Contribution to a public good with altruistic preferences

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Abstract. This paper presents a model of private provision of a public good where individuals have altruistic preferences and care about the private and public good consumption of the other members of their group. I compare the Nash level of the public good to the benchmark level of provision by a social planner who aggregates the preferences of group members. I find that income inequality can cause overprovision of the public good as compared to the planner's benchmark. To understand overprovision, I examine a second model where, in addition to contributing to the public good, members can give private transfers of income to other members they care about. The Nash equilibrium of the model with transfers is found to be closely connected to the equilibrium of the model without transfers. Overprovision can occur in the model without transfers because the richer individuals contribute to the public good as a way to improve the welfare of the poor (noncontributors to the public good) in the absence of private transfers. These results indicate that public goods cannot substitute the role of income transfers to the poor, even when individuals are altruistic, if there is extreme income inequality.

Résumé. Contribution à un bien public avec des préférences altruistes. Cet article présente un modèle de fourniture privée d'un bien public où des personnes ont des préférences altruistes et se soucient de la consommation de biens privés et publics des autres membres de leur groupe. Je compare le niveau de bien public atteint dans un équilibre de Nash à un niveau de référence établi par un planificateur social qui agrège les préférences des membres du groupe. Je constate que les inégalités de revenus peuvent entraîner une surabondance du bien public par rapport au niveau de référence du planificateur. Pour mieux comprendre cette surabondance, j'examine un second modèle où les membres peuvent effectuer des transferts de revenus privés à d'autres membres qu'ils apprécient, en plus de contribuer au bien public. L'équilibre de Nash du modèle avec transferts est étroitement lié à celui du modèle sans transferts. La surabondance peut se produire dans le modèle sans transfert, car les personnes les plus riches contribuent au bien public dans le but d'améliorer le bien-être des plus pauvres (qui ne contribuent pas au bien public) en l'absence de transferts privés. Ces résultats indiquent qu'en cas d'inégalités extrêmes de revenus, les biens publics ne peuvent pas remplacer parfaitement les transferts de revenus vers les plus pauvres.

JEL classification: C72, H41, D64

1. Introduction

INDIVIDUALS WHO CONTRIBUTE to a public good are often part of a community and care about the other members that benefit from it. This may be true if the group members are friends, family, or even larger communities—as is the case for local relief aid in the wake of pandemics or the efforts of environmental or human rights groups. Moreover, the role

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of the public good may be of particular interest both in cases where private transfers of income between individuals are not possible due to transaction costs or social norms and in situations where it is possible to directly give money to others that one cares about. The current paper addresses this topic by developing a model of private provision of a public good where individuals have altruistic preferences and care about all other members of their group. This model allows for two different settings—one where private transfers between members are not possible, and another where such transfers are allowed—in order to understand the implications of altruism for the overall level of provision and members' welfare.

The paper shows that while caring about others is beneficial for public good provision, contrary to the common expectation of under-provision under a Nash equilibrium, being altruistic may lead to overprovision of the public good as compared to the welfare benchmark of a social planner in the model without transfers. The intuition behind overprovision is connected to the redistributive role played by a public good in a society where there is income inequality. Overprovision only occurs when there are individuals who benefit from but do not contribute to the public good. When there are noncontributors in equilibrium, the richest members of the group are the contributors and it is the poorest who do not contribute. Since the richer members of the group care about the welfare of the other members (including the poorer members) but have no means of transferring money privately to them, they compensate by contributing more to the public good. However, since the welfare of poorer members depends on their private consumption as well as their consumption of the public good, I show that modifying the model to allow for private transfers of income may prevent overprovision when the distribution of income is highly unequal.

The paper makes three contributions to the literature. It introduces, to my knowledge, one of the first analyses of a model of private provision of a public good with altruistic preferences. Within this context, it develops and connects two alternative models of giving: one where individuals cannot make private transfers to people they care about, and the case when such private transfers are possible. Second, this research also has policy implications. In the model without transfers, overprovision may occur if the poorest in the society do not contribute and the rich give as a way to improve the welfare of the poor. However, since the richer members cannot directly increase the private consumption of others, the outcome may be a level of provision that is both "too much" and inefficient compared to a welfare benchmark. This indicates that public goods may not perfectly substitute the role of government income transfers to the poor when there is income inequality. Finally, the paper shows that when the redistributive role of a public good is limited due to income inequality, private transfers of income may play a complementary role to public good provision in the interest of achieving redistribution in a society. This research thus also serves as a preliminary step of a broader agenda that seeks to understand the connections between public good provision and private transfers.

The analysis of the paper unfolds in three steps. I first develop a model of private provision of a public good, where members of a group have altruistic preferences. Following Becker (1974), the social utility of a group member is modelled as a linear combination of their own private utility as well as the private utility of other group members. Within this setting, I show that the analysis of altruistic preferences has consequences for the welfare of individuals. I compare the Nash level of the public good to the level provided by a social planner who aggregates the preferences of the group. I show that at a strictly interior equilibrium, the level of the public good provided is always lower in the Nash equilibrium than the social planner's public good allocation. A counter example subsequently provides conditions on the income distribution such that if there are non-contributors, there can be overprovision of the public good compared to the social planner's level. In the second step, I study the model where, in addition to contributing to the public good, members can transfer money privately to other members of the group. Third and finally, I find a necessary and sufficient condition under which the equilibrium of the model without transfers also serves as the equilibrium of the model with transfers. The two models are thus found to be interconnected. I subsequently illustrate, with the help of an example, how the existence of private transfers and the level of provision of the public good depend upon the relative difference in income between the richest and poorest individuals in the group. Under the same conditions on the income distribution, the amount of the public good provided with altruistic preferences is identical to the social planner's level when we allow for private transfers between individuals.

The paper relates to the literature on private provision of public goods, initiated largely by Bergstrom et al. (1986). The paper presented "altruistic" donations to charity as an example of a public good. Donations to charity, however, do not directly benefit the donors unless they get some benefit from the "joy of giving" (Ribar and Wilhelm 2002). Recognizing this, Andreoni (1988) and Andreoni (1989), among others, proposed models of impure altruism as an alternative motivation for donations.[1]

Another, more recent branch of this literature does consider relational aspects of charitable donations. For example, Scharf and Smith (2016) suggest that individuals may donate to charity because they care about the individual organizing the donation event, and Hungerman and Ottoni-Wilhelm (2021) model the effect of sponsors matching the contributions of donors to charity. These papers focus on forms of directed altruism that are very specific to the case of charity, in contrast to the current paper's model, which is concerned with the redistributive implications of a general public good in an altruistic society. Although the model à la Bergstrom et al. (1986) continued to be considered as the benchmark general model for altruistic behavior in the subsequent large body of research on alternative motivations for charity,[2] the implications of caring about the overall level of well-being of individuals, rather than about only the amount of the public good provided, were largely neglected. Existing papers consider specific forms of these altruistic preferences, such as Echazu and Nocetti (2015) construct a model of caring donors who give to a public good that is consumed only by beneficiaries who do not contribute at all, in contrast with the model in the current paper, where everyone can contribute to the public good.

The analysis in Ley (1997) is the closest to the current paper, where agents' utilities are a weighted sum of two components. The first component is the private utility of the individual from their consumption of a private good and the public good, and the second component is the private utility of other individuals in the group. Ley (1997), however, does not establish either existence or uniqueness of the Nash equilibria, nor does he examine the properties of such a Nash equilibrium. The paper shows that Pareto-efficient equilibria of an economy where individuals have such altruistically interlinked preferences remain Pareto-efficient in the selfish economy with no altruism. Ley also shows that for the subclass of additively separable preferences, the optimal level of the public good does not change

1 See also Andreoni (1990), Steinberg (1987), and Cornes and Sandler (1994) for models where individuals gain some private utility from their own contribution to the public good as well as derive utility from the total amount of contributions. Such preferences may exist because individuals get a "warm glow" from charitable contributions: They may take pride in contributing to charity, may want to show off their social status by donations or simply feel joyful about the intrinsic act of giving (Andreoni, 1990).

2 Such motivations can include social pressure (DellaVigna et al. 2012, Name-Correa and Yildirim 2016), conformism (Rotemberg 2014), and signaling (Glazer and Konrad 1996).

with the degree of altruism. In contrast, the objective of the current paper is to examine the comparative statics of the Nash equilibrium and examine how altruism can impact the level of provision of the public good and welfare.

The paper also relates to the study of altruism in economics, initiated largely by the seminal papers by Becker (1974) and Barro (1974).[3] Recent papers have looked at a number of broad applications of this field, ranging from intergenerational altruism (see, e.g., Gonzalez et al. [2018] and Galperti and Strulovici [2017]) to games where the payoffs of players are interdependent (Ray and Vohra, 2020), to private transfers over a fixed social network of individuals with altruistic preferences (Bourlès et al., 2017). However, these papers do not include any analysis of public goods.

Finally, the paper contributes to the literature on interconnections between public good provision and private transfers. Buchholz and Konrad (1995) study the case of two countries with different marginal costs of production of the public good, where the country with the higher cost has the incentive to make private transfers to the country with the lower cost, in a two-stage game where private transfers are decided in the first stage and contributions to the public good are decided in the second stage. Their results rely on differing marginal costs, distinguishing it from both the classic Bergstrom et al. (1986) as well as the current paper. The current paper does not rely on differing productivities and, indeed, is the first to establish that transfers can occur even with identical marginal productivities if individuals are altruistic. Very few papers look at connections between public good provision and private transfers in the context of altruism. The implications of altruistic preferences on private transfers were first modeled by Arrow (1981), but his paper has no analysis of public goods. Arora and Sanditov (2016) consider a model where individuals with altruistic preferences, embedded in a fixed network, contribute to a public good for the specific case of constant relative risk aversion utility functions. They do not examine the implications of a general model, and they do not include an analysis of private transfers, as considered by the current paper. Even more recently, Bommier et al. (2019) consider a model involving altruism, public good provision, and private transfers with one developed country and many developing countries. The developed nation has altruistic preferences towards other developing nations and may transfer money to developing nations. All countries derive utility from abatement of pollution, the public good. Unlike the setting of the current paper where both private transfers and public good provision are determined simultaneously, Bommier et al. (2019) model their game as a two-stage process, where the amount of public good provided is determined by a simultaneous game in the first stage, and private transfers are decided only by the developed country in the second stage.

The rest of the paper is organized as follows: Section 2 presents the model and the comparative statics results of increasing the level of altruism. Section 3 presents the analysis on the welfare implications of the model, following which Section 4 discusses the model with transfers. Section 5 concludes the paper.

2. The model

This section discusses a model of contribution to a public good with altruism. There is a group of $N = \{1, 2, \dots, n\}$ individuals with $n \geq 2$. A member $i \in N$ has an income $y_i \in$

3 Becker's famous "Rotten Kid" result showed that in the presence of a benevolent family member who transfers income to other members, even selfish, "rotten" members would maximize family income. Some early papers focus solely on whether the presence of a public good may cause the Rotten Kid theorem to fail (Bergstrom 1989) or hold (Cornes and Silva 1999 without considering the larger implications for public good provision.

