Y(t))(\bar{Z} - Y(t)) \quad (A2)
\]

\[
Y(0) = \bar{Z} - Z_0 \quad (A3)
\]

\[
-Y(t) \leq 0 \quad \forall \ t \quad (A4)
\]

\[
\lim_{t \to \infty} \mu(t)Y(t)e^{-\rho t} = 0. \quad (A5)
\]

As the problem contains an additional state-space constrained, we use the current-value Lagrangian

\[
L_c = H_c + \lambda(t)(-Y(t)) = B(X(t)) - \tilde{D}(Y(t)) + \mu(t)\left[ -X(t) + \delta(\bar{Z} - Y(t))(\bar{Z} - Y(t)) \right] + \lambda(t)(-Y(t)) \quad (A6)
\]

with the following conditions (assuming an interior solution):

\[
\frac{\partial L_c}{\partial X(t)} = \frac{\partial B(X(t))}{\partial X(t)} - \mu(t) = 0 \quad \Leftrightarrow \quad \mu(t) = \frac{\partial B(X(t))}{\partial X(t)} \quad (A7)
\]

\[
\frac{\partial L_c}{\partial \mu(t)} = \dot{Y}(t) = -X(t) + \delta(\bar{Z} - Y(t))(\bar{Z} - Y(t)) \quad (A8)
\]

\[
\dot{\mu}(t) = \rho \mu(t) - \frac{\partial L_c}{\partial Y(t)} = \rho \mu(t) + \frac{\partial \tilde{D}(Y(t))}{\partial Y(t)} + \mu(t)\left[ \frac{\partial \delta(\bar{Z} - Y(t))}{\partial Y(t)}(\bar{Z} - Y(t)) + \delta(\bar{Z} - Y(t)) \right] - \lambda(t) \quad (A9)
\]

\[
\frac{\partial L_c}{\partial \lambda(t)} = Y(t) \geq 0, \quad \lambda(t) \geq 0, \quad \lambda(t) \frac{\partial L_c}{\partial \lambda(t)} = 0. \quad (A10)
\]

We define \( \tilde{t} \) as the point in time, where the budget is depleted \( Y(\tilde{t}) = 0 \). Time \( \tilde{t} \) may be indefinite if this never occurs \( Y(t) > 0 \quad \forall \ t \). For \( t = [0, \tilde{t}] \), the constraint in Eq (A4) is nonbinding, \( Y(t) > 0 \) and thus, as in Eq (A10) required, \( \lambda(t) = 0 \). From this follows the growth rate of the shadow price \( \mu(t) \) (omitting time dependency \( t \) for better readability):
\[ \dot{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta (Z - Y) + \frac{\partial \delta (Z - Y)}{\partial Y} (Z - Y) + \frac{\partial \tilde{D} (Y)}{\partial B (X)} . \tag{A11} \]

For \( t = [\tilde{t}, \infty) \), the constraint in Eq. (A4) is binding, \( Y (t > \tilde{t}) = 0 \), and thus, as in Eq. (A10) required, \( \lambda (t > \tilde{t}) > 0 \). The growth rate of the shadow price \( \mu (t) \) then is:

\[ \dot{\mu} = \frac{\dot{\mu}}{\mu} = \rho + \delta (Z - Y) + \frac{\partial \delta (Z - Y)}{\partial Y} (Z - Y) + \frac{\partial \tilde{D} (Y)}{\partial B (X)} - \frac{\lambda}{\mu} . \tag{A12} \]

Note, the variable \( \mu (t) \) is the co-state variable for the budget \( Y (t) \) here; it denotes a shadow price that measures the social value of the remaining resource stock \( Y (t) = Z - Z (t) \). That means, it measures the marginal change in social welfare for a marginal change in \( Y \).

As \( \tilde{D}' (Y (t)) = -D' (Z (t)) \), the expressions above are structurally the same as in the pollution problem formulation in Section 2.2. The optimization problem without exogenous limit (i.e. without the constraint Eq (A4)) yields the same results as shown in Eq (A11).

