Macroscopic Stochastic Model for Economic Cycle Dynamics

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We present a stochastic dynamic model which can explain economic cycles. We show that the macroscopic description yields a complex dynamical landscape consisting of multiple stable fixed points, each corresponding to a split of the population into a large low and a small high income group. The stochastic fluctuations induce switching between the resulting metastable states and excitation oscillations just below a deterministic bifurcation. The shocks are caused by the decisions of a few agents who have a disproportionate influence over the macroscopic state of the economy due to the unequal distribution of wealth among the population. The fluctuations have a long-term effect on the growth of economic output and lead to business cycle oscillations exhibiting coherence resonance, where the correlation time is controlled by the population size which is inversely proportional to the noise intensity.

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The complex networks approach is a transdisciplinary paradigm to capture the nonlinear dynamics of a multitude of natural, technological, or social systems. In order to predict and help to understand the effects of economic crises or shocks, and guide policymakers to handle such situations [1–3], the economy should be modeled as a complex socioeconomic system with a plethora of network interactions between agents (households) and market institutions, taking into regard that wealth and power are heterogeneously and unequally distributed among the population.

The classical approach in economics is to assume complete rationality and only consider a single representative agent, who then solves a long-term optimization problem in order to maximize the long-term benefits of increased consumption. The typical use of convex functions results in the existence of a unique fixed point that is then disturbed by external shocks [1,4,5]. These models in general lack the dynamical complexity needed to describe the economic reality observed, and completely disregard the highly nonuniform distribution of wealth in typical modern economies [6]. The long-lasting effects of the 2008 financial crisis have led to the paradigm shift in economics that business cycles and random fluctuations are interdependent in economic growth theory [7,8].

This work addresses the question of how stochastic interactions between individuals can give rise to fluctuations of macroeconomic quantities and the associated long-term effects on economic growth.

We start from a modified version of the agent-based model for business cycles and economic inequality presented in [5], but develop a macroscopic stochastic model using the Langevin equation approach [9,10] and a moment closure for a description of the underlying agent-based model for large but finite population size. This approach allows us to first study the deterministic system for an infinite population and then use the Langevin approach to understand the effect of finite-size fluctuations. Such macroscopic models are advantageous with respect to comparing them to data, since they only deal with average and aggregate quantities, which in reality are much easier to obtain than the refined data necessary to specify the initial conditions of an agent-based model. The method could be applied to other problems, e.g., neural systems [11] or network motifs [12] as well.

Agent-based "micro" model—The starting point is a stochastic model of $N \gg 1$ households i ("agents") in a fully connected network, characterized by two dynamic variables (K_i, S_i). Household capital $K_i \geq 0$ is accumulated by saving a fraction of household income given by its current saving rate S_i. Although in principle S_i could be any real number in [0, 1], we assume S_i is one of $M > 1$ discrete saving rate levels $s_1 < \cdots < s_M$. Agent i independently and stochastically updates S_i at random times given by a Poisson process with common jump rate $1/\tau$. At each update, i either *explores* or *imitates*. With probability ϵ, i switches to any of the saving rate levels uniformly at random ("exploration"). With probability $1-\epsilon$, i will instead copy the saving rate S_j of any agent j, drawn with a probability that depends on j's current consumption C_j ("imitation"). We assume that the probability to choose agent j for imitation is governed by a Boltzmann distribution with inverse temperature β,

$$P(S_i \to S_j) = \frac{1}{Z}\exp(\beta C_j), \qquad Z = \sum_{j=1}^{N}\exp(\beta C_j), \tag{1}$$

resulting in a voter model with coevolving transition probabilities [13–15]. In the low temperature limit $\beta \to \infty$, this "softmax policy" converges to the imitate-the-best ("argmax") policy used in [5], where agents deterministically adopt the saving rate of the agent with the highest consumption in their neighborhood. Our generalization to a stochastic softmax policy can be interpreted as representing rational decision making under uncertain measurements of others' consumption, similar to [16], and it is a common assumption in behavioral economics and machine learning.