$[0, \overline{y}]$, where $\overline{y}$ is some strictly positive number. They must choose how to divide their income between their consumption of a private good, x_i, and their contribution to a public good, g_i. A given profile of contributions $(g_1, g_2, \dots, g_n)$ produces an amount of public good $G = \sum_i g_i$. An individual i's preferences have two components. The first is a private component: They derive utility from their consumption of the private good x_i as well as the amount of the public good G. This private component is represented by the same utility function $U : [0, \overline{y}] \times [0, n\overline{y}] \rightarrow \mathbb{R}$ for all individuals in the group. In addition, individuals are altruistic, and their preferences have a social component: They care *equally* about the levels of private utilities of *all* the other members in their group. We assume that private transfers of money between individuals are not possible in this model, either due to transaction costs, or social norms that prevent such transfers.

Individual i's preferences are represented by the following function V:

$$V(x_1, \dots, x_n, G, \alpha) = U(x_i, G) + \alpha \sum_{j \in N, j \neq i} U(x_j, G), \tag{1}$$

where $\alpha \in (0, 1)$ is a parameter measuring the degree of altruism. I assume that α is positive, meaning that individuals benefit when the private utility of group members increases. The assumption $\alpha < 1$ implies that the individual values their private utility strictly more than their social utility. U is also taken to be increasing and strictly concave.[4] I impose, in addition, the following assumption on U:

ASSUMPTION 1. *For any combination* $(x, G) \in [0, \overline{y}] \times [0, n\overline{y}]$,

$$U_{xG}(x, G) \geq 0.$$

Assumption 1 means that private consumption and the public good are complementary. This is satisfied for many commonly used utility functions: for instance, for Cobb-Douglas utility, where $U(x, G)$ takes the form $U(x, G) = x^a G^{1-a}$ with $0 < a < 1$. Assumption 1, combined with a second assumption outlined below, is a sufficient condition that guarantees the results of this paper.

ASSUMPTION 2. *For any* $G \in [0, n\overline{y}]$ *and* $\alpha \in (0, 1)$,

$$\begin{aligned} U_x(0,0) + (1-\alpha)U_G(0,0) + \alpha \sum_{i \in N} U_G(0,0) > 0 > U_x(\overline{y}, G) \\ + (1-\alpha)U_G(\overline{y}, G) + \alpha \sum_{i \in N} U_G(\overline{y}, G). \end{aligned}$$

An implication of this assumption is that there exists at least one contributor in any equilibrium. We impose this condition to rule out equilibria where nobody contributes. This assumption will help us in establishing the uniqueness of the Nash equilibria of the model. Denote by $\mathcal{U}$ the class of utility functions that satisfy the assumptions above.

A group of n members, the payoff function V and the members' choice of contribution $(g_1, \dots, g_n)$ and private consumptions $(x_1, \dots, x_n)$ define a game in normal form. For a given distribution of incomes denoted by $\mathbf{y} = (y_1, y_2, \dots, y_n)$ and a coefficient of caring α,

4 Derivatives are denoted by subscript—that is, U_x denotes the first order (partial) derivative of U with respect to x.

denote the normal form game described above by $\mathcal{G}(\mathbf{y},\alpha)$. The objective of this paper is to examine the properties of the Nash equilibrium of this game.

Since the social utility function V is strictly concave, finding the Nash equilibrium corresponds to solving the following maximization problem for every individual i, given the contributions of members other than i:

$$\max_{x_i,g_i} \; U(x_i, g_i + \sum_{j\in N, j\neq i} g_j) + \alpha \sum_{j\in N, j\neq i} U(x_j, g_i + \sum_{j\in N, j\neq i} g_j),$$

subject to

$$x_i + g_i \leq y_i \tag{2}$$

$$and \; g_i \geq 0 \;, \; x_i \geq 0$$

Since the constraint in equation 2 must always hold with equality, it is possible to transform the maximization problem above to

$$\max_{g_i} \; U(y_i - g_i, g_i + \sum_{j\neq i}^{n} g_j) + \alpha \sum_{j\in N, j\neq i} U(x_j, g_i + \sum_{j\neq i}^{n} g_j),$$

subject to

$$0 \leq g_i \leq y_i.$$

The first-order condition (necessary and sufficient for a solution to the above program) for any $i \in N$ is

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j\in N, j\neq i} U_G(x_j, G) \leq 0 \,\forall\, i \in N \tag{3}$$

and

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j\in N, j\neq i} U_G(x_j, G) = 0 \;\; if \; g_i > 0. \tag{4}$$

Using this setting, we start by proving the existence and uniqueness of the Nash equilibrium in the following proposition.

PROPOSITION 1. *The game $\mathcal{G}(\mathbf{y},\alpha)$ admits a unique Nash equilibrium. All members who contribute to the public good consume the same level of the private good, denoted by $\tilde{x}$. Agent i is a contributor to the public good if and only if $y_i > \tilde{x}$ in which case, agent i 's contribution is $g_i^* = y_i - \tilde{x}$.*

All proofs are provided in the Appendix. Proposition 1 indicates that $\tilde{x}$ is also the critical level of income above which an agent starts contributing. We can see this in two parts: First, we show that an individual with an income $y_i \leq \tilde{x}$ cannot be a contributor. To show this, suppose for contradiction that $y_i \leq \tilde{x}$ for some i and i is still a contributor. Then the private good consumption for agent i is $x_i^* < y_i$, and hence $x_i^* < \tilde{x}$. Since all contributors at any equilibrium have the same private good consumption, this violates what is proved in the proof of proposition 1.

We can next show that anyone with an income higher than $\tilde{x}$ must be a contributor. To show this, assume for contradiction that there exists some i such that $y_i > \tilde{x}$ and i does not contribute. Then $x_i^* = y_i > \tilde{x}$, again violating what was just proved above. Finally, note that if $y_1 > y_2 > \tilde{x}$, for two agents 1 and 2, it follows that $y_1 - \tilde{x} = g_1^* > g_2^* = y_2 - \tilde{x}$. Hence, the contributions of agents are ordered in increasing order of their income.

The fact that $\tilde{x}$ is also the critical level of income above which an agent starts contributing is driven by the assumption that everyone has the same preferences. This assumption implies that the threshold level of income above which members start contributing is the same for everyone and contributing members consecrate all of their income that crosses this threshold to the public good. The role played by $\tilde{x}$ as the critical level of income thus reveals two more insights about the model. First, contributing to a public good serves the purpose of redistributing income in an altruistic society. It is easy to see that when all members contribute to the public good, the redistribution achieved is complete—all the members consume the same level of the private good and the public good. Second, the threshold $\tilde{x}$ depends upon the distribution of income, and it is possible that there are noncontributors in equilibrium when there is income inequality. In this case, the wealthier individuals in the society contribute to the public good, and the poorer individuals free-ride. Free-riding behavior in an altruistic society is hence purely based on income inequality.

Proposition 1 implies that the amount of the public good provided at the equilibrium does not change with any redistribution of income that keeps the set of contributors unchanged. Since every contributor i donates $y_i - \tilde{x}$, if we redistribute income in a way that does not change the set of contributors, the threshold $\tilde{x}$ does not change and, hence, neither does the amount of the public good provided. This is formally stated without proof in the following corollary:

Corollary 1. *Any redistribution in incomes that does not change the number of contributors at the equilibrium leaves the Nash level of the public good unchanged.*

The neutrality to redistribution of income as predicted in corollary 1 is not specific to the model of public good provision with altruistic preferences; it holds true as well in particular for the model in Bergstrom et al. (1986). While the model in Bergstrom et al. (1986) had strong neutrality results, such as predicting that the total amount of charity does not change when income is redistributed among the set of donors and complete crowding out of private donations by government provision, the joy-of-giving models of altruistic giving were thought to avoid these neutrality results (Ribar and Wilhelm (2002)). Corollary 1 reveals that the specification of altruism can indeed matter for determining the neutrality of income redistribution among the set of contributors. It should also be noted that neutrality may not hold if the current model is modified to the situation where individuals do not have the same level of altruism, making the case where government contributions are neutral an extremely specific case rather than the general one proposed by Bergstrom et al. (1986).

Thanks to proposition 1, we can now denote by $\mathbf{g}^*(\mathbf{y}, \alpha) = (g_1^*, \dots, g_n^*)$ the unique Nash equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$ and by $G^*(\mathbf{y}, \alpha)$ the aggregate level of the public good at such an equilibrium. We next examine a comparative statics result that determines how a change in the degree of altruism α affects contributions. The following proposition shows that a weak (or strict) increase in altruism weakly (or strictly) increases the contributions of all contributors to the public good at the Nash equilibrium.

Proposition 2. *For any two levels of the altruism coefficient $\alpha_1, \alpha_2 \in (0, 1)$, whenever $\alpha_1 \leq \alpha_2$, we have that $G^*(\mathbf{y}, \alpha_1) \leq G^*(\mathbf{y}, \alpha_2)$. Moreover, whenever $\alpha_1 < \alpha_2$, we have that $G^*(\mathbf{y}, \alpha_1) < G^*(\mathbf{y}, \alpha_2)$. The set of contributors may increase when the level of altruism α increases.*

The structure of the proof of proposition 2 shows that when the level of altruism strictly increases, the threshold level of private consumption strictly decreases, meaning that the individuals who contributed before the increase in altruism strictly increase their contributions, and in addition, there may be new contributors. A strict increase in altruism therefore not only strictly increases the total amount of the public good, it strictly increases the contributions of all existing contributors. Being altruistic in the current model helps improve public good provision.

While the amount of the public good is higher when the group is more altruistic, it is not clear from proposition 2 alone that when the group is more altruistic, the level of public good provision is social-welfare maximizing. Indeed, this is not necessarily the case, as seen in the next section on welfare implications of the model.

3. Welfare

Comparisons of well-being between Nash equilibria of public good models can be complex (see, e.g., Banerjee and Gravel 2020). In the case of the current paper, while the level of the public good is higher when the group's level of altruism increases, the levels of some of the individuals' private consumption decrease and, hence, the net effect on well-being can be ambiguous. A different metric of welfare to consider is the Pareto-efficiency of the Nash equilibrium. While it is widely understood that the Nash equilibrium level of a public good is typically Pareto-inefficient (Cornes and Sandler 1996), this is true if there exists a Pareto-optimal allocation with a higher level of the public good than in the Nash equilibrium. This may not happen in many cases, such as when the Nash equilibrium is not strictly interior and there are noncontributors in the equilibrium (Buchholz and Peters 2001).

Despite these complexities, it is nevertheless conceivable to compare the Nash outcome against the metric of a benevolent social planner's preferred allocation of resources. The first step in this endeavor is to define the social planner's objective function. There are two possible ways of formulating the social planner's function. In the first formulation, the benevolent social planner aggregates only the private preferences of the individuals, making the social planner's objective function:

$$\sum_{i \in N} U(x_i, G). \tag{5}$$

Alternatively, one might think that the social planner takes both the private and the social component of the individual utilities into account when aggregating the preferences of all the individuals in the group. The social planner's function can be then written as

$$\sum_{i \in N} (U(x_i, G) + \alpha \sum_{j \in N, j \neq i} U(x_j, G)). \tag{6}$$

Removing the inner summation sign, the function in equation 6 can rewritten as

$$(1 + (n-1)\alpha) \sum_{i \in N} U(x_i, G). \tag{7}$$

Since the function in expression 7 is just a monotonic transformation of the function in expression 5, it follows that irrespective of whether we choose the formulation in expression 5 or expression 6 (or any convex combination of the two), the social planner's preferred solution to the public good allocation problem remains unchanged. The strict concavity of the social planner's function ensures that the social planner's program as presented in expression 5 has a unique interior solution. Hence the social planner would allocate the same, strictly positive level of the private good to all the members of the group, and we denote this level of private good consumption by $\overline{x}$. This also makes intuitive sense because the utility function is identical for all members of the group. We denote the level of public good allocated by the social planner by $\overline{G}$. It is also worth remarking that this solution is by definition Pareto-efficient. The following proposition looks at how the Nash level of provision of the public good compares to the level $\overline{G}$ chosen by the social planner.