**A2: Derivation of optimality conditions for the regional–global model with cooperation (Section 4.1):**

\[
\max_{X_1, \ldots, X_n} \int_0^\infty \left[ \sum_i [B_i (X_i (t)) - D_i (W (t)) - G_i (W_i (t))] e^{\rho t} \right] dt \tag{A13}
\]

subject to

\[
\dot{W} (t) = \sum_i X_i (t) - \delta (W (t)) W (t) \tag{A14}
\]

\[
\dot{W}_i (t) = X_i (t) - \varepsilon_i (W_i (t)) W_i (t) - \delta_i (W (t)) W_i (t) \quad \forall i = \{1, \ldots, n\} \tag{A15}
\]

\[
W_{PB} - W (t) \geq 0 \quad \& \quad W_{PB, i} - W_i (t) \geq 0 \quad \forall i = \{1, \ldots, n\} \tag{A16}
\]

\[
\lim_{t \to \infty} \mu (t) W (t) e^{\rho t} = 0, \quad \lim_{t \to \infty} \lambda_i (t) W_i (t) e^{\rho t} = 0. \tag{A17}
\]

As the problem contains an additional state-space constrained, we use the current-value Lagrangian.
\[
L_c = \sum_i [B_i(X_i(t)) - D_i(W(t)) - G_i(W_i(t))] + \mu(t) \left[ \sum_i X_i(t) - \delta(W(t))W(t) \right] \\
+ \theta(t) (W_{PB} - W(t)) + \sum_i \lambda_i(t) [X_i(t) - \varepsilon_i(W_i(t))W_i(t) - \delta_i(W(t))W_i(t)] \\
+ \sum_i \gamma_i(t) (W_{PB,i} - W_i(t))
\]

(A18)

with the following conditions (assuming an interior solution):

\[
\frac{\partial L_c}{\partial X_i(t)} = B_i'(X_i(t)) + \mu(t) + \lambda_i(t) = 0 \iff \mu(t) = -B_i'(X(t)) - \lambda_i(t)
\]

(A19)

\[
\frac{\partial L_c}{\partial \mu(t)} = \sum_i X_i(t) - \delta(W(t))W(t)
\]

(A20)

\[
\dot{\mu}(t) = \rho \mu(t) - \frac{\partial L_c}{\partial W(t)} \\
= \rho \mu(t) + \sum_i D_i'(W(t)) - \mu(t) [\delta'(W(t))W(t) - \delta(W(t))] - \sum_i \lambda_i(t) \\
\times [-\delta_i(W(t))W_i(t)] - \theta(t)
\]

(A21)

\[
\frac{\partial L_c}{\partial \theta(t)} = W_{PB} - W(t) \geq 0, \quad \theta(t) \geq 0, \quad \theta(t) \frac{\partial L_c}{\partial \theta(t)} = 0
\]

(A22)

\[
\frac{\partial L_c}{\partial \lambda_i(t)} = X_i(t) - \varepsilon_i(W_i(t))W_i(t) - \delta_i(W(t))W_i(t)
\]

(A23)

\[
\dot{\lambda}_i(t) = \rho \lambda_i(t) - \frac{\partial L_c}{\partial W_i(t)} \\
= \rho \lambda_i(t) + G_i'(W_i(t)) - \lambda_i(t) [\varepsilon_i'(W_i(t))W_i(t) - \varepsilon_i(W_i(t)) - \delta_i(W(t))] \\
- \gamma_i(t)
\]

(A24)

\[
\frac{\partial L_c}{\partial \gamma_i(t)} = W_{PB,i} - W_i(t) \geq 0, \quad \gamma_i(t) \geq 0, \quad \gamma_i(t) \frac{\partial L_c}{\partial \gamma_i(t)} = 0.
\]

(A25)

We define \( \tilde{t} \) as the point in time, where one of the limits is reached (\( W(\tilde{t}) = W_{PB} \) or \( W_i(\tilde{t}) = W_{PB,i} \)). Time \( \tilde{t} \) may be indefinite if the limit is never reached (\( W(t) < W_{PB} \) and \( W_i(t) < W_{PB,i} \) \( \forall t \)). For \( t \in [0, \tilde{t}] \), the constraints in Eq. (A16) are nonbinding, \( W(t) < W_{PB} \) and \( W_i(t) < W_{PB,i} \) and thus, \( \theta(t) = 0 \) and \( \gamma_i(t) = 0 \) \( \forall i = \{1, \ldots, n\} \).