Household consumption $C_i = (1-S_i)I_i$ is that part of income I_i which is not saved. Income depends on gross economic production Y, determined by a Cobb-Douglas production function [17] $Y = AK^{\epsilon_K}L^{\epsilon_L}$, where $K = \sum_{i=1}^{N} K_i$ is aggregate capital, L is aggregate labor, A is a constant, and ϵ_L and ϵ_K are elasticities; here, $\epsilon_K = \epsilon_L = 1/2$. Household i supplies their capital and fixed labor $l_i = L/N$ to the economy for production and is compensated at wage $w = \partial Y/\partial L$ and capital return $r = \partial Y/\partial K$, resulting in an income [18] of

$$I_i = rK_i + wL/N = A\sqrt{L}(K_i/\sqrt{K} + \sqrt{K}/N)/2. \tag{2}$$

Investing the saved fraction S_i of I_i into capital growth results in a coupled, nonlinear evolution of capital stocks,

$$\dot{K}_i = S_i I_i - \kappa K_i = (rS_i - \kappa)K_i + wS_iL/N, \tag{3}$$

where $\kappa > 0$ is the common capital *depreciation rate*.

Macro model—In the following we study this model in the large system limit $N \to \infty$, so we focus on a few aggregate quantities: the vector of *occupation numbers* $\boldsymbol{n} = (n_1, \ldots, n_M)$ of all saving rate levels, and the capital distribution in each of these levels. This admits an approximation via a chemical Langevin equation [9,10] combined with a moment closure approach for the capital distributions in each saving rate level.

The Langevin equation incorporates fluctuations in the transition rates due to the finite size of the system. This contrasts the usual ways fluctuations are introduced into macroeconomic growth models based on demand shocks, credit defaults, or technological progress [7]. The time evolution of the occupation numbers follows an Itô stochastic differential equation,

$$d\boldsymbol{n} = \sum_{k,l=1}^{M}\alpha_{kl}\boldsymbol{\nu}_{kl}dt + \sum_{k,l=1}^{M}\sqrt{\alpha_{kl}}\boldsymbol{\nu}_{kl}dB_{kl}. \tag{4}$$

$\boldsymbol{\nu}_{kl} = \boldsymbol{e}_k - \boldsymbol{e}_l$ indicates a transition between levels $s_k \to s_l$, where $\boldsymbol{e}_k$ is the kth unit vector in $\mathbb{R}^M$, and dB_{kl} are the increments of the Wiener process. Because of imitation and exploration, the transition rate for $k \to l$ is

$$\alpha_{kl} = (1-\epsilon)\frac{n_k}{\tau Z}n_l\langle\exp(\beta C_i)\rangle_l + \epsilon\frac{n_k}{\tau M}, \tag{5}$$

where $\langle X_i\rangle_l = (n_l)^{-1}\sum_{\{i:S_i=s_l\}}X_i$ denotes the population average of agents in saving rate level l.

For the moment closure, we consider the pth moment ($p \geq 1$) of the capital distribution among those households whose saving rates are in level l: $m_l^p = \langle K_i^p\rangle_l$. We cannot directly compute the evolution of the capital moments using Eq. (3), since when a household switches to a different saving rate, it takes its capital stock with it. This leads to correction terms [19–21] that directly couple Eq. (4) with the evolution of the capital moments:

$$dm_l^p = \left(p(rs_l-\kappa)m_l^p + pws_l\frac{L}{N}m_l^{p-1} + \sum_{k=1}^{M}\frac{m_k^p - m_l^p}{n_l}\alpha_{kl}\right)dt + \sum_{k=1}^{M}\frac{m_k^p - m_l^p}{n_l}\sqrt{\alpha_{kl}}dB_{kl}. \tag{6}$$

Since $\tau \gg 1$, this results in a slow-fast system, where the occupation numbers are the slow variables. We apply a Taylor approximation of the exponential in Eq. (5), and in order to better capture the maximum consumption in each level, we expand about the mean consumption $\langle C_i(t)\rangle_l$:

$$\langle\exp(\beta C_i)\rangle_l = \exp(\beta\langle C_i\rangle_l)\sum_{p=0}^{\infty}\frac{\beta^p}{p!}\langle(C_i - \langle C_i\rangle_l)^p\rangle_l. \tag{7}$$

This reduces the systematic error from underestimating the maximal consumption, when using finitely many terms, but introduces a further nonlinearity. The moments of the

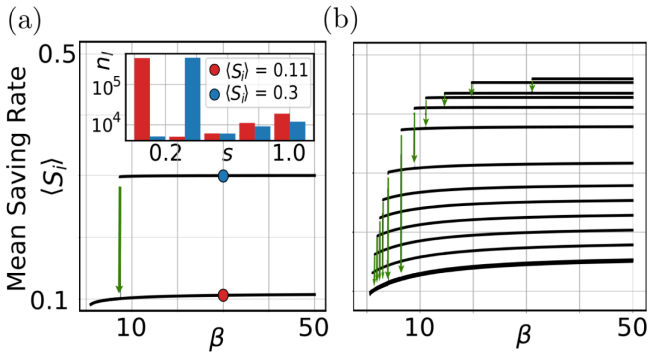

FIG. 1. Bifurcation diagram of the deterministic approximation for (a) $M = 5$ levels. For small β, there is only one stable state with a very low mean saving rate. For $\beta > 7.35$, another stable state appears with a higher mean saving rate. The inset shows the two saving rate distributions (red and blue) associated with the two states. (b) For $M = 30$ there is a cascade of bifurcations. This highlights the possibility of irreversible tipping transitions indicated by green arrows. Parameters are $L = N = 2.5 \times 10^5$, $\tau = 300$, $\kappa = \epsilon = 0.05$, $A = 1$.