PROPOSITION 3. *If the game $\mathcal{G}(\mathbf{y},\alpha)$ has a strictly interior equilibrium, then $G^* < \overline{G}$. However, if there are noncontributors to the public good in the Nash equilibrium, then there may be overprovision of the public good in the Nash equilibrium compared to the social planner's solution.*

The proof of the first part of the proposition is provided in the appendix. The second part of the proposition related to overprovision of the public good can be easily demonstrated with an example of two agents and additively separable preferences, as shown below:

EXAMPLE 1. The private utility function of any player i takes the form

$$U(x_i, G) = a \log x_i + b \log G \text{ with } 0 < a < 1,\ 0 < b < 1,\ a + b = 1. \tag{8}$$

Two agents 1 and 2 have incomes y_1 and y_2. Assume without loss of generality that $y_1 \leq y_2$. The Nash equilibrium has only one contributor when $y_1 < \frac{ay_2}{a+b(1+\alpha)}$. We know from proposition 1 that in the case when there is only one contributor, only the richer individual, member 2, will contribute. A comparison of the levels of private and public good provided under the social planner's regime and in the Nash equilibrium is presented in the first two columns of table 1. The level of provision of the public good by the social planner is strictly lower than the level of provision with altruistic preferences if $y_1 < \frac{\alpha a y_2}{a+b(1+\alpha)}$.

Proposition 3 offers two key insights into the welfare implications of contributing to a public good with altruistic preferences. First, the intuition behind the under-provision of the public good is closely related to proposition 1. Since all contributors must have the same level of consumption of the private good, contributing to a public good acts as a means of redistribution in a society. However, given that the objective of redistribution is achieved, as shown in table 1, the social planner's solution will allocate a larger fraction of the total resources $y_1 + y_2$ to the public good because of the free-riding problem, which typically makes the provision of a public good inefficient. Second, proposition 3 also shows

TABLE 1
Levels of provision of the public and the private good with Example 1

	Social planner's solution	Nash solution
		Both members contribute when
		$y_1 > \frac{ay_2}{a+b(1+\alpha)}$
x_1^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$
x_2^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$
G^*	$\frac{b(y_1+y_2)}{a+b}$	$\frac{b(1+\alpha)(y_1+y_2)}{2a+b(1+\alpha)}$
		Only member 2 contributes when
		$y_1 \leq \frac{ay_2}{a+b(1+\alpha)}$
x_1^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	y_1
x_2^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{ay_2}{a+b(1+\alpha)}$
G^*	$\frac{b(y_1+y_2)}{a+b}$	$\frac{b(1+\alpha)y_2}{a+b(1+\alpha)}$

that the presence of noncontributors can impede redistribution and lead to inefficiency by overprovision, even when individuals are altruistic.

While the idea that the amount of public good provided under the Nash equilibrium can exceed the social planner's allocation is by no means new (Buchanan and Kafoglis 1963), there is very little analysis in the existing literature that explains when overprovision occurs, or why. Buchholz and Peters (2001) suggest that the existence of overprovision might depend on non-normality of preferences, but this explanation does not address agents' preferences for altruism despite the common interpretation of charitable donations as a public good. As further shown by the example in this paper with log preferences, the presence of noncontributors is endogenous and may depend on the distribution of income in the society. Although inequality in income is a known source of the overprovision anomaly even in the model with standard preferences, the presence of altruistic preferences plays a special role in this context. As presented in proposition 2, increasing the level of the altruism parameter α lowers the threshold level of income that determines the number of contributors, potentially expanding the set of contributors. Compared to the standard model, the threshold level of income is therefore lower when individuals are altruistic, and individuals are hence more likely to contribute in the latter case. The persistence of the overprovision phenomenon under altruism is therefore a response to the existence of stark income inequality in society—a point that is further elaborated upon in the discussion at the end of section 4.

A key takeaway of proposition 3, as demonstrated by Example 1, is that the extent of redistribution possible is limited when poorer individuals do not contribute to the public good. This raises the question: is it possible to redistribute resources more efficiently? The most direct way to redistribute income would be through a direct income transfer: however, such transfers may not be always be possible due to transaction costs, as seen in the model presented so far. For a more complete understanding of preferences (and consequences) for redistribution, it is therefore essential to consider the setting of an altruistically minded individual, who cares about both the total amount of the public good as well as the private consumption of other individuals, and has the choice of making both contributions to the public good as well as private transfers. With this framework in mind, the next section presents a second possible model with altruistic preferences: one that incorporates the possibility of making private transfers.

4. Model with private transfers

In this section we consider the model of public goods contribution with altruistic preferences while allowing for private transfers between any two individuals in the group. That is, members of a group of n individuals have incomes $(y_1, \ldots, y_n)$ and the social utility functions of the form V as defined in Section 3. We assume, as before, that the function V is defined using private utilities U belonging to the class of functions $\mathcal{U}$:

$$V(x_1, \ldots x_n, G, \alpha) = U(x_i, G) + \alpha \sum_{j \in N, j \neq i} U(x_j, G).$$

As before, every agent simultaneously chooses how to divide the income net of transfers between his consumption of a private good, x_i, and his contribution to a public good g_i. In addition, an individual i may now transfer an amount $t_{ij} \geq 0$ out of his income y_i to some individual j in his group. The individual's budget constraint gets modified from that in Section 2. It now reads

$$x_i + g_i = y_i - \sum_j t_{ij} + \sum_k t_{ki}.$$

The collection of all bilateral transfers is a matrix $\mathbf{T}$ whose elements t_{ij} are transfers from member i to member j. By convention, $t_{ii} = 0 \; \forall \; i$. The transfers are a means of redistribution of income within the group without changing the total income of the group, $Y = \sum_{i=1}^{n} y_i$.

Denote the game as described above by $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$, where we use as before $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ to denote a given profile of income distributions. Our objective is to characterize the Nash equilibria of this game. Finding the Nash equilibrium corresponds to solving the following maximization problem for every individual i given the contributions to the public and private transfers by members others than i:

$$\max_{g_i, t_{i1}, \ldots, t_{ij}, \ldots t_{in}} U\left(y_i - \sum_j t_{ij} + \sum_k t_{ki} - g_i, g_i + \sum_{j \in N, j \neq i} g_j\right)$$
$$+ \alpha \sum_{j \in N, j \neq i} U\left(y_j - g_j - \sum_h t_{jh} + \sum_k t_{kj}, g_i + \sum_{j \in N, j \neq i} g_j\right), \tag{9}$$

subject to

$$g_i \geq 0 \text{ and } x_i \geq 0 \;\; \forall \; i$$

and

$$t_{ij} \geq 0 \; \forall \; i, j, \;\; t_{ii} = 0 \; \forall \; i.$$

The first order conditions for agent i are necessary and sufficient for the solution to the program above.

With respect to g_i,

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) \leq 0 \;\; \forall \; i \in N \tag{10}$$

and

$$-U_x(x_i, G) + U_G(x_i, G) + \alpha \sum_{j \in N, j \neq i} U_G(x_j, G) = 0 \quad if \; g_i > 0; \tag{11}$$

and with respect to t_{ij}:

$$-U_x(x_i, G) + \alpha U_x(x_j, G) \leq 0 \quad \forall \; i, j \in N \tag{12}$$

and

$$-U_x(x_i, G) + \alpha U_x(x_j, G) = 0 \quad if \;\; t_{ij} > 0. \tag{13}$$

While transfers are potentially unconstrained, we must have that the sum of all consumptions, net transfers made and the contributions to the public good cannot exceed the sum of all incomes available. This is because if individual i transfers a certain amount to individual j, individual j receives the same amount of money. Hence, transfers given, net of transfers received, must equal zero.

$$\sum_i (x_i + g_i) - \sum_i \sum_j t_{ij} + \sum_k \sum_i t_{ki} = \sum_i y_i$$

We will now establish some properties regarding the Nash equilibrium for this modified setting of the model with altruism. We have not yet proved the existence of this equilibrium. Denote by $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$, $\mathbf{T}^*$, and G^*, the levels of private good consumption, the matrix of transfers, and the level of the public good, respectively, at a Nash equilibrium.

Lemma 1. *If there is at least one donor and one receiver of transfers in any Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$, then there exist threshold levels of consumption $\underline{x} \in \mathbb{R}$ and $\overline{x} \in \mathbb{R}$, $\underline{x} < \overline{x}$ such that,*

a. All receivers of transfers have a private good consumption equal to the minimum of all private good consumptions in equilibrium, $\underline{x}$, and all donors have the same private good consumption, denoted by $\overline{x}$.
b. There are no transfer intermediaries in equilibrium—no individual who both gives and receives transfers. The set of donors and the set of receivers of transfers are disjoint.
c. $\underline{x} \le x_i^* \le \overline{x}\ \forall\, i \in N$.

The thresholds in consumption $\underline{x}$ and $\overline{x}$ correspond to the thresholds in consumption established for the charity game by Arrow (1981). Indeed, the model of private transfers considered here is the extension of the charity game, to the case where individuals have two ways to give to individuals they care about—either transfer money privately or contribute to the public good that everyone benefits from. Lemma 1 establishes that the result in Arrow (1981) establishing the thresholds in private consumption still holds true for the extension of this model with private transfers. Another implication of lemma 1 is that an individual in an equilibrium with transfers is going to be in one of the following mutually cases: He will either give or receive transfers, or do neither. This implication, together with the next result, will be very useful in further characterizing the equilibrium with transfers. The next lemma shows that a receiver cannot contribute to the public good at an equilibrium.

Lemma 2. *If there is at least one donor and one receiver of transfers in any Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$, then at the Nash equilibrium a receiver of a transfer cannot contribute to the public good.*

Lemma 1 shows that at equilibrium, there must be threshold levels of consumption for donors and receivers of transfers when there are transfers in equilibrium. However, it is not clear whether these thresholds in consumption are, in fact, thresholds in income. Lemma 2 will help establish the next result of this section: Lemma 3, which explores the implications of the thresholds in consumption established in lemma 1 in terms of thresholds in income. Let $\mathbf{g}^* = (g_1^*, \ldots, g_n^*)$ denote as before the profile of contributions to the public good at the Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ defined with a function $U \in \mathcal{U}$.

Lemma 3. *If there exist donors and receivers in a Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$, then for any agent $i \in N$,*

a. If $y_i < \underline{x}$ then i is a receiver.
b. If $\underline{x} \le y_i \le \overline{x}$ then i is neither a donor, nor a receiver.
c. If $y_i > \overline{x}$ then i is in one of the mutually exclusive cases:

- *i is donor.*
- *i is neither a donor nor a receiver and contributes exactly $g_i^* = y_i - \overline{x}$ to the public good.*

The thresholds in consumption $\underline{x}$ and $\overline{x}$, in Arrow (1981)'s classic paper on the charity game, translate to thresholds in income. Lemma 3 establishes that his result does not follow through quite exactly for the case of private transfers, since now contributions to the public good are possible. We next establish that there exists a common threshold level of consumption that all contributors to the public good enjoy. Lemma 4 is essentially a modification of the threshold income discussed in proposition 1, for the model with transfers. While it is still true that all contributors enjoy the same level of private consumption, this common level of consumption does not translate exactly to a threshold in income as in proposition 1.