From this follow the growth rates of the shadow prices \( \mu(t) \) and \( \lambda_i(t) \) (omitting time dependency \( t \) for better readability):
\begin{align}
\hat{\mu} = \frac{\dot{\mu}}{\mu} &= \rho + \delta(W) + \delta'(W)W + \frac{1}{\mu} \sum \delta_i(W)W_i + \frac{1}{\mu} \sum \lambda_i \delta_i'(W)W_i \\
&= \rho + \delta(W) + \delta'(W)W - \frac{\sum \delta_i(W)}{B_i(X_i) - \lambda_i} + \sum \frac{\lambda_i \delta_i'(W)W_i}{\mu}. \tag{A26}
\end{align}

\begin{align}
\hat{\lambda} = \frac{\dot{\lambda}_i}{\lambda_i} &= \rho + \frac{1}{\lambda_i} G'_i(W_i) + \varepsilon_i(W_i)W_i + \varepsilon_i(W_i) + \delta_i(W) \\
&= \rho + \varepsilon_i(W_i) + \delta_i(W) + \varepsilon_i(W_i)W_i - \frac{G'_i(W_i)}{B_i(X_i) + \mu}. \tag{A27}
\end{align}

**A3: Derivation of optimality conditions for the non-cooperative regional–global model (Section 4.2):**

\begin{align}
\max_{X_i} \int_0^\infty \sum \left[ B_i(X_i(t)) - D_i(W(t)) - G_i(W_i(t)) \right] e^{-\rho t} dt \tag{A28}
\end{align}

subject to

\begin{align}
\dot{W}(t) &= \sum X_i(t) - \delta(W(t))W(t) \tag{A29}
\end{align}

\begin{align}
\dot{W}_i(t) &= X_i(t) - \varepsilon_i(W_i(t))W_i(t) - \delta_i(W(t))W_i(t) \tag{A30}
\end{align}

\begin{align}
W_{PB} - W(t) \geq 0 & \quad & W_{PB,i} - W_i(t) \geq 0 \quad \forall \, t \tag{A31}
\end{align}

\begin{align}
\lim_{t \to \infty} \mu(t)W(t)e^{-\rho t} = 0, \quad \lim_{t \to \infty} \lambda_i(t)W_i(t)e^{-\rho t} = 0. \tag{A32}
\end{align}

As the problem contains an additional state-space constrained, we use the current-value Lagrangian

\begin{align}
L_c = B_i(X_i(t)) - D_i(W(t)) - G_i(W_i(t)) + \mu(t) \left[ X_i(t) - \delta(W(t))W(t) \right] + \theta(t) \\
\times (W_{PB} - W(t)) + \lambda_i(t) \left[ X_i(t) - \varepsilon_i(W_i(t))W_i(t) - \delta_i(W(t))W_i(t) \right] \\
+ \gamma_i(t)(W_{PB,i} - W_i(t)) \tag{A33}
\end{align}

with the following conditions (assuming an interior solution):

\begin{align}
\frac{\partial L_c}{\partial X_i(t)} = B'_i(X_i(t)) + \mu(t) + \lambda_i(t) = 0 \quad \Leftrightarrow \quad \mu(t) = -B'_i(X(t)) - \lambda_i(t) \tag{A34}
\end{align}

\begin{align}
\frac{\partial L_c}{\partial \mu(t)} = X_i(t) - \delta(W(t))W(t) \tag{A35}
\end{align}
\[ \dot{\mu}(t) = \rho \mu(t) - \frac{\partial L_c}{\partial W(t)} = \rho \mu(t) + D'_i(W(t)) - \mu(t) \left[ - \dot{\delta}(W(t))W(t) - \delta(W(t)) \right] \]

\[ - \lambda_i(t) \left[ - \delta'_i(W(t))W_i(t) \right] - \theta(t) \]

(A36)

\[ \frac{\partial L_c}{\partial \theta(t)} = W_{PB} - W(t) \geq 0, \quad \theta(t) \geq 0, \quad \theta(t) \frac{\partial L_c}{\partial \theta(t)} = 0 \quad (A37) \]

\[ \frac{\partial L_c}{\partial \lambda_i(t)} = X_i(t) - \varepsilon_i(W_i(t))W_i(t) - \delta_i(W(t))W_i(t) \]

(A38)

\[ \dot{\lambda}_i(t) = \rho \lambda_i(t) - \frac{\partial L_c}{\partial W_i(t)} \]

\[ = \rho \lambda_i(t) + G'_i(W_i(t)) - \lambda_i(t) \left[ - \varepsilon'_i(W_i(t))W_i(t) - \varepsilon_i(W_i(t)) - \delta_i(W(t)) \right] \]

\[ - \gamma_i(t) \]