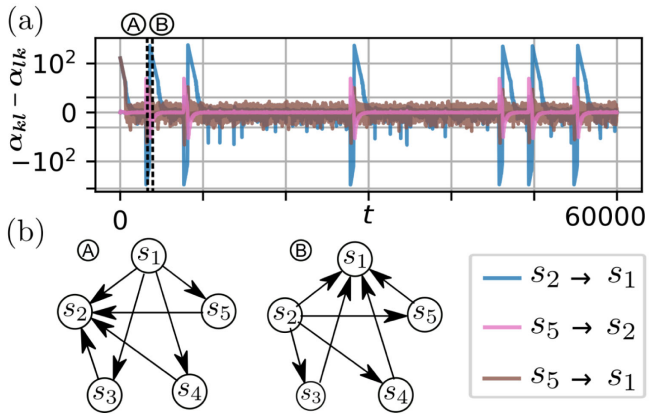

FIG. 2. (a) Net transition rates $\alpha_{kl} - \alpha_{lk}$ between the saving rate levels, just below the bifurcation in Fig. 1(a). The system displays excitation oscillations, where each oscillation starts with agents switching from $s_1 \to s_2$ and $s_5 \to s_2$. The arrows in the legend denote positive net transition rates. (b) Direction of the net flows at times A (excitation) and B (recovery). Although most agents transit from $s_1 = 0.05$ to $s_2 = 0.27$ during the spike, the path they take can be complicated. After the majority of agents have changed their saving rate to s_2, the flows turn around and almost symmetrically bring the system back to the original distribution of saving rates. Parameters as in Fig. 1(a) with $M = 5$ and $\beta = 5$.

consumption distributions are easily computed from the moments of the capital distribution using $C_i = (1 - S_i)I_i$ and Eq. (2); see Ref. [21].

We choose to include the third moment of the consumption and capital distributions, since the micro model displays significant amounts of skewness [5]. We hence truncate the moment closure at $p = 3$. This truncation affects the dynamics only by decreasing the accuracy of the maximum consumption estimate, while the evolution equations for the capital moments are unaffected.

Results—In our simulations, we restrict ourselves to equidistant saving rate levels s_l within the interval [0.05, 0.95].

Let us first explore the basic phase space structure by ignoring the noise terms. For sufficiently large number of levels M and sufficiently high inverse temperature β, this deterministic approximation displays a complex dynamical landscape with several fixed points, each corresponding to a different distribution of saving rates (Fig. 1); see also Ref. [21]: we observe a split of the population into two groups as in [5]. Most agents sit in a low saving rate level with a very low capital stock, but a small group of agents has very high saving rates and owns most of the capital; this state is prominent for all parameters considered. This gives the few "high savers" crucial influence on the overall dynamics.

With the addition of noise, we switch to a mesoscale for a large but finite number of households. Here the multistability on the macroscale results in excitation oscillations and switching between the now metastable states.

In the following we will focus on the case $M = 5$. Above the bifurcation in Fig. 1(a) the fluctuations lead to switching between the metastable states. Just below the bifurcation, i.e., below the critical inverse temperature β, the system takes an excursion through the phase space, before returning to the original stable state. This is evidenced by the net transition rates shown in Fig. 2.

Therefore, the presence of intrinsic fluctuations induces macroeconomic shocks. This stands in contrast to the deterministic model, which does not produce these shocks.

The split of the population into two groups of agents with high and low capital stock, respectively, leads to a disparity of influence that drives the transitions between the two metastable states of the system.