LEMMA 4. *At a Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$, all members who contribute a positive amount to the public good enjoy a common level of private good consumption, denoted by $\hat{x}$. Further, the following is true for any $i \in N$:*

a. If $y_i \leq \hat{x}$, then i is not a contributor.
b. If $y_i > \hat{x}$ then i is in one of the following mutually exclusive cases:

– i is a contributor to the public good.
– i does not contribute to the public good, and i is a donor and $x_i^* = \hat{x} = \overline{x}$.

In the charity game presented by Arrow (1981), the thresholds in terms of consumption were thresholds in terms of incomes as well. Lemma 3 and lemma 4 together lead to a useful result—indeed, that there is a common level of consumption that both givers of transfers and contributors to the public good enjoy, and that this is the common threshold level of income required to become either a donor of transfers or a contributor to the public good, as we establish in proposition 4, which also summarizes the common implications from lemma 3 and lemma 4.

PROPOSITION 4. *At a Nash equilibrium of the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$, the following is true: For any $i \in N$,*

- *If $y_i \leq \underline{x}$, then i is not a contributor and i is a receiver, and $x_i^* = \underline{x}$.*
- *If $\underline{x} \leq y_i \leq \overline{x}$, then i is neither a contributor to the public good, nor a donor, nor a receiver of transfers, and $x_i^* = y_i$.*
- *If $y_i > \overline{x}$ then i is either a contributor to the public good, or a donor of transfers, or both, with $x_i^* = \overline{x}$.*

Proposition 4 shows that the minimum level of income required to contribute to a public good and the threshold income required to become a donor are the same. This, of course, does not imply that the set of contributors to the public good needs to be identical to the set of givers of transfers, just that their levels of private consumption must be the same at the equilibrium. Another implication is that the set of donors of transfers at an equilibrium, and contributors to a public good, may not be unique even if the thresholds $\underline{x}$ and $\overline{x}$ are uniquely defined. Proposition 4 provides an intuition for the following result that holds true at any equilibrium for the model with transfers:

LEMMA 5. *At a Nash equilibrium for the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$, if every member contributes a positive amount to the public good, then private transfers between individuals cannot exist.*

Intuitively, lemma 5 is an implication of proposition 4. Since the threshold level of consumption required to be a donor or a contributor to the public good is, in fact, a threshold in terms of income, if everyone has an income that is higher than the threshold, then there is no need for transfers of income. In this sense, transfers of income are supplementing the role of the public good: Even when there are no transfers, individuals would like to consume the public good. However, individuals only give transfers when incomes or consumptions of individuals that they care about are sufficiently low.

Note that proposition 4 holds true for an equilibrium with transfers, if it exists. It is also a counterpart to Arrow (1981)'s result establishing common thresholds in income for donors and receivers of transfers for the model without public good provision. Deriving the uniqueness of the thresholds $\underline{x}$ and $\overline{x}$ for the general model requires more detailed analysis meriting further research, which is beyond the scope of the current paper. For the purposes of this paper, I provide the characterization, existence, and uniqueness of the equilibrium of the model with transfers for the case of additively separable utility functions. As presented below, the model with additive separability, despite being a special case of the more general model, provides us with considerable insight regarding the interactive role played by the public goods and private transfers in achieving redistribution.

4.1. Analysis for additively separable utility

This section provides the analysis for the case of additively separable utility functions. Since this is only a special case of the more general function form U examined in the rest of the paper, the results established in the paper so far still hold true. If U is additively separable, then the private utility function U can be expressed as $U(x_i, G) = U^1(x_i) + U^2(G)$. We assume that U belongs to the class of function $\mathcal{U}$ as before. The maximization problem as shown in (9) transforms into

$$\max_{g_i, t_{i1}, \dots, t_{ij}, \dots t_{in}} U^1(y_i - \sum_j t_{ij} + \sum_k t_{ki} - g_i) + U^2\left(g_i + \sum_{j \in N, j \neq i} g_j\right)$$
$$+ \alpha \sum_{j \in N, j \neq i} U^1\left(y_j - g_j - \sum_h t_{jh} + \sum_k t_{kj}\right) + U^2(g_i + \sum_{j \in N, j \neq i} g_j),$$

subject to

$$g_i \geq 0 \text{ and } x_i \geq 0 \ \ \forall\, i$$

and

$$t_{ij} \geq 0 \,\forall\, i, j, \ \ t_{ii} = 0 \,\forall\, i.$$

The first-order conditions for agent i must hold at a Nash equilibrium:
With respect to g_i,

$$-U^{1\prime}(x_i) + (1 + \alpha(n-1))U^{2\prime}(G) \leq 0 \ \forall\, i \in N \tag{14}$$

and

$$-U^{1\prime}(x_i) + (1 + \alpha(n-1))U^{2\prime}(G) = 0 \quad if\ g_i > 0; \tag{15}$$

with respect to t_{ij},

$$-U^{1\prime}(x_i) + \alpha U^{1\prime}(x_j) \leq 0 \quad \forall\, i, j \in N \tag{16}$$

and

$$-U^{1\prime}(x_i) + \alpha U^{1\prime}(x_j) = 0 \quad if\ \ t_{ij} > 0. \tag{17}$$

Since the function $U^{1\prime}$ is strictly increasing in equation 16, we can define its inverse: $U^{1\prime -1}$. We can then define the function $\epsilon : \mathbb{R} \to \mathbb{R}$ as follows:

$$x_i \leq U^{1\prime -1}(\alpha U^{1\prime}(x_j)) = \epsilon(x_j). \tag{18}$$

Since we must have at least one contributor to the public good, and by proposition 4, this individual's level of private good consumption must equal $\overline{x}$, and equation 15 holds for this individual:

$$
\begin{aligned}
&-U^{1'}(\overline{x}) + (1+\alpha(n-1))U^{2'}(G) = 0 \\
&\implies (1+\alpha(n-1))U^{2'}(G) = U^{1'}(\overline{x}) \\
&\implies G = U^{2'^{-1}}\left(\frac{U^{1'}(\overline{x})}{1+\alpha(n-1)}\right) = \lambda(\overline{x}).
\end{aligned} \tag{19}
$$

Similarly, at an equilibrium with transfers, there must be at least one donor and one receiver, and equation 17 holds for this pair of members:

$$
\overline{x} = \epsilon(\underline{x}). \tag{20}
$$

We can thus write $G = \gamma(\underline{x})$ where $\gamma(\underline{x}) = \lambda(\epsilon(\underline{x}))$. We add an additional assumption to ensure that the functions ϵ and λ are always defined over their respective domains:

ASSUMPTION 3. *For every $x \in [y_{(1)}, y_{(n)}]$ where $y_{(1)} = \min{}_i y_i$ and $y_{(n)} = \max{}_i y_i$, and every $\alpha \in (0,1)$,*

$$
\lim_{x\to\infty} U^{1'}(x) = \lim_{x\to\infty} \alpha U^{1'}(x)
$$

and

$$
\lim_{x\to\infty} U^{2'}(x) = \lim_{x\to\infty} \frac{U^{1'}(x)}{1+\alpha(n-1)}.
$$

If the above assumption does not hold, the functions ϵ and λ may not be defined for every $x \in [0, \overline{y}]$. Following Arrow (1981), define the function x^+ such that:

$$
x^+ = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}
$$

The sum of all transfers given, net of contributions to the public good, and transfers received, must be zero. We can rewrite this constraint as

$$
\sum_i (y_i - \epsilon(\underline{x}) - g_i)^+ - \sum_i (\underline{x} - y_i)^+ = 0.
$$

Alternatively,

$$
\sum_i (y_i - \epsilon(\underline{x}))^+ - \sum_i (\underline{x} - y_i)^+ - \gamma(\underline{x}) = 0.
$$

We use the notations defined above to characterize the equilibrium of the model with transfers in the following proposition:

PROPOSITION 5. *If the function $U \in \mathcal{U}$ is additively separable, an equilibrium for the game $\mathcal{G}^{\mathcal{T}}(\mathbf{y}, \alpha)$ always exists.*

- *If $\tilde{x} \leq \epsilon(y_{(1)})$ where $y_{(1)} = \min{}_i y_i$ and $\tilde{x}$ is the unique threshold level of income required to become a contributor for the game $\mathcal{G}(\mathbf{y}, \alpha)$, the equilibrium for the model without transfers $\mathbf{g}^*(\mathbf{y}, \alpha)$ is also the equilibrium for the model with transfers. The equilibrium level of the public good G^* as well as individual levels of contributions to the public good g_i^* are uniquely determined.*

- *If $\tilde{x} > \epsilon(y_{(1)})$, there exists an equilibrium with transfers, where the threshold levels of private consumption for receivers ($\underline{x}$), for donors and contributors to the public good ($\overline{x}$), and the aggregate public good G^* are uniquely determined. Individual contributions to the public good and transfers, however, may not be unique.*

The proof of proposition 5 involves three steps. It is first shown that for the case of additively separable utility functions, an equilibrium with transfers and an equilibrium without transfers are mutually exclusive. We derive a necessary and sufficient condition ($\tilde{x} \leq \epsilon(y_{(1)})$) under which there exists an equilibrium without transfers. When the model with transfers has an equilibrium without transfers, of course, the equilibrium of the model with transfers coincides with the equilibrium of the model without transfers. We have already shown in section 2 that the equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$ is unique. Secondly, we derive a necessary and sufficient condition under which $\underline{x}$, $\overline{x}$ and G^* are uniquely determined for the model with transfers. In our last step, we show that the latter necessary and sufficient condition condition holds if and only if $\tilde{x} \leq \epsilon(y_{(1)})$.

Proposition 5 offers some important insights. If everyone has the same income, private transfers cannot exist (i.e., it is easy to see that $\tilde{x} \leq \epsilon(y_{(1)})$ holds by default). The existence of transfers hence depends on the extent of inequality in private consumption between the richest and the poorest individual. This result bears similarity to the structure of the equilibria in Arrow (1981), where inequality in incomes motivated the rich to give to the poor, and is clearly driven by the assumption that individuals are all equally altruistic. Proposition 5 shows that when individuals derive benefit from a public good, the existence of transfer depends on the inequality of private consumptions present in the society, not just the inequality in incomes.

Proposition 5 thus offers three additional insights to Arrow's result. First, as mentioned, even when private transfers are permitted, they still do not always happen. The intuition behind this is as follows: Whereas everyone benefits from the public good, private transfers benefit the utility of individuals only via the private consumption of the receiver and the giver of transfers. In a society where everyone cares about everyone else, if the difference in private consumption is not very high, the richest individual may want to contribute to the public good rather than make a private transfer. Second, transfers were the sole means of redistribution in Arrow's altruistic society. Public good provision is also an important means of redistribution in society, as demonstrated by Bergstrom et al. (1986). The current proposition shows us insight into the interaction between two alternative means of redistribution—public good provision and transfers. It shows us both that the methods of distribution are important and that whether or not private transfers are chosen as a means of redistribution in society depends on the extent of inequality in private consumption, which may, in turn, depend on how much inequality in consumption in society remains despite the redistributive effect of contributions to the public good. Finally, the importance of public good provision as a means of redistribution is highlighted when the implications of proposition 5 are combined with those of lemma 5. Since inequality in private consumption is necessary for the existence of transfers, if everyone is contributing to the public good, then private transfers are unnecessary.