(A39)

\[ \frac{\partial L_c}{\partial \gamma_i(t)} = W_{PB,i} - W_i(t) \geq 0, \quad \gamma_i(t) \geq 0, \quad \gamma_i(t) \frac{\partial L_c}{\partial \gamma_i(t)} = 0. \]

(A40)

We define \( \hat{t} \) as the point in time, where one of the limits is reached \( (W(\hat{t}) = W_{PB} \) or \( W_i(\hat{t}) = W_{PB,i} \)). Time \( \hat{t} \) may be indefinite if the limit is never reached \( (W(t) < W_{PB} \) and \( W_i(t) < W_{PB,i} \) \( \forall t \)). For \( t = [0, \hat{t}] \), the constraints in Eq. (A31) are nonbinding, \( W(t) < W_{PB} \) and \( W_i(t) < W_{PB,i} \) and thus, \( \theta(t) = 0 \) and \( \gamma_i(t) = 0 \) \( \forall i = \{1, \ldots, n\} \). From this follow the growth rates of the shadow prices \( \mu(t) \) and \( \lambda_i(t) \) (omitting time dependency \( t \) for better readability):

\[ \ddot{\mu} = \dot{\mu} + \rho \delta(W) + \dot{\delta}(W)W + \frac{1}{\mu} D'_i(W) + \frac{1}{\lambda_i} \delta'_i(W)W_i \]

\[ = \rho + \delta(W) + \dot{\delta}(W)W - \frac{D'_i(W)}{B'_i(X_i) - \lambda_i} + \frac{\lambda_i}{\mu} \delta'_i(W)W_i \]

(A41)

\[ \ddot{\lambda}_i = \dot{\lambda}_i + \frac{1}{\lambda_i} G'_i(W_i) + \varepsilon'_i(W_i)W_i + \varepsilon_i(W_i) + \delta_i(W) \]

\[ = \rho + \varepsilon_i(W_i) + \delta_i(W) + \varepsilon'_i(W_i)W_i - \frac{G'_i(W_i)}{B'_i(X_i) + \mu} \]

(A42)
## A4: Literature used for the assessment in Table 1

<table>
<thead>
<tr>
<th>Planetary boundary</th>
<th>Main activities altering natural system</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Climate change</td>
<td>Fossil fuel burning, Agriculture, Deforestation</td>
<td>Blanco et al. (2014)</td>
</tr>
<tr>
<td>Biosphere integrity</td>
<td>Genetic diversity: Habitat loss through Land-system change (esp. deforestation), Pollution (esp. N and P), Overexploitation, Non-native species introduction</td>
<td>Sodhi et al. (2009)</td>
</tr>
<tr>
<td>Functional diversity</td>
<td>Habitat loss through Land-system change (esp. deforestation), Pollution (esp. N and P), Harvesting (hunting, poaching etc.)</td>
<td>Brodie et al. (2021) and Sodhi et al. (2009)</td>
</tr>
<tr>
<td>Land-system change</td>
<td>Deforestation, Wet-/Peatland conversion, Soil sealing (urbanization and infrastructure building)</td>
<td>DeFries et al. (2010) and Lambin and Meyfroidt (2011)</td>
</tr>
<tr>
<td>Freshwater use</td>
<td>Agricultural irrigation, Industrial water use, Household water use</td>
<td>FAO (2021)</td>
</tr>
<tr>
<td>Biochemical flows</td>
<td>Phosphorous: Application of synthetic fertilizer</td>
<td>Bouwman et al. (2013)</td>
</tr>
<tr>
<td></td>
<td>Nitrogen: Application of fertilizer, Fossil fuel burning</td>
<td>Battye et al. (2017)</td>
</tr>
<tr>
<td></td>
<td>Ocean acidification: CO₂ emissions from Fossil fuel burning, Agriculture, Deforestation</td>
<td>Blanco et al. (2014) and Doney et al. (2009)</td>
</tr>
<tr>
<td></td>
<td>Atmospheric aerosol loading: Fossil fuel burning, Deforestation through burning</td>
<td>Boucher et al. (2013)</td>
</tr>
<tr>
<td></td>
<td>Release of CFCs and HCFCs</td>
<td>2018</td>
</tr>
</tbody>
</table>
### A5: Literature used for the assessments in Table 2