Each switching transition is preceded by an abrupt spike in mean consumption in the saving rate level to which the agents then switch. This is shown in Fig. 3 for β above the deterministic bifurcation but otherwise same parameters as in Fig. 2, and for two saving levels s_2 [Figs. 3(a), 3(c), and 3(e)] and s_5 [Figs. 3(b), 3(d), and 3(f)]. This spike exponentially increases the transition rate into that level according to Eq. (5). The preceding spike for a switch to higher mean saving rate is depicted in Fig. 3(a) for level s_2, which most agents will finally adopt, and in Fig. 3(b) for the highest saving rate level s_5. The enlargements Figs. 3(c) and 3(d) show that the spike in s_2 happens on a much faster timescale than the spike in s_5 and cannot be attributed to changes in the economic variables, i.e., the market dynamics, since the capital return r (and thus also the wages) is almost constant during the spike. Figure 3(e) visualizes that during the spike significant amounts of capital are transferred by a small amount of agents (0.1% of the population [21]) switching from s_5 to s_2 and all other capital flows are significantly smaller or reduce the average capital in s_2.

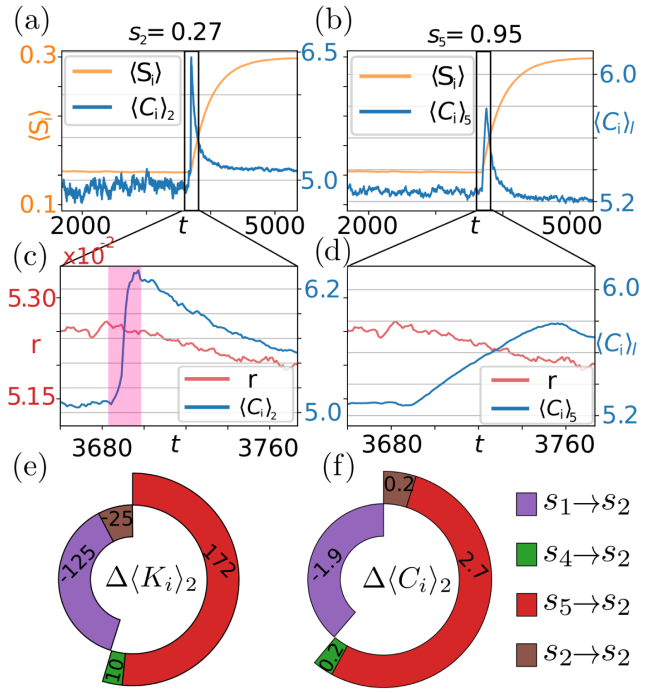

FIG. 3. Mean consumption $\langle C_i(t)\rangle_l$ in the saving rate levels s_2 (a),(c),(e) and s_5 (b),(d),(f), and mean saving rate $\langle S_i(t)\rangle$. (c), (d) Enlargements of (a) and (b), respectively, showing also the return $r(t)$. (e) Change of capital $\langle K_i\rangle_2$ and (f) of consumption $\langle C_i\rangle_2$ in level s_2 due to transitions from the other levels. (The transition $s_3 \to s_2$ is negligible and invisible.) Parameters as in Fig. 1(a) with $M = 5$, $\beta = 15$.

Figure 3(f) shows that this capital flow leads to the increase in average consumption, which then draws all the agents into this level.

This shows the disparity of influence, which is generated from the average capital difference for high and low savers. The decisions of a tiny fraction of influential agents can lead to tipping of the entire macroeconomy.

The small increase of "high savers" moving to the lower saving rate is due to the finite-size fluctuations of the transition rates, which are then amplified by the economic inequality and the fact that the level s_2 has a very low occupation. This combination creates a timescale separation, allowing for the sudden increase in mean capital.

The consumption spike in the high saving rate level s_5 in Fig. 3 happens on the much slower timescale which is associated with the economic variables, mainly the depreciation rate κ. The change in occupation numbers then follows on the slow timescale.

Our model exhibits hysteresis of business cycles, referring to long-term effects of fluctuations on economic growth, which corroborates a recent paradigm shift in economic growth theory [7,8]. We find that fluctuations can lead to a long-term change in production, if the system is in the multistable regime and population growth is introduced [4]. For details and empirical literature, see [21].