This result has some policy implications. Reducing inequality in incomes should not be a policy objective for its own sake but a means to the end of reducing inequality in consumptions. Other than income taxation, provision of a public good can be an alternative means of redistribution, and as the example below shows, a progressive income taxation policy may actually improve public good contribution by reducing the necessity to make private transfers.

4.2. Welfare with the model with transfers

With proposition 5 in mind, it is possible to revisit Example 1 with a view to understand why overprovision happens in the absence of private transfers. Consider a modification of Example 1. In addition to the existing model of altruistic preferences in public good provision, we now introduce the possibility of the two individuals to make private transfers of income to each other. The preferences of the two individuals i and j stay the same, and the private utility functions take the same form of log preferences as seen in Example 1. Adding private transfers changes the model by affecting net income, so the budget constraint of the individuals changes. Example 2 summarizes the new model for an agent i.

EXAMPLE 2. The maximization problem for any agent i is given by

$$\max_{x_i, t_{ij}, g_i} a \log x_i + b \log G + \alpha a \log x_j + \alpha b \log G, \tag{21}$$

subject to

$$x_i + g_i = y_i - t_{ij} + t_{ji}$$

and

$$x_i \geq 0,\ g_i \geq 0,\ t_{ij} \geq 0,\ t_{ji} \geq 0,\ t_{ii} = 0.$$

The third column of table 2 shows the levels of private and public good provided under this modified model, which allows for private transfers, as well the conditions necessary and sufficient for the existence of transfers, together with a comparison of the levels of public good provided in the model without transfers, as well as the social planner's solution.

It is straightforward to see that, given y_1, α, a and b, we must have $\frac{\alpha a}{a+b(1+\alpha)} y_1 < \frac{a}{a+b(1+\alpha)} y_1 < \frac{a}{a+b} y_1$. This gives us a additional insight into proposition 5. Given y_1 and y_2, when there is only one contributor to the public good in Bergstrom et al. (1986), or the model without transfers, if we allow for transfers, it does not mean that transfers will exist. Transfers exist only when individual 1 is "sufficiently poorer" than individual 2 (i.e.,

TABLE 2

Levels of provision of the public and the private good with example 1

	Social planner's solution	Nash solution	Model with transfers
		Both members contribute when $y_1 > \frac{ay_2}{a+b(1+\alpha)}$	$y_1 > \frac{ay_2}{a+b(1+\alpha)}$
x_1^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$
x_2^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$	$\frac{a(y_1+y_2)}{2a+b(1+\alpha)}$
G^*	$\frac{b(y_1+y_2)}{a+b}$	$\frac{b(1+\alpha)(y_1+y_2)}{2a+b(1+\alpha)}$	$\frac{b(1+\alpha)(y_1+y_2)}{2a+b(1+\alpha)}$
t_{21}			-
		Only member 2 contributes when $y_1 \leq \frac{ay_2}{a+b(1+\alpha)}$	$y_1 \leq \frac{ay_2}{a+b(1+\alpha)}$
			Transfers exist when $y_1 \leq \frac{\alpha a y_2}{a+b(1+\alpha)}$
x_1^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	y_1	$\frac{\alpha}{1+\alpha}\frac{a(y_1+y_2)}{a+b}$
x_2^*	$\frac{1}{2}\frac{a(y_1+y_2)}{a+b}$	$\frac{ay_2}{a+b(1+\alpha)}$	$\frac{1}{1+\alpha}\frac{a(y_1+y_2)}{a+b}$
G^*	$\frac{b(y_1+y_2)}{a+b}$	$\frac{b(1+\alpha)y_2}{a+b(1+\alpha)}$	$\frac{b(y_1+y_2)}{a+b}$
t_{21}			$\frac{a\alpha y_2-[a+b(1+\alpha)]y_1}{(1+\alpha)(a+b)}$

$y_1 \leq \frac{\alpha a}{a+b(1+\alpha)} y_2$). Three further implications of the model with transfers in particular are worth noting:

- The threshold level of income required to become a contributor $\tilde{x} = \frac{a y_2}{a+b(1+\alpha)}$ in our model with altruistic preferences is relatively defined in terms of the income of the richer member (member 2). Moreover, this threshold does not change once we allow for private transfers. Hence when $y_1 > \frac{a y_2}{a+b(1+\alpha)}$, both the model with and without transfers have two contributors, and no private transfers occur between individuals.
- Transfers do not always exist. Transfers exist when $y_1 \leq \frac{a \alpha y_2}{a+b(1+\alpha)}$, and this threshold is strictly lower than the threshold $\tilde{x}$. In other words, it is possible for the distribution of income to be unequal, but not enough merit a transfer; in this case, the level of public good provided does not exceed the social planner's level of provision.
- When transfers exist, the level of public good provided under the model with transfers is exactly equal to the social planner's level of provision. In other words, there is no longer overprovision of the public good. Hence, allowing for private transfers removes the need for the level of public good provision to be inefficiently high. We also observe that the levels of private good at the equilibrium of the model with transfers is not the same as the social planner's level, even when the levels of public good provision under the two models are the same.

The intuition provided above has policy implications for government cash transfers to the poor. If the distribution of income in society is highly unequal and private transfers of income are not possible, an overprovision of the public good may occur when richer individuals contribute to the public good as a way to improve the welfare of the poor. Since the poor care about the consumption of both the private and the public good, the public good may not directly improve the private consumption of the poor. The outcome may be a level of provision that is both "too much" and inefficient. Proposition 3 thus indicates that public goods may not perfectly substitute the role of government income transfers to the poor.

The question of the efficiency of government transfers of income versus private transfers of income when there is a public good involved is an important one, but beyond the scope of the current analysis. While example 2 is a very specific case of a more general model of altruistic preferences with transfers, it is still useful in gaining an understanding of preferences for redistribution. Moreover, these results underline the idea that public goods and private transfers play a complementary role in achieving redistribution and that the private provision of a public good as the sole "second-best" means of redistribution (first-best being the social planner) is only the case if the initial extent of income inequality in a society is not very high. This is of special relevance in light of the fact that income inequality is on the rise in many economies across the world (Piketty et al. 2019, Hoffmann et al. 2020).

5. Conclusion

This paper examines the implications of a model wherein individuals with altruistic preferences contribute to a public good. Two alternative versions of this model are studied and found to be closely related: In the first model, individuals cannot transfer money privately to other people they care about; in the second model, such private transfers are allowed. There are many examples of situations where each model holds relevance. People may care about other individuals in a society who benefit from the public good while being unable to privately transfer money; at the same time, in circles such as family and friends, the possibility of transferring income privately may change the behaviour of individuals with regard to public good contributions. Although altruism in itself might positively influence contributions to a public good, the possibility of giving private transfers causes a conflict: Is altruism best expressed by contributing to the public good or by transferring money privately? The paper finds that both means of redistribution matter and that income inequality in society plays a key role in determining which one is preferred.

The conclusions of this paper also raise important questions relevant for future research. Since private transfers are a (direct) means of redistributing income, the effect of this on labour market decisions is pertinent. As Konrad and Lommerud (2011) show, the decision to participate more in the labour market may well be interconnected with the decision to contribute to the public good, and hence deserves serious consideration in further research. Another important question is the effect on the model of increasing the population. Andreoni (1988) shows that free-riding dominates in large economies; whether the same effect persists in economies that allow for altruism and transfers is worth consideration. In addition, the analysis of the equilibrium/equilibria of the model with transfers for the general case of nonadditively separable preferences is another context that may have interesting connections to the results of this paper. Last, the case where individuals differ in the degree of their altruism (with or without including the possibility of private transfers) is another notable avenue for further research.

Appendix A: Proofs

A.1. Proof of proposition 1

Proof. We first show that a Nash equilibrium of the game $\mathcal{G}(\mathbf{y}, \alpha)$ always exists. For a given α, $\mathbf{y}$, the function

$$V(g_1, \dots, g_n, \mathbf{y}, \alpha) = U(y_i - g_i, g_i + G_{-i}) + \alpha \sum_{j \in N, j \neq i} U(x_j, g_i + G_{-i})$$

is continuous, and given that $g_i \in [0, \overline{\mathbf{y}}]$ is a compact set for any agent i, then by Weierstrass theorem, the maximization problem with V admits a solution. Define for every i, $G_{-i} = \sum_{j \neq i} g_j$ and $\mathbf{x_{-i}} = (x_1, \dots x_{i-1}, x_{i+1}, \dots, x_n)$ (the profiles of total public good contributions and individual private good consumption of members other than i). By Berge maximum theorem, the best response functions $g_i(G_{-i}, \mathbf{x_{-i}}, \mathbf{y}, \alpha)$ are continuous. Hence, by Brouwer's fixed point theorem, an equilibrium exists. We then show that all agents who contribute to the public good have the same level of private good consumption, which will be denoted by $\tilde{x}$. Denote by $\mathbf{x}^* = (x_1^*, \dots, x_n^*)$ the levels of private good consumption at an equilibrium corresponding to the levels of contribution to the public good $\mathbf{g}^* = (g_1^*, \dots, g_n^*)$ for the n group members at the equilibrium. For contradiction, assume that in an equilibrium there exist arbitrary agents 1 and 2 who contribute to the public good and without loss of generality, that $x_1^* < x_2^*$. Since condition 4 holds at the equilibrium for both agents,

$$\begin{aligned} &- U_x(x_1^*, G^*) + U_G(x_1^*, G^*) + \alpha \sum_{j \in N, j \neq 1} U_G(x_j^*, G^*) \\ &= 0 = -U_x(x_2^*, G^*) + U_G(x_2^*, G^*) + \alpha \sum_{j \in N, j \neq 2} U_G(x_j^*, G^*). \end{aligned}$$

Cancelling the common terms out:

$$\begin{aligned} &- U_x(x_1^*, G^*) + U_G(x_1^*, G^*) + \alpha U_G(x_2^*, G^*) \\ &= -U_x(x_2^*, G^*) + U_G(x_2^*, G^*) + \alpha U_G(x_1^*, G^*). \end{aligned}$$

Rearranging

$$U_x(x_2^*, G^*) - U_x(x_1^*, G^*) + (1-\alpha)[U_G(x_1^*, G^*) - U_G(x_2^*, G^*)] = 0.$$

Since we assumed that $x_1^* < x_2^*$,

$$U_x(x_2^*, G^*) - U_x(x_1^*, G^*) < 0$$

by strict concavity of U.

By assumption 1,

$$U_G(x_1^*, G^*) - U_G(x_2^*, G^*) \leq 0$$

as well, leading to a contradiction. A similar argument holds if $x_1^* > x_2^*$.

Next, we show uniqueness. We just showed above that all contributors must have the same level of private consumption—the threshold level of income $\tilde{x}$ that must hold in any equilibrium. For contradiction, suppose that in fact there are two equilibria for a game $\mathcal{G}(\mathbf{y}, \alpha)$. Then in fact, there must exist two distinct thresholds; let us denote them by $\tilde{x}_1 \neq \tilde{x}_2$, with two sets of contributors, denoted by C_1 and C_2. Let us note the number of contributors by c_1 and c_2, and the amount of the public good G as G_1^* and G_2^* respectively as well. Without loss of generality, assume that $\tilde{x}_1 < \tilde{x}_2$.