<table>
<thead>
<tr>
<th>Planetary boundary</th>
<th>Main activities altering natural system</th>
<th>References</th>
<th>Global tipping points</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stratospheric ozone depletion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Novel entities</td>
<td>– Chemical pollution</td>
<td>Steffen et al. (2015)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>– GMOs</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>– Release of heavy metals</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Planetary boundary

| Climate change | Slow (CO₂) to medium (Methane) | Eby et al. (2009) and Matthews et al. (2009) | Yes | IPCC (2021) and Wunderling et al. (2021) |
| Novel entities | – Climate response to CO₂ pulse approx. constant over centuries | IPCC (2014) | – 1.5–2 °C (disintegration of Greenland and West Antarctic ice sheets as starters of domino effect) |            |
|                | – Half-life time of methane in the atmosphere is 12 years |            | – See Lenton et al. (2008) for further tipping points |            |

#### Biosphere integrity

|                  | – One lineage split every two million years | Sepkoski (1998) |            |            |
|                  | – 3–6 species/year for complete biota |            |            |            |

| Functional diversity | Slow to medium | Own judgement | No | Rockström et al. (2009) and Steffen et al. (2015) |
|                      | – Highly variable depending on ecosystem | Liebsch et al. (2008) |            |            |
|                      | – Rainforest may recover within 100–300 years |            |            |            |

| Land-system change | Slow to medium | Own judgement | No | Rockström et al. (2009) and Steffen et al. (2015) |
|                    | – Highly variable and depends on kind, duration, and degree of change |            |            |            |
### Planetary boundary

<table>
<thead>
<tr>
<th>Regeneration rate of system</th>
<th>References</th>
<th>Global tipping points</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Freshwater use</strong></td>
<td></td>
<td><strong>No</strong></td>
<td></td>
</tr>
<tr>
<td>Fast</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Surface freshwater is renewable resource and usually re-generates within years</td>
<td>Own judgement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- However, fossil freshwater does not regenerate at all</td>
<td>Own judgement</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Biochemical flows**

<table>
<thead>
<tr>
<th>Type</th>
<th>Regeneration rate of system</th>
<th>References</th>
<th>Global tipping points</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phosphorous</strong></td>
<td>Slow to medium</td>
<td></td>
<td><strong>No</strong></td>
<td></td>
</tr>
<tr>
<td>- Pp to 10,000 years in deep ocean</td>
<td>Colman and Holland (2000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Ten years half-life time for pyrophosphates, 20 years half-life time for mono- and di-esters in freshwater bodies</td>
<td>Ahlgren et al. (2005)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Nitrogen</strong></td>
<td>Fast to medium</td>
<td></td>
<td><strong>No</strong></td>
<td></td>
</tr>
<tr>
<td>- Globally, denitrification fluxes seem to match anthropogenic nitrogen fixation</td>
<td>Battye et al. (2017)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Regionally, accumulation in terrestrial biomass and soils is likely</td>
<td>Schlesinger (2008)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ocean acidification</strong></td>
<td>Slow</td>
<td></td>
<td><strong>Yes</strong></td>
<td></td>
</tr>
<tr>
<td>- Dissolved carbon remains &gt;10,000 years in deep sea</td>
<td>Cartapanis et al. (2016)</td>
<td>350 ppm CO₂ (mass loss of coral reefs)</td>
<td>Veron et al. (2009)</td>
<td></td>
</tr>
<tr>
<td><strong>Atmospheric aerosol loading</strong></td>
<td>Fast</td>
<td></td>
<td><strong>No</strong></td>
<td></td>
</tr>
<tr>
<td>- Atmospheric aerosol lifetime is typically less than a month</td>
<td>Kristiansen et al. (2016)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Stratospheric ozone depletion</strong></td>
<td>Medium</td>
<td></td>
<td><strong>Yes</strong></td>
<td></td>
</tr>
<tr>
<td>- 18–31% recovery in 31 years; full recovery expected within 80 years (1980–2060)</td>
<td>WMO (2018)</td>
<td>275 O₃ concentration in Dobson units</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Novel entities**

|                      |                      | **?**      | **?**                |            |
References


**Supplementary Material:** This article contains supplementary material (https://doi.org/10.1515/jbnst-2022-0022).