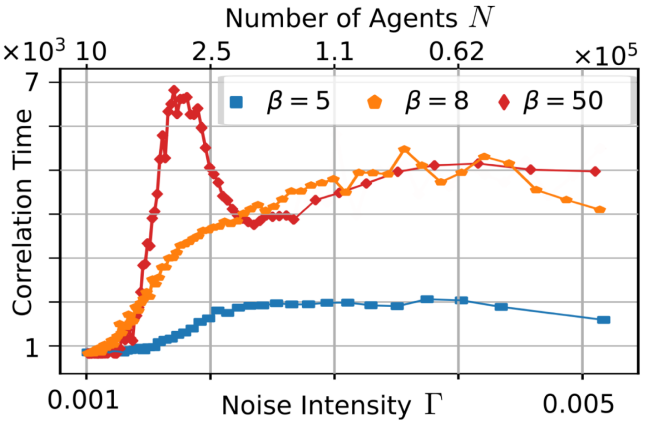

FIG. 4. Correlation time for the economic production Y vs noise intensity $\Gamma = 1/\sqrt{N}$ for different values of β below the bifurcation ($\beta = 5$) and above the bifurcation ($\beta = 8$, $\beta = 50$). For $\beta = 50$ the peak at optimum noise intensity $\Gamma \approx 1.7 \times 10^{-3}$ or $N \approx 3.5 \times 10^5$ indicates coherence resonance. Parameters as in Fig. 1 with $L = N$.

Coherence resonance is a common phenomenon in noisy excitable nonlinear systems. It describes the nonmonotonic dependence of the coherence of noise-induced oscillations upon noise intensity; i.e., there exists an optimum noise intensity maximizing coherence [39]. A common measure of coherence is correlation time [40–42]. Coherence resonance is visible as maximum correlation time [21] for nonzero noise intensity.

Since population growth is a major driver of economic growth, it is natural to ask how the oscillatory behavior changes as the system size increases. After rescaling Eqs. (4) and (6) to densities $\boldsymbol{c} = \boldsymbol{n}/N$, the population size N directly corresponds to a noise intensity parameter $\Gamma = 1/\sqrt{N}$ if $L/N = \text{const}$.

In Fig. 4 the correlation time of the economic production is plotted vs noise intensities for several inverse temperatures β. For $\beta = 5$, i.e., below the bifurcation in Fig. 1(a), the correlation time exhibits a very flat region of increased coherence. Above the bifurcation ($\beta = 8$) the correlation time increases drastically. For $\beta = 50$ we have a clear peak of optimum coherence at $\Gamma \approx 1.7 \times 10^{-3}$, and a very broad second maximum upon further increase of noise intensity. Note that no deterministic bifurcation is involved in the dramatic change of the behavior of the correlation time, when going to larger inverse temperatures. However, the time spent near each metastable state changes drastically [see Figs. 2(b) and 2(c) in Ref. [21]].

This illustrates that the precision, with which the agents imitate the behavior of the agents with the highest consumption, can have a strong effect on the coherence of the business cycle oscillations. The presence of coherence resonance indicates that fluctuations in the system can dramatically affect the business cycles and make them more coherent at a certain noise intensity defined by the size of

the population. In more realistic models stochastic fluctuations can arise from other sources as well, and although we are dealing with multiplicative noise, it seems plausible that other sources of noise might lead to oscillatory behavior that has similar characteristics, because the underlying phase space structure strongly influences the response to fluctuations.

In conclusion, we have developed a macroscopic model which captures the characteristic dynamical features of the agent-based model proposed in [5], and beyond that includes specific effects of stochastic fluctuations like coherence resonance. The decision-making process of the households creates a high degree of multistability in the infinite population limit, particularly when many saving rate levels are allowed. The multistable states result in a split of the population into a small group with high saving rate and high capital, and a large group of low savers with low capital stock, where the smaller group can exert great influence on the entire economy.

The effect of finite-size fluctuations arising in the case of a large but finite population size leads to the possibility of excitation oscillations and stochastic switching between metastable states, which correspond to a synchronized change of saving strategy of a majority of agents in the population. Compared to [5], the increased population size results in business cycles that are much more abrupt and more akin to rare isolated events than sustained oscillations.

In going beyond the agent-based model [5] we are able to deal with a substantially larger population and show that the capital inequality leads to timescale separation, which can cause rapid changes in macroscopic variables. We also find that only about 0.1% of the population [21] are responsible for triggering a recession period.

With the introduction of economic growth through a growing population, we show that the fluctuations can lead to long-lasting recessions in economic production, which is commonly discussed as hysteresis in the economics community. Hysteresis of business cycles has typically been linked to fluctuations in financing, debt and monetary policy, and only in a few cases with heterogeneous agents [21]. Our model is considerably simpler, and explains the metastability as well as the fluctuations solely from the decision-making process of heterogeneous households, in contrast to external sources of noise. To the best of our knowledge, such long-term effects on growth solely resulting from the collective saving behavior of households have not been noted before.

In our model, we find coherence resonance and a qualitative change in the correlation time when the system switches from excitation oscillation to the stochastic switching regime for larger β, which may elucidate also the effects of other sources of stochastic fluctuations. An application to brain analysis might open future research.

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