We first argue that we must have $C_2 \subseteq C_1$. To see this, we use the fact that contributions are ordered in increasing order of income. Suppose we order individuals in increasing order of income (i.e., $y_1 \leq y_2 \leq \cdots \leq y_n$). Then if agent n, the richest individual, is a contributor under the threshold $\tilde{x}_2$, they must be a contributor under $\tilde{x}_1$. Since the threshold $\tilde{x}_1$ is lower than $\tilde{x}_2$, the number of contributors under the threshold $\tilde{x}_1$ must be at least as large as under $\tilde{x}_2$.

We next argue that $G_1^* > G_2^*$. We use that $\tilde{x}_1 < \tilde{x}_2$, hence $y_i - \tilde{x}_1 > y_i - \tilde{x}_2$ for any $y_i \geq 0$. Summing these over the sets C_1 and C_2 and using that $C_2 \subseteq C_1$, we will have that $G_1^* = \sum_{i \in C_1}(y_i - \tilde{x}_1) > \sum_{i \in C_2}(y_i - \tilde{x}_2) = G_2^*$.

We next use that equation 4 must hold for any contributor in an equilibrium. We know from assumption 2 that there must be at least one contributor in any equilibrium. Therefore, equation 4 will hold with equality for any contributor under the thresholds $\tilde{x}_1$ and $\tilde{x}_2$, giving us

$$\begin{aligned} - U_x(\tilde{x}_1, G_1^*)) + U_G(\tilde{x}_1, G_1^*) + \alpha \sum_{j \neq i} U_G(x_j, G_1^*)) = 0 = -U_x(\tilde{x}_2, G_2^*) \\ + U_G(\tilde{x}_2, G_2^*)) + \alpha \sum_{j \neq i} U_G(x_j, G_2^*). \end{aligned}$$

We add and subtract a term $\alpha U_G(\tilde{x}_1, G_1^*)$ on the left hand side, and we do the same on the right hand side with the term $\alpha U_G(\tilde{x}_2, G_2^*)$, giving us

$$\begin{aligned} & - U_x(\tilde{x}_1, G_1^*)) + (1-\alpha) U_G(\tilde{x}_1, G_1^*) + \alpha \sum_{j \in N} U_G(x_j, G_1^*) \\ & = -U_x(\tilde{x}_2, G_2^*) + (1-\alpha) U_G(\tilde{x}_2, G_2^*) + \alpha \sum_{j \in N} U_G(x_j, G_2^*). \end{aligned}$$

Rearranging, and collecting common terms,

$$\begin{aligned} & \underbrace{U_x(\tilde{x}_2, G_2^*) - U_x(\tilde{x}_1, G_1^*)}_{<0} + (1-\alpha)\{\underbrace{U_G(\tilde{x}_1, G_1^*) - U_G(\tilde{x}_2, G_2^*}_{<0}\} \\ & + \alpha \sum_{j \in N} \underbrace{U_G(x_j, G_1^*) - U_G(x_j, G_2^*)}_{?} = 0 \end{aligned}$$

Given our assumptions on the increasingness and concavity of U and the cross-derivative $U_{xG} = U_{Gx} \geq 0$ (the assumption 1), the first two pair of terms of the above equation are clearly negative. For the first pair of terms, we use that

$$\tilde{x}_2 > \tilde{x}_1 \text{ and } G_1^* > G_2^*.$$

Since U_x is decreasing in x and increasing in G, we have $U_x(\tilde{x}_2, G_2^*) < U_x(\tilde{x}_1, G_1^*)$. The same type of argument establishes the inequality for the second pair of terms. The only term whose sign is difficult to determine is the third term of the above equation. We already know that $C_2 \subseteq C_1$, or $c_1 \geq c_2$. Any $j \in N$ is in one of three categories:

- j is a contributor under both $\tilde{x}_1$ and $\tilde{x}_2$; in this case the sign of the third pair of terms is negative exactly by the same arguments by which the second pair of terms is negative for that agent j;
- j is a noncontributor under both equilibria, in which case the third pair of terms is signed negative because $x_j = y_j$ under both thresholds and $G_1^* > G_2*$; or
- j is a noncontributor under x_2 and a contributor under x_1. In this case, we use that

$$U_G(x_j, \sum_{i \in C_1} (y_i - \tilde{x}_1)) - U_G(x_j, \sum_{i \in C_2} (y_i - \tilde{x}_2)) = U_G(\tilde{x}_1, \sum_{i \in C_1} (y_i - \tilde{x}_1)) - U_G(y_j, \sum_{i \in C_2} (y_i - \tilde{x}_2)) < 0,$$

using $\tilde{x}_1 < y_j$ and similar arguments as before.

We will then have that the left hand side is strictly negative and the right hand side is 0, giving us a contradiction. Hence, the threshold must be unique for any set of contributors C. ■

A.2. Proof of proposition 2

Proof. Proposition 1 tells us that there exists a threshold level of private good consumption that determines the set of contributors at an equilibrium. We name these thresholds with a slight abuse of notation $\tilde{x}_1$ and $\tilde{x}_2$ for the levels of altruism α_1, α_2, and the aggregate levels of the public good by G_1 and G_2 respectively. To show the result it suffices to show that $\tilde{x}_1 \geq \tilde{x}_2$.

Assume for contradiction that $\tilde{x}_1 < \tilde{x}_2$. Since there must be at least one contributor at any Nash equilibrium, and $\tilde{x}_1 < \tilde{x}_2$, the individual contributing in the game $\mathcal{G}(\mathbf{y}, \alpha_2)$ must also be contributing in the game $\mathcal{G}(\mathbf{y}, \alpha_1)$. Condition 4 must hold for the same contributor, giving us

$$-U_x(\tilde{x_1}, G_1) + U_G(\tilde{x_1}, G_1) + \alpha_1 \sum_{j \in N, j \neq 1} U_G(x_{j_1}^*, G_1)$$
$$= 0 = -U_x(\tilde{x_2}, G_2) + U_G(\tilde{x_2}, G_2) + \alpha_2 \sum_{j \in N, j \neq 2} U_G(x_{j_2}^*, G_2),$$

where $x_{j_i}^*$, $i = 1, 2$ is the equilibrium private contribution of individual j under the altruism level α_i. Or, alternatively,

$$U_x(\tilde{x_2}, G_2) - U_x(\tilde{x_1}, G_1) + \{U_G(\tilde{x_1}, G_1) - U_G(\tilde{x_2}, G_2)\} + \alpha_1 \sum_{j \in N} U_G(x_{j_1}^*, G_1) - \alpha_2 \sum_{j \in N} U_G(x_{j_2}^*, G_2) = 0. \tag{A1}$$

By our assumption for the proof $\tilde{x}_1 < \tilde{x}_2$, and this implies straightforwardly that that $G_1 > G_2$. It is clear also that for individuals j who are contributors under both the levels of altruism α_1 and α_2, $x^*_{j_1} = \tilde{x}_1 < \tilde{x}_2 = x^*_{j_2}$. For noncontributors under the altruism regime α_1 who also evidently stay noncontributors under α_2, $x^*_{j_1} = y_j = x^*_{j_2}$. This leaves only the subgroup of noncontributors under the altruism regime α_2 who may become contributors under the regime α_1. For any such person, we have that $x^*_{j_1} = \tilde{x}_1 < y_j = x^*_{j_2}$. Hence we can conclude that $x^*_{j_1} \leq x^*_{j_2}$ for all individuals $j \in N$.

Using the strict concavity of U in x and assumption 1,

$$U_x(\tilde{x_2}, G_2) - U_x(\tilde{x_1}, G_1) < 0;$$

and using the strict concavity of U in G and assumption 1,

$$U_G(\tilde{x_1}, G_1) - U_G(\tilde{x_2}, G_2) < 0;$$

and, finally, using the same reasoning, for all j

$$U_G(x^*_{j_1}, G_1) - U_G(x^*_{j_2}, G_2) < 0;$$

and using that $\alpha_1 \leq \alpha_2$, for all j

$$\alpha_1 U_G(x^*_{j_1}, G_1) - \alpha_2 U_G(x^*_{j_2}, G_2) < 0,$$

giving us

$$\alpha_1 \sum_{j \in N} U_G(x^*_{j_1}, G_1) - \alpha_2 \sum_{j \in N} U_G(x^*_{j_2}, G_2) < 0.$$

The left hand side of equation A1 is thus strictly less than zero, giving us a contradiction. Similar arguments hold when $\alpha_1 < \alpha_2$. Note that since $\tilde{x}_1 \geq \tilde{x}_2$, the set of contributors under α_1 must be at least as large as the set of contributors under α_2, and may be larger. This establishes the final part of proposition 2. ∎

A.3. Proof of proposition 3

Proof. Proposition 3 has two parts. Here I provide the proof of the first part of proposition 3 establishing under-provision of the public good under the Nash equilibrium. The strict concavity of the utility functions U ensures that there is a unique maximizer to expression 5.

According to proposition 1, at a strictly interior Nash solution, every contributor will have the same level of consumption of the private good, and let us denote this by $x^*_i = x^*$ for all i, and the level of the public good is denoted by G^* as before.

Suppose for contradiction that $G^* \geq \overline{G}$ when the Nash is strictly interior. It is then straightforward to show that $x^* \leq \overline{x}$. Since the three different formulations of the social planner's objective function yield the same solution, I use the social planner's objective function as per expression 5:

$$\sum_{i \in N} U(x_i, G).$$

We can use the function above to formulate the Lagrangian for the social planner's problem with λ as the Lagrangian multiplier:

$$\mathcal{L} = \sum_{i \in N} U(x_i, G) + \gamma(\sum_i y_i - G - \sum_i x_i).$$

Using the Lagrangian to solve for the maximum demonstrates quite easily that the social-planner solution would allocate the same, strictly positive level of the private good to all the members of the group, and we denote this level of private good consumption by $\overline{x}$. We also denote the level of public good allocated by the social planner by $\overline{G}$. The maximum of this function gives us that the social planner's solution is characterized by the familiar Samuelson condition,

$$\sum_{i \in N} \frac{U_G(\overline{x}, \overline{G})}{U_x(\overline{x}, \overline{G})} = 1, \tag{A2}$$

which can be rewritten as

$$\frac{U_G(\overline{x}, \overline{G})}{U_x(\overline{x}, \overline{G})} = \frac{1}{n}.$$

Using the Lagrangian to solve for the Nash solution gives us a similar condition:

$$\frac{U_G(x^*, G^*)}{U_x(x^*, G^*)} = \frac{1}{1 + \alpha(n-1)}. \tag{A3}$$

Note that if $\alpha = 1$, then the conditions in equations A2 and A3 are identical, and the Nash equilibrium attains Pareto-efficiency. $\alpha < 1$ implies that $\frac{1}{1+\alpha(n-1)} > \frac{1}{n}$ and we note

$$\frac{U_G(\overline{x}, \overline{G})}{U_x(\overline{x}, \overline{G})} - \frac{U_G(x^*, G^*)}{U_x(x^*, G^*)} < 0.$$

Given our assumptions, the marginal rate of substitution $\frac{U_G(x,G)}{U_x(x,G)}$ is decreasing in G and increasing in x, meaning that given $G^* \geq \overline{G}$ the expression on the left hand side must be non-negative, giving us a contradiction. ■

A.4. Proof of lemma 1

Proof. Part (i). First, we establish the existence of $\underline{x}$. Suppose an equilibrium with transfers exists. Then $t^*_{ij} > 0$ for some i and j and let k be any other player. Then we have from equations 13 and 12 that

$$\alpha U_x(x^*_j, G^*) = U_x(x^*_i, G^*) \geq \alpha U_x(x^*_k, G^*).$$

Since k was any player, we will have by the concavity of U that
$x^*_j \leq \min_k x^*_k \quad \forall k$. Denote $\min_k x^*_k = \underline{x}$. Therefore, if j is a receiver, $x^*_j = \underline{x}$. All receivers will have a final private good consumption equal to the minimum of all private good consumptions.

Establishing the existence of $\overline{x}$. Suppose there are two donors i and j who give to players k and l. Then, by equation 13 and proposition 1 (a),

$$U_x(x^*_i, G^*) = \alpha U_x(\underline{x}, G^*) = \alpha U_x(\underline{x}, G^*) = U_x(x^*_j, G^*).$$

Hence, $x^*_i = x^*_j = \overline{x}$.

It follows immediately from equation 13 that the equilibrium private good consumption of a donor must be higher than the equilibrium private good consumption of a receiver, hence $\underline{x} < \overline{x}$.

Part (ii). Suppose there is some player j such that he both gives and receives in equilibrium (i.e., $\exists\, i, k$ such that $t^*_{ij} > 0$ and $t^*_{jk} > 0$). Then equation 13 will hold for both players:

$$U_x(x^*_i, G^*) = \alpha U_x(x^*_j, G^*) < U_x(x^*_j, G^*) = \alpha U_x(x^*_k, G^*).$$

Violating equation 12 for the pair (i,k). This implies that the set of givers (donors) of transfers and receivers are disjoint in equilibrium.

Part (iii). In equilibrium, $\underline{x} \leq x_i^* \leq \overline{x} \, \forall \, i$.

We already know the result for the donors and the receivers. We only have to show it for a player k who is neither a donor nor a receiver in equilibrium. We know that the minimum consumption possible is that of a receiver, so someone who neither gives nor receives cannot have a level of private good consumption lower than $\underline{x}$. We need to now show that his equilibrium consumption cannot exceed that of a donor. It suffices to show that from equation 12 that for any donor i and any individual k,

$$U_x(\overline{x}, G^*) \geq \alpha U_x(x_k^*, G^*).$$

From the concavity of U in x, $x_k^* \leq \overline{x}$. ■

A.5. Proof of lemma 2

Proof. Suppose for contradiction that a receiver contributes to the public good (i.e., for some receiver j, $\exists \, i$ such that $t_{ij} > 0$ and $g_j > 0$). Then equation 13 for individuals i and j and equation 11 for individual j will be the following:

$$U_x(\overline{x}, G^*) = \alpha U_x(\underline{x}, G^*)$$

$$-U_x(\underline{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) = 0.$$

Rearranging the above equation,

$$U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) = U_x(\underline{x}, G^*).$$

Using that $U_x(\underline{x}, G^*) > \alpha U_x(\underline{x}, G^*)$, since $\alpha < 1$ and equation 13:

$$U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) = U_x(\underline{x}, G^*) > \alpha U_x(\underline{x}, G^*) = U_x(\overline{x}, G^*).$$

Or,

$$-U_x(\overline{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) > 0.$$

Rewriting,

$$-U_x(\overline{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) > 0.$$

We now add and subtract the terms $U_G(\overline{x}, G^*)$ and $\alpha U_G(\underline{x}, G^*)$:

$$\begin{aligned} &- U_x(\overline{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*) + U_G(\overline{x}, G^*) \\ &- U_G(\overline{x}, G^*) + \alpha U_G(\underline{x}, G^*) - \alpha U_G(\underline{x}, G^*) > 0. \end{aligned}$$

Rearranging terms,

$$\begin{aligned} &- U_x(\overline{x}, G^*) + U_G(\overline{x}, G^*) + \alpha \sum_{k \neq i} U_G(x_k^*, G^*) > U_G(\overline{x}, G^*) \\ &- U_G(\underline{x}, G^*) - \alpha U_G(\overline{x}, G^*) + \alpha U_G(\underline{x}, G^*), \end{aligned}$$

and then

$$- U_x(\overline{x}, G^*) + U_G(\overline{x}, G^*) + \alpha \sum_{k \neq i} U_G(x_k^*, G^*) > (1 - \alpha) \left(U_G(\overline{x}, G^*) \right.$$
$$\left. - U_G(\underline{x}, G^*) \right).$$

We have that the right hand side is positive by assumption 1, and hence the left hand side is positive as well, violating the first-order condition equation 10 for the donor i. Hence we cannot be at an equilibrium, giving us a contradiction. ■

A.6. Proof of lemma 3

Proof. We know by lemma 1 that an individual in an equilibrium with transfers has to be in one of the following three mutually exclusive cases: He will give transfers, receive transfers or do neither.

Lemma 3 therefore has three parts. We will first prove part (i): the case when $y_i < \underline{x}$. There are two possible cases, i either is, or is not, a contributor to the public good. In either case, $x_i^* \leq y_i - \sum_j t_{ij} + \sum_k t_{ki}$. If i is either a donor or in the case that he is neither a donor, nor a receiver, net transfers are negative or equal to zero. Hence, $x_i^* \leq y_i$. We must have that $x_i^* \leq \underline{x} < \overline{x}$. Then, by lemma 1 i cannot be a donor, nor an individual who neither gives nor receives. Hence, i has to be a receiver.

For part (ii), consider any individual with income y_i such that $\underline{x} \leq y_i \leq \overline{x}$. As before, there are two possibilities: i either contributes or does not contribute to the public good. In either case, as before, $x_i^* \leq y_i - \sum_j t_{ij} + \sum_k t_{ki}$. If i is a donor, net transfers are negative; hence $x_i^* < y_i < \overline{x}$. Therefore, i cannot be a donor, by lemma 1. Next, we show that i cannot be a receiver. Assume for contradiction that i is a receiver. Then $y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i$. There are two cases to consider here: In the first case, i does contribute to the public good, and, thus, $x_i^* = y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i \geq \underline{x}$. Then, i is not a receiver by lemma 1. The other possibility is that i is a contributor to the public good. In this case, it may so happen that $x_i^* \leq \underline{x} < y_i - \sum_j t_{ij} + \sum_k t_{ki}$, while $g_i^* = y_i - \sum_j t_{ij} + \sum_k t_{ki} - x_i^*$. However, if $x_i^* < \underline{x}$, then i will be a receiver who contributes to the public good, and a receiver cannot contribute to the public good by lemma 2, and we have a contradiction.

For the final part, consider an individual i such that $y_i > \overline{x}$. Suppose for contradiction, first, that such an individual is a receiver. Then, $y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i > \overline{x}$. There are two possibilities as before—i either contributes or does not contribute to the public good. If i does not contribute to the public good, then $y_i - \sum_j t_{ij} + \sum_k t_{ki} = x_i^* > \overline{x} > \underline{x}$, contradicting lemma 1. If i does contribute to the public good, it contradicts lemma 2. Hence, such an individual cannot be a receiver. Next, suppose again for contradiction that this individual neither receives nor gives transfers. Then $y_i - \sum_j t_{ij} + \sum_k t_{ki} = y_i$. As before, there are two possibilities: i contributes or does not contribute to the public good. If i does not contribute, $x_i^* = y_i > \overline{x}$, contradicting lemma 1. If i contributes, then x_i^* certainly cannot be less than $\underline{x}$ because of the arguments previously made. That leaves only two possibilities:

- i has a consumption x_i^* where $\underline{x} \leq x_i^* \leq \overline{x} < y_i$ with $g_i^* = y_i - x_i^*$.
- i has a consumption x_i^* where $\overline{x} < x_i^* < y_i$ with $g_i^* = y_i - x_i^*$.

The second case of the above two is immediately ruled out by lemma 1. In the first case, we show that $x_i^* < \overline{x}$ is impossible for the individual i. Since we are considering what happens in an equilibrium with transfers, there must exist at least one donor. By lemma 1 we know that his level of private consumption will equal $\overline{x}$.

We will thus have that for this donor j that

$$-U_x(\overline{x}, G^*) + U_G(\overline{x}, G^*) + \alpha \sum_{k \in N, j \neq j} U_G(x_k^*, G^*) \leq 0.$$

Whereas for individual i it is the case that

$$-U_x(x_i^*, G^*) + U_G(x_i^*, G^*) + \alpha \sum_{j \in N, j \neq i} U_G(x_j^*, G^*) = 0$$

for some $x_i^* \leq \overline{x}$.
We thus have that

$$- U_x(x_i^*, G^*) + U_G(x_i^*, G^*) + \alpha \sum_{j \in N, j \neq i} U_G(x_j^*, G^*) \geq -U_x(\overline{x}, G^*)$$
$$+ U_G(\overline{x}, G^*) + \alpha \sum_{k \in N, j \neq j} U_G(x_k^*, G^*).$$

Cancelling the common terms and rearranging,

$$U_x(\overline{x}, G^*) - U_x(x_i^*, G^*) + (1 - \alpha)(U_G(x_i^*, G^*) - U_G(\overline{x}, G^*)) \geq 0.$$

This gives us an immediate contradiction for any $x_i^* < \overline{x}$ by assumption 1 and the concavity of U. The above equation is only true if $x_i^* = \overline{x}$. When i is neither a donor nor a receiver, this is only possible when he is contributing to the public good to an amount $g_i^* = y_i - \overline{x}$.

Since we have eliminated all other possibilities, the only case that remains is that when $y_i > \overline{x}$, i is a donor. This completes the final part of lemma 3. ■

A.7. Proof of lemma 4

Proof. We prove this in three parts. First, we show that every person who contributes to the public good must have the same private good consumption. Denote this level of consumption by $\hat{x}$. Suppose for contradiction that there are two contributors to the public good with private consumption, x_i^* and x_j^*, $x_i^* \neq x_j^*$.

Without loss of generality assume $x_i^* < x_j^*$. Since both of them contribute to the public good, equation13 holds for both the agents:

$$- U_x(x_i^*, G^*) + U_G(x_i^*, G^*) + \alpha \sum_{k \neq i} U_G(x_k^*, G^*) = -U_x(x_j^*, G^*)$$
$$+ U_G(x_j^*, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*),$$

which can equivalently be written as

$$- U_x(x_i^*, G^*) + (1 - \alpha) U_G(x_i^*, G^*) + \alpha \sum_{k} U_G(x_k^*, G^*) = -U_x(x_j^*, G^*)$$
$$+ (1 - \alpha) U_x(x_j^*, G^*) + \alpha \sum_{k} U_G(x_k^*, G^*).$$

Cancelling the common terms and rearranging,

$$\underbrace{U_x(x_j^*, G^*) - U_x(x_i^*, G^*)}_{<0} + (1 - \alpha)\underbrace{\{U_G(x_i^*, G^*) - U_G(x_j^*, G^*)\}}_{\leq 0} = 0.$$

Which is a contradiction, given our assumptions on the class of functions $\mathcal{U}$.

For the next part of the lemma, we need to establish that if $y_i \leq \hat{x}$, then i cannot be a contributor to the public good. Suppose for contradiction that i still is a contributor to the public good. Thanks to lemma 1, there are three mutually exclusive possibilities in an equilibrium with transfers: an individual is either a donor, a receiver, or neither.

- If i is neither a donor nor a receiver, his income after transfers, $y_i - \sum_j t_{ij} + \sum_k t_{ki}$, is the same as his income before transfers, y_i, and hence $x_i^* < y_i = y_i - \sum_j t_{ij} + \sum_k t_{ki} \leq \hat{x}$, violating the first part of lemma 4, where we show that all contributors to the public good have the same equilibrium private good consumption, $x_i^* = \hat{x}$. Thus, an individual who neither gives nor receives transfers cannot contribute when $y_i \leq \hat{x}$.
- If i is a donor, then, $y_i - \sum_j t_{ij} + \sum_k t_{ki} < y_i \leq \hat{x}$. If i is a contributor to the public good, then i's equilibrium private good consumption x_i^* must be strictly lower than $y_i - \sum_j t_{ij} + \sum_k t_{ki}$ and, hence, lower than $\hat{x}$, violating the first part of lemma 4, where we showed above when all contributors have the same private good consumption.
- If i is a receiver, we know from lemma 2 that in this case, i cannot contribute to the public good.

For the final part of the lemma, we need to establish that if $y_i > \hat{x}$, then i is in one of the following mutually exclusive cases: Either i is a contributor to the public good, or i does not contribute to the public good, is a donor, and $x_i^* = \hat{x}$.

Suppose for contradiction that i is not a contributor to the public good despite his income $y_i > \hat{x}$. As before, there are three cases to consider:

- If i is a receiver who does not contribute to the public good, then i consumes his entire income after transfers: $\underline{x} = y_i - \sum_j t_{ij} + \sum_k t_{ki}$. Since $y_i - \sum_j t_{ij} + \sum_k t_{ki} > y_i > \hat{x}$ by the assumptions for this particular case, it must be that $\underline{x} > \hat{x}$.
  By the assumptions on the class of functions U, there must be at least one contributor to the public good, with a level of private consumption $\hat{x}$; denote such an individual as individual j. Since j contributes but i does not, it holds that
  $$-U_x(\underline{x}, G^*) + U_G(\underline{x}, G^*) + \alpha \sum_{k \neq i} U_G(x_k^*, G^*) < 0 = -U_x(\hat{x}, G^*) + U_G(\hat{x}, G^*) + \alpha \sum_{k \neq j} U_G(x_k^*, G^*).$$
  Rearranging and cancelling the common terms, as in previous proofs in this paper,
  $$U_x(\hat{x}, G^*) - U_x(\underline{x}, G^*) + (1-\alpha)(U_G(\underline{x}, G^*) - U_G(\hat{x}, G^*)) < 0.$$
  The expression on the left hand side is strictly positive when $\underline{x} > \hat{x}$, giving us a contradiction. Hence, i cannot be a receiver if $y_i > \hat{x}$.
- If i is neither a donor nor a receiver, his income after transfers $y_i - \sum_j t_{ij} + \sum_k t_{ki}$ is the same as his income before transfers: y_i. By arguments similar to the one made above for this case, this implies that $x_i^* = y_i = y_i - \sum_j t_{ij} + \sum_k t_{ki} > \hat{x}$, giving us a contradiction exactly in the same way as the case of receivers. Hence, i cannot be an individual who is neither a donor nor a receiver if $y_i > \hat{x}$.
- This leaves the last case where i is a donor and does not contribute. We next need to show that in this case, i's level of private consumption equals exactly $\hat{x}$. There are two possibilities for i's level of private consumption:

– $\hat{x} < x_i^*$, and this is the case where $\hat{x} < x_i^* = y_i - \sum_j t_{ij} + \sum_k t_{ki} < y_i$. In this case, if i does not contribute to the public good, then by exactly the argument that we made for the receiver and the individual who neither gives nor receives, we arrive at a contradiction.
– $x_i^* \leq \hat{x}$. This is the case where $y_i - \sum_j t_{ij} + \sum_k t_{ki} = x_i^* \leq \hat{x} < y_i$: Where perhaps, after making the transfers, the donor doesn't have a high enough level of income left to have the level of consumption enjoyed by contributors to the public good. It remains to be shown that $x_i^* < \hat{x}$ is impossible. There will always be at least one contributor to the public good, and since, by the arguments above, such a person cannot be a receiver or someone who neither gives nor receives, this person must be a donor. Since all donors enjoy the same level of private consumption $\overline{x}$, it must be the case that $\hat{x} = \overline{x}$, and the level of private consumption enjoyed by a donor who does not contribute is $\hat{x}$.

■

A.8. Proof of proposition 4

Proof. The first part and second part of proposition 4 are direct implications of combining results established in lemma 3 and lemma 4. For the final part, it must be established that $\overline{x} = \hat{x}$.

By lemma 4, we know that this is true when $y_i > \overline{x}$ and i does not contribute to the public good. By combining lemma 4 with lemma 3 we know that this also holds for the case when $y_i > \overline{x}$ and i is not a donor. We also know from lemma 3 that it is not possible that when $y_i > \overline{x}$, i neither contributes, nor donates. The only case left is when $y_i > \overline{x}$ and i both contributes and donates. In this case, if $\hat{x} \neq \overline{x}$, i has two different levels of private consumption $\overline{x}$ and $\hat{x}$ at the same equilibrium, which is a contradiction. ■

A.9. Proof of lemma 5

Proof. The first part of the proof of lemma 4 is that if everybody is contributing to the public good, then $x_i^* = \hat{x} \; \forall \, i$.

Then, for arbitrary players i and j:

$$U_x(x_i^*, G^*) = U_x(x_j^*, G^*).$$

Suppose now that, to the contrary, there exist individuals i and j such that $t_{ij} > 0$. Then, from equation 13 it will also hold that

$$U_x(x_i^*, G^*) = \alpha U_x(x_j^*, G^*).$$

Clearly, the above equations are contradictory when $\alpha < 1$. ■

A.10. Proof of proposition 5

Proof. We tackle the proof in three parts. We first show that when the model with transfers has the same equilibrium as the model without transfers, we have $\tilde{x} \leq \epsilon(y_{(1)})$, and this equilibrium is unique

Let us denote by $(x_1^o, \ldots .., x_n^o)$ the unique levels of private good consumption for the n members of the group at the unique equilibrium of the model without transfers, $\mathbf{g}^*(\mathbf{y}, \alpha)$. For this to also be an equilibrium of the model with transfers, a necessary and sufficient condition is that equation 18 must hold for every i with $x_i = x_i^o$. Specifically, for any i and j,

$$x_i^o \leq \epsilon(x_j^o). \tag{A4}$$

Since ϵ is a strictly increasing function of x, for it to hold true for all i and j it must hold true for the largest possible x_i^o and the least possible x_j^o. The least possible private good consumption is $x_j^o = y_{(1)}$ (i.e., the income of the poorest member if he does not contribute). Since the game $\mathcal{G}(\mathbf{y}, \alpha)$ always has an equilibrium with at least one contributor, the maximum possible consumption is $x_i^o = \tilde{x}$ (since contributions are arranged in increasing order of income, by proposition 1, and the consumption of a contributor is the highest possible level of private good consumption). This case includes the case of an interior equilibrium where everybody contributes to the public good (including the poorest individual) and there are no transfers (as shown in lemma 5): $\tilde{x} < y_{(1)} < \epsilon(y_{(1)})$. The uniqueness of the equilibrium without transfers is guaranteed because the game $\mathcal{G}(\mathbf{y}, \alpha)$ has a unique equilibrium. We can further deduce from equation A4 being a necessary and sufficient condition that an equilibrium with transfers and an equilibrium without transfers are mutually exclusive.

We next show that the model with transfers has an equilibrium with transfers iff $\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \geq 0$, and in this case $\underline{x}$, $\overline{x}$ and G^* are determined uniquely. Given the concavity of U^1, $\overline{x}$ is a strictly increasing function of $\underline{x}$. Note also that, due to the concavity of the functions U^1 and U^2, G is a strictly increasing function of $\overline{x}$. This is not specific to the model with transfers; this property would hold even for the model without transfers. Since G is a strictly increasing function of $\overline{x}$ and $\overline{x}$ is a strictly increasing function of $\underline{x}$, $G = \gamma(\underline{x})$ is strictly increasing in $\underline{x}$. Using the function x^+, we rewrite the constraint that the sum of all transfers given, net of the sum of transfers received and contributions to the public good, must be zero:

$$\sum_i (y_i - \epsilon(\underline{x}))^+ - \sum_i (\underline{x} - y_i)^+ - \gamma(\underline{x}) = 0. \tag{A5}$$

Equation A5 is entirely defined in terms of $\underline{x}$. Define the function

$$F(\underline{x}) = \sum_i (y_i - \epsilon(\underline{x}))^+ - \sum_i (\underline{x} - y_i)^+ - \gamma(\underline{x}).$$

Finding an equilibrium for the model with transfers thus corresponds to finding a root for the equation $F(\underline{x}) = 0$. The domain of $F(\underline{x})$ is $[y_{(1)}, y_{(n)}]$ (the minimum, and the maximum incomes for the group members). Given our assumptions, $F(\underline{x})$ is a strictly decreasing function of $\underline{x}$ if $\underline{x} > y_i$ for at least one i. For a unique solution to exist to $F(\underline{x}) = 0$, we should have that $F(x_1) > 0$ and $F(x_2) < 0$ for some x_1 and x_2 in the domain of $F(\underline{x})$ with $x_1 < x_2$. Consider any $x_2 > \max{}_i y_i$. It is easy to show that $F(x_2)$ is strictly negative given our assumptions. Thus, $F(x_2) < 0$. The lower bound for $\underline{x}$ (or the minimum possible value) is $y_{(1)}$, the minimum income for the group. At $\underline{x} = y_{(1)}$, $\sum_i (y_{(1)} - y_i)^+ = 0$. However, $\sum_i (y_i - \epsilon(y_{(1)}))^+ \geq 0$, and $-\gamma(y_{(1)}) < 0$. This difference is positive iff

$$\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \geq 0.$$

Otherwise, this difference is strictly negative, and the function $F(\underline{x})$ lies everywhere below zero under the domain of F and there is no solution to the equation $F(\underline{x}) = 0$.

Hence, if $\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \geq 0$, the equation $F(\underline{x}) = 0$ has a unique solution. The thresholds $\underline{x}$, $\overline{x}$ and the amount of public good G are uniquely determined, even though individual levels of contributions and transfers may not be unique. If $\sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) < 0$, $F(\underline{x}) = 0$ has no solution, and $\underline{x}$ is undetermined.

In the final step of the proof we show that $\tilde{x} \leq \epsilon(y_{(1)}) \iff \sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}) \leq 0$.

$$\begin{aligned} &\tilde{x} \leq \epsilon(y_{(1)}) \\ &\iff \sum_i (y_i - \tilde{x})^+ \geq \sum_i (y_i - \epsilon(y_{(1)}))^+. \end{aligned}$$

Since λ and ϵ are both increasing functions,

$$\iff \sum_i (y_i - \tilde{x})^+ - \lambda(\tilde{x}) \geq \sum_i (y_i - \epsilon(y_{(1)}))^+ - \lambda(\epsilon(y_{(1)})).$$

We know from proposition A4 that if $\tilde{x} \leq \epsilon(y_{(1)})$, then there are no transfers. Thus, $\sum_i (y_i - \tilde{x})^+ = G = \lambda(\tilde{x})$ and $\sum_i (y_i - \tilde{x})^+ - \lambda(\tilde{x}) = 0$. Then,

$$0 \geq \sum_i (y_i - \epsilon(y_{(1)}))^+ - \lambda(\epsilon(y_{(1)})) = \sum_i (y_i - \epsilon(y_{(1)}))^+ - \gamma(y_{(1)}).$$

And that completes the proof. ■